5. Unions of Conjunctive Queries and Queries with Disequalities

1. Conjunctive Queries with Disequalities

Disequalities are atoms of the form $s \neq t$.

1. Generalize the concept of a query homomorphism to conjunctive queries with disequalities.

2. Show that for queries with disequalities existence of such a homomorphism is a sufficient condition for containment.

3. Is it also a necessary condition?

2. Containment of Unions of Conjunctive Queries

A query Q is a *union of conjunctive queries* if there are conjunctive queries $Q_1, \ldots, Q_n$ such that

$$Q(\mathbf{I}) := \bigcup_{i=1}^{n} Q_i(\mathbf{I})$$

for every instance $\mathbf{I}$. We also write in such a case $Q = \bigcup_{i=1}^{n} Q_i$.

Note that all queries Q_i must have the same arity so that this definition makes sense. Thus, in general we can assume that each Q_i is defined by a rule of the form

$$Q_i(\bar{x}) :\!\!- L_i, M_i,$$

where all the queries Q_i have the same vector of distinguished variables.

We say that Q is a *union of relational conjunctive queries* if $M_i = \emptyset$ for every $i = 1, \ldots, n$.

How can we check the containment of unions of conjunctive queries?

1. Give characterizing conditions for the containment of conjunctive queries.
   Hint: Distinguish between the relational and the general case.

2. Use these conditions to find out which containment and equivalence relationships hold between $Q_1 \cup Q_3$ and $Q_2 \cup Q_4$ for Q_1, Q_2, Q_3, Q_4 defined on Slide 45 of part 1 of the lecture.

3. What is the complexity of the problem in the relational case and what is it in the general case?