Introduction to Database Systems

The Relational Data Model

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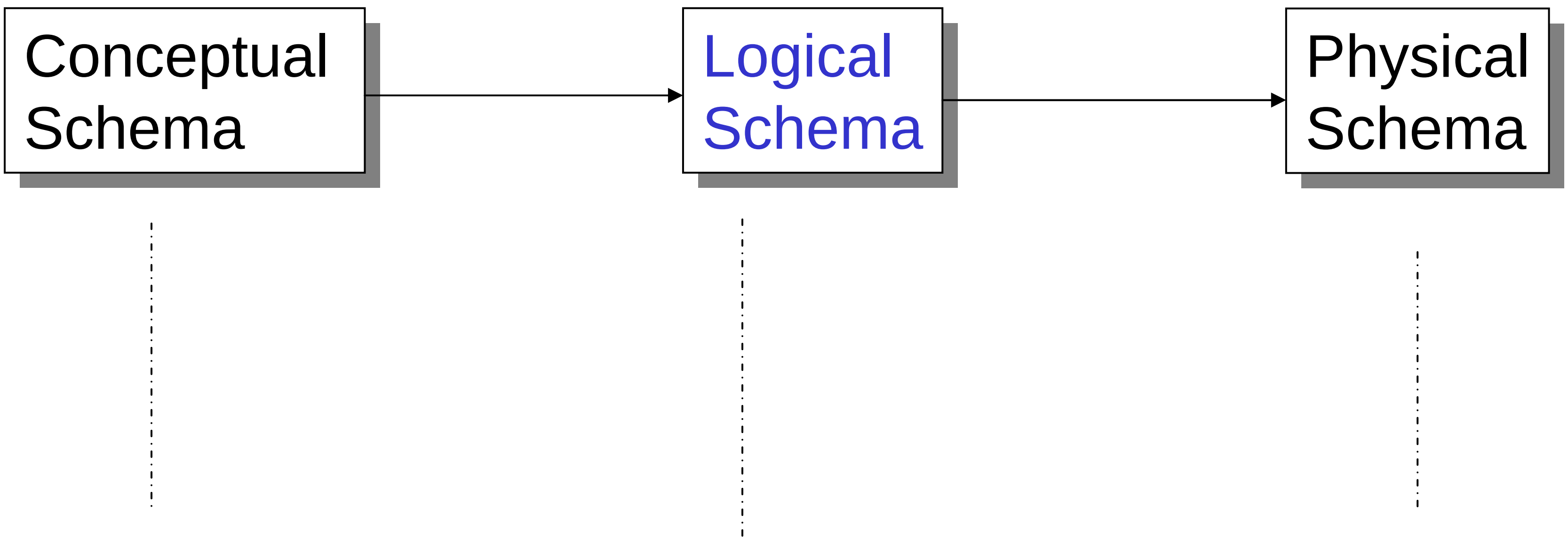
4. The Relational Data Model

4.1 Schemas

Different Schemas are Based on Different Concepts



ER:
- Entities,
- Relationships,
- Attributes

Tables/Relations:
- column names/attributes
- rows/tuples

File organisation:
- File types
- Index structures

A Table

Table name

Column names

Product:

Name	Price	Category	Manufacturer
gizmo	$19.99	gadgets	GizmoWorks
Power gizmo	$29.99	gadgets	GizmoWorks
SingleTouch	$149.99	photography	Canon
MultiTouch	$203.99	household	Hitachi

Rows

Review of Mathematical Concepts (1)

- Sets: $S, T, S_1, \ldots, S_n, T_1, \ldots, T_n, \{\ \}$
  Cardinality of a set S denoted as $|S|$

- Cartesian Product of sets (also *cross product*):
  $S \times T$ set of all pairs (s,t) where $s \in S$ and $t \in T$
  $S_1 \times \ldots \times S_n$ set of all n-tuples $(s_1,\ldots, s_n)$
  where $s_i \in S_i$

- Relation R over S, T:
  subset of $S \times T$, written $R \subseteq S \times T$
  We write $(s,t) \in R$ or, equivalently, sRt

Review of Mathematical Concepts (2)

- Relation R over $S_1, ..., S_n$:

  subset $R \subseteq S_1 \times ... \times S_n$

  The number n is the arity of R
  (R is *binary* if n=2 and *ternary* if n=3)

