

## 2. Properties of Conjunctive Queries

### 1. Satisfiability of Conjunctive Queries with Equalities and Comparisons

We consider conjunctive queries that in their body may contain comparisons, that is, atoms with the predicates “=”, “≠”, “≤”, and “<” and we are interested in finding out how difficult it is to decide whether a conjunctive query with comparisons is satisfiable.

(Of course, for queries with the predicates “≤” and “<” we have to specify which is the ordered domain over which those comparisons range. We will distinguish the two cases that they range over the rational numbers or over the integers.)

For each of the following classes of queries, describe a method by which one can check satisfiability: Conjunctive queries with

1. “=”
2. “=” and “≠”
3. “≤”, ranging over the rational numbers
4. “≤”, ranging over the integers
5. “≤” and “≠”, ranging over the rational numbers
6. “≤” and “<”, ranging over the rational numbers
7. “≤” and “<”, ranging over the integers.

### 2. Reducing the Hamiltonian Path Problem to Conjunctive Query Evaluation

Let  $G = (V, E)$  be an undirected graph, where  $V$  is a finite set, the elements of which are called *vertices*, and  $E \subseteq \mathcal{P}_2(V)$  is a collection of two-element subsets of  $V$ , the elements of which called the *edges* of  $G$ .

A path in  $G$  is a sequence  $v_1, \dots, v_n$  of vertices such that  $\{v_i, v_{i+1}\} \in E$  for  $i = 1, \dots, n - 1$  (that is, each node is connected to the next by an edge). A path  $v_1, \dots, v_n$  is *Hamiltonian* if in addition we have

1.  $v_i \neq v_j$  if  $i \neq j$  (that is, all vertices on the path are distinct)
2.  $\{v_1, \dots, v_n\} = V$  (that is,  $v_1, \dots, v_n$  enumerates all vertices of  $G$ ).

The Hamiltonian Path Problem is defined as follows:

**Given:** An undirected graph  $G = (V, E)$ .

**Question:** Does there exist a Hamiltonian Path in  $G$ ?

This problem is known to be NP-complete.

Show that the combined complexity of conjunctive query evaluation is NP-hard by reducing the Hamiltonian Path Problem to the evaluation problem.

Is it also possible to show NP-hardness for conjunctive queries without built-in predicates?