

### 3. Containment of Conjunctive Queries

**Instructions:** Work in groups of 2 students. You can write up your answers by hand (provided your handwriting is legible) or use a word processing system like Latex or Word. However, experience shows that Word is in general difficult to use for this kind of task. Please, include name and email address in your submission.

#### 1. Evaluation of Conjunctive Queries with Unary Relation Symbols

Recall that relational conjunctive queries have only relational atoms in their body, and no equalities or inequalities. We know that the combined complexity of evaluating relational conjunctive queries is NP-complete. However, the reduction used queries with binary relation symbols.

What can you say about the difficulty of evaluating relational conjunctive queries that have only unary relations in their body (that is, relations of arity 1)? Is this an NP-hard, a coNP-hard, or a polynomial time problem?

Distinguish between the problem for

1. relational queries (that is, queries without built-in predicates),
2. general conjunctive queries, which may contain built-in predicates.

To prove NP-hardness or coNP-hardness of a problem, provide a reduction from a known NP-hard or coNP-hard problem to the new one. To prove that it is in polynomial time, give an algorithm, show that it solves the problem, and explain why it runs in polynomial time.

**Hint:** Recall that 3-UNSAT, the problem to decide whether a set of 3-clauses is unsatisfiable, is complete for the class coNP.

(15 Points)

## 2. Conjunctive Queries without Self Joins

A conjunctive query has a *self join* if its body contains two relational atoms with the same relation symbol. Thus, in the body of a query without self join, any two relational atoms have distinct relation symbols.

We know that containment is NP-complete for arbitrary relational conjunctive queries and that containment is  $\Pi_2^P$ -complete for conjunctive queries with comparisons.

**Question:** How difficult is it to decide containment of conjunctive queries (possibly with comparisons) that have no self join? Can this problem be solved in polynomial time? Or is it NP-complete? Or even  $\Pi_2^P$ -complete? (Consider only the case that the comparisons range over the rational numbers.)

**Hint 1:** First, find a characterizing property of containment for this class of queries. Then assess the difficulty of checking this property.

**Hint 2:** Consider first the case of queries without comparisons, and then study queries with comparisons. For queries with comparisons you can assume that queries and comparison sets are *reduced* in the following sense: a set of comparisons  $M$  is *reduced*, if for all terms  $s, t$  it holds that  $M \models s = t$  only if  $s$  and  $t$  are syntactically equal; a conjunctive query is *reduced* if its comparisons are reduced. Note that every satisfiable query can be equivalently rewritten as a reduced query in polynomial time.

(15 Points)

Submission: At the first lecture after 30 April 2012 (FUB is closed on 30 April).