- Function f from S to T, written $f: S \to T$

  associates to every element $s \in S$
  exactly one element of T, denoted as $f(s)$

Review of Mathematical Concepts (3)

- Partial function f from S to T, written $f: S \sim\!\rightarrow T$

  associates to *some* element $s \in S$
  exactly one element of T, still denoted as $f(s)$

  We write $f(s) = \perp$ (read "bottom")
  if f does not associate any element of T to s

- A relation R over $S_1,\ldots,S_n$ is total on S_i
  if for every $s_i \in S_i$
  there are $s_j \in S_j$, $j \neq i$, such that $(s_1,\ldots, s_n) \in R$

  In other words:
  every element of S_i occurs in some tuple of R

Review of Mathematical Concepts (4)

- A relation R over S and T is functional in its first argument if

  sRt_1 and sRt_2 implies that $t_1 = t_2$

  for all $s \in S$, $t_1, t_2 \in T$.

  In other words, for every $s \in S$,
  there is at most one $t \in T$ related by R to s

- Analogously, a relation R over $S_1, ..., S_n$ can be functional
  - in an argument i, or
  - in a tuple of arguments, say (i,j,k)

How Many ... ?

Consider sets

S, T with $|S| = N$ and $|T| = M$

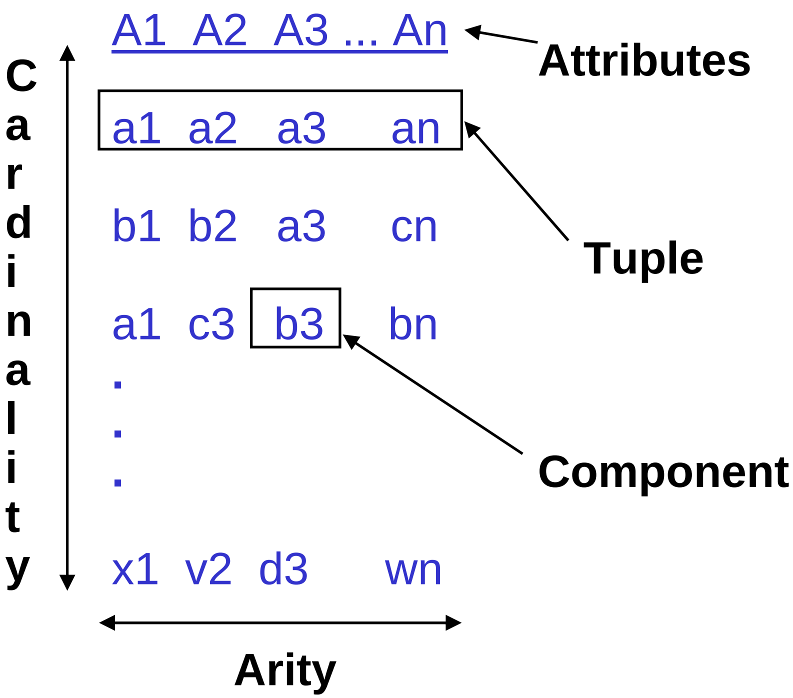
$S_1, \ldots, S_n$ with $|S_i| = N_i$

- How many elements can a relation over $S_1, \ldots, S_n$ have? At least? At most?
- How many relations over S, T are there? How many over $S_1, \ldots, S_n$?
- How many functions from S to T are there?
- How many partial functions from S to T are there?

How Many … ? (Cntd.)

- How many relations are there over S and T that are functional in the first argument?
- How many relations are there over S and T that are total on S?

Tables Look Like Relations …



{(a1, a2, a3, …, an),
(b1, b2, a3, …, cn),
(a1, c3, b3, …, bn),
. . .
(x1, v2, d3, …, wn)}

Over which sets does this relation range?

In Databases: Distinguish between

- *Schema* (structure) and
- *Instance* (content)

Relation Schemas

A **relation schema**

$$R(A_1:D_1, ..., A_n:D_n)$$

consists of

- a *name*, say R
- a nonemtpy set of *attributes*, say $A_1,..., A_n$
- a *type* or *domain,* say $D_i = dom(A_i)$, for each attribute A_i

Example: Product (Prodname: Name,
Price: DollarPrice,
Category: Name,
Manufacturer: Name)

Types and Domains

Type: Class of atomic values, e.g.,

- integers, reals, strings
- integers between 15 and 80,
  strings of (up to) 50 characters

Domain: Set of atomic values, that have a specific meaning in an application, e.g.,

  – Name, EmployeeAge

- Domains have a type, e.g.,
  – EmployeeAge = Int[15,80]
- Domains may have default values

Domains allow for an additional layer of abstraction

4. The Relational Data Model

4.2 Instances

1. Schemas
2. **Instances**
3. Integrity Constraints

Relation Schema and Instance

- A tuple of values $(v_1, \ldots, v_n)$ **satisfies** the relation schema $R(A_1:D_1, \ldots, A_n:D_n)$ if $v_i \in D_i$ $(i=1,\ldots n)$

- An **instance** of R is a set of tuples that satisfy the schema of R
  (i.e., a relation over $D_1, \ldots, D_n$)

- ***Analogy*** with programming languages:
  - schema = type
  - instance = value

Example

Domain declaration:

Name=String(30), DollarPrice=Decimal (10,2),

Relation schema:

Product(Prodname: Name, Price: DollarPrice,
Category: Name, Manufacturer: Name)

Instance:

Prodname	Price	Category	Manufacturer
gizmo	19.99	gadgets	GizmoWorks
Power gizmo	29.99	gadgets	GizmoWorks
SingleTouch	149.99	photography	Canon
MultiTouch	203.99	household	Hitachi

Database Schema and Instance

Database Schema

Set of relation schemas, e.g.,

Product (Productname, Price, Category, Manufacturer),

Vendor (Vendorname, Address, Phone), ...

To keep things simple, we have dropped types/domains

Database Instance

Set of relation instances,
one for each relation in the schema

Important *distinction*:

– Database Schema = stable over long periods of time

– Database Instance = changes constantly

Updates

A database reflects the state of an aspect of the real world:
The world changes ➔ the database has to change

Updates to an instance:

1) adding a tuple
2) deleting a tuple
3) modifying an attribute of a tuple

What could be updates to a schema?

- Updates to the data happen very frequently.
- Updates to the schema: relatively rare, rather painful.

*Why**?***

Null Values

Attribute values

- are atomic
- have a known domain
- can sometimes be "*null*"

Three meanings of null values

1. not applicable
2. not known
3. absent (not recorded)

Student

studno	name	hons	tutor	year	thesis title
s1	jones	ca	bush	2	*null*
s2	brown	cis	kahn	2	*null*
s3	smith	*null*	goble	2	*null*
s4	bloggs	ca	goble	1	*null*
s5	jones	cs	zobel	1	*null*
s6	peters	ca	kahn	3	*"A CS Survey"*

Order and Duplication

In tables:

- *Order of attributes* is fixed
- *Order of rows* is fixed (i.e., tables with different order of rows are different)
- *Duplicate rows* matter

In mathematical relations:

- Order of tuples and duplicate tuples do not matter
- Order of attributes is still fixed

Question:

Can we model relations so that we get rid of attribute order?

Reminder: Relations as Subsets of Cartesian Products

- Tuple as elements of String x Int x String x String

  E.g., t = (gizmo, 19.99, gadgets, GizmoWorks)

- Relation = subset of String x Int x String x String

- Order in the tuple is important !
  - (gizmo, 19.99, gadgets, GizmoWorks)
  - (gizmo, 19.99 , GizmoWorks, gadgets)

- No explicit attributes, hidden behind positions

Alternative Definition: Relations as Sets of Functions

- Fix the set A of attributes, e.g.

  A = {Name, Price, Category, Manufacturer}

- Fix D as the union of the attribute domains, e.g.,

  D = dom(Name) $\cup$ dom(Price) $\cup$ dom(Category) $\cup$ dom(Manufacturer)

- A tuple is a function $t: A \rightarrow D$

- E.g.

  {Prodname $\rightarrow$ gizmo,
  Price $\rightarrow$ 19.99,
  Category $\rightarrow$ gadgets,
  Manufacturer $\rightarrow$ GizmoWorks}

This is the model underlying SQL

- Order in a tuple is not important, attribute names are important!

Notation

Schema $R(A_1, ..., A_n)$, tuple t that satisfies the schema

Then:

- $t[A_i]$ = value of t for attribute A_i
- $t[A_i, A_j, A_k]$
  = subtuple of t, with values for A_i, A_j, A_k

Example: t = (gizmo, 19.99, gadgets, GizmoWorks)

- t[Price] = 19.99
- t[ProdName, Manufacturer] = (gizmo, GizmoWorks)

Two Definitions of Relations

- Positional tuples, without attribute names
- Tuples as mappings/functions of attributes

In theory and practice, both are used, e.g.,

- SQL: tuples as functions
- QBE (query by example): positional tuples

We will switch back and forth between the two…

Why Relations?

- Very simple model
- *Often* a good match for the way we think about our data
- Foundations in logic and set theory
- Abstract model that underlies SQL, the most important language in DBMSs today
  - But SQL uses “bags” while the abstract relational model is set-oriented

4. The Relational Data Model

4.3 Integrity Constraints

1. Schemas
2. Instances
3. **Integrity Constraints**

Integrity Constraints

Ideal: DB instance reflects the real world

In real life: This is not always the case

Goal: Find out, when DB is out of sync

Observation:

Not all mathematically possible instances make sense

Idea:

- Formulate conditions that hold for all plausible instances
- Check whether the condition holds after an update

Such conditions are called integrity constraints!

Common Types of Integrity Constraints

- Functional Dependencies (FDs)
  - "Employees in the same department have the same boss"
- Superkeys and keys (special case of FDs)
  - "Employees with the same tax code are identical"
- Referential Integrity (also "foreign key constraints")
  - "Employees can only belong to a department that is mentioned in the Department relation"
- Domain Constraints
  - "No employee is younger than 15 or older than 80"

Integrity constraints (ICs) are part of the schema

We allow only instances that satisfy the ICs

Functional Dependencies (Example)

Emp (Name, taxCode, Dept, DeptHead)

A state of Emp that contains two tuples with

- the same Dept, but different DeptHead
- the same taxCode, but different Name, Dept, or DeptHead

is definitely out of sync.

We write the desired conditions symbolically as

- Dept $\rightarrow$ DeptHead
- taxCode $\rightarrow$ Name, Dept, DeptHead.

We read:

- "Dept functionally determines DeptHead", or
- "Name, Dept, and DeptHead functionally depend on taxCode"

Functional Dependencies

DB relation R.

A functional dependency on R is an expression

$$A1,...,Am \rightarrow B1, ...,Bn$$

where $A_1,...,A_m$ and $B_1, ...,B_n$ are attributes of R.

An instance r of R satisfies the FD if for all tuples t1, t2 in R

t1[A1,...,Am] = t2[A1,...,Am]
implies
t1[B1, ...,Bn] = t2[B1, ...,Bn]

How many FDs are there on a given relation?

FDs: Example

Emp (EmpID, Name, Phone, Position)

with instance

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E1847	Jones	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Brown	1234	Lawyer

Which FDs does this instance satisfy?

- EmpID $\rightarrow$ Name, Phone, Position
- Position $\rightarrow$ Phone
- Phone $\rightarrow$ Position

General Approach for Checking FDs

To check $A \rightarrow B$ on an instance,

- erase all other columns

…	A	…	B	
	X1		Y1	
	X2		Y2	
	…		…	

- check if the remaining relation is functional in A

Why is that correct?

FDs: Example (Cntd.)

Check Position $\rightarrow$ Phone !

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E1847	Jones	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Brown	1234	Lawyer

Is the white relation functional in Position?

FDs, Superkeys and Keys

Person (SSN, Name, DOB)
SSN $\rightarrow$ Name, DOB

Product (Name, Price, Manufacturer)
Name $\rightarrow$ Price, Manufacturer
Name $\rightarrow$ Name, Price, Manufacturer
Name, Price $\rightarrow$ Name, Price, Manufacturer

Book (Author, Title, Edition, Price)
Author, Title, Edition $\rightarrow$ Price

- A set of attributes of a relation is a superkey if it functionally determines all the attributes of the relation
- A superkey is a candidate key if none of its subsets is a superkey

Candidate keys are minimal superkeys

Keys: Definitions

- Superkey
  - a *set* of attributes whose values together *uniquely* identify a tuple in a relation
- Candidate Key
  - a superkey for which no proper subset is a superkey: a superkey that is *minimal*
  - *Can be more than one for a relation*
- Primary Key
  - a candidate key chosen to be the main key
  - *One for each relation,* indicated by underlining the key attributes

Student(<u>studno</u>,name,tutor,year)

Example: Multiple Keys

Student (Lastname, Firstname, MatriculationNo, Major)

Candidate key (2 attributes) — {Lastname, Firstname}

Candidate key — {MatriculationNo}

Superkey — {MatriculationNo, Major}

Note: There are alternate candidate keys

- Candidate keys are

  {Lastname, Firstname} and

  {StudentID}

Foreign Keys

A set of attributes in a relation that exactly matches the primary key in another relation

– the names of the attributes don't have to be the same but must be of the same domain

Student (studno, name, hons, tutor, year)

Staff (lecturer, roomno, appraiser)

Notation:

FK1: Student (tutor) references Staff (lecturer)

FK2: Staff (appraiser) references Staff (lecturer)

Satisfaction of Foreign Key Constraints

"FK: R(A) references S(B)"

is satisfied by an instance of R and S if
for every t1 in R there is a t2 in S such that
t1[A] = t2[B],
provided t1[A] is not null

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Foreign key constraints are also called "referential integrity constraints."

Updates May Violate Constraints …

Updates are

Insertions, Deletions, Modifications

of tuples

Example DB with tables as before:

Student (studno, name, hons, tutor, year)

Staff (lecturer, roomno, appraiser)

The DB has key and foreign key constraints

Questions:

- *What can go wrong?*
- *How should the DBMS react?*

Insertions (1)

If the following tuple is inserted into Student, what should happen? Why?

(s1, jones, cis, capon, 3)

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Insertions (2)

If the following tuple is inserted into Student, what should happen? Why?

(null, jones, cis, capon, 3)

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Insertions (3)

If the following tuple is inserted into Student, what should happen? Why?

(s7, jones, cis, null, 3)

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Insertions (4)

If the following tuple is inserted into Student, what should happen? Why?

(s7, jones, cis, calvanese, 3)

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Deletions (1)

If the following tuple is deleted from Student,
is there a problem? And what should happen?

(s2, brown, cis, kahn, 2)

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Deletions (2)

And if this one is deleted from Staff ?

(kahn, IT206, watson)

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Modifications (1)

What if we change in Student

(s1, jones, ca, bush, 2)

to

(s1, jones, ca, watson, 2) ?

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Modifications (2)

And what if we change in Student

(s2, brown, cis, kahn, 2)

to

(s1, jones, ca, bloggs, 2) ?

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Modifications (3)

And what if we change in Staff

(lindsey, 2.10, woods)

to

(lindsay, 2.10, woods) ?

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

Staff

lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
goble	2.82	capon
zobel	2.34	watson
watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Modifications (4)

Now, let's change in Staff

(goble, 2.82, capon)

to

(gobel, 2.82, capon) …

Student

studno	name	hons	tutor	year
s1	jones	ca	bush	2
s2	brown	cis	kahn	2
s3	smith	cs	goble	2
s4	bloggs	ca	goble	1
s5	jones	cs	zobel	1
s6	peters	ca	kahn	3

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lecturer	roomno	appraiser
kahn	IT206	watson
bush	2.26	capon
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watson	IT212	barringer
woods	IT204	barringer
capon	A14	watson
lindsey	2.10	woods
barringer	2.125	*null*

Summary: Reactions to Integrity Violations

If an update violates an IC, the DBMS can

- Reject the update
- Repair the violation by
  - *inserting* null
  - *inserting* a default value
  - *cascading* a deletion
  - *cascading* a modification

Summary

- The relational model is based on concepts from set theory (and logic)
- It formalises the concept of a *table*
- Distinguish:
  - relation schema: relation name, attributes, domains/types
  - relation instance: relation over domains of attributes
- Two formalisations of tuples
  - positional tuples vs. tuples as functions on attributes
- Integrity constraints: Domain cs, FDs, Keys, FKs
- Updates may violate ICs

  — and the DMBS has to react