# *Introduction to Database Systems*

## Functional Dependencies

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# Functional Dependencies

## 1. Meaning of FDs

1. **Meaning of FDs**
2. Keys and Superkeys
3. Inferring FDs

# Functional Dependencies

- A FD is written

$$X \to A \quad \text{or} \quad X \to Y$$

- Notation: $X$, $Y$, $Z$ represent sets of attributes; $A$, $B$, $C$,… represent single attributes
- $X \to A$ ("X determines A") is an assertion about a relation $R$: whenever two tuples of $R$ agree on all the attributes of $X$, then they must also agree on the attribute $A$, or

t1[X] = t2[X] implies t1[A] = t2[A] for all t1, t2 in R

(analogously for $X \to Y$)

- Convention:We say "$X \to A$ holds in $R$"
- Notation: No set braces in sets of attributes: just $ABC$, rather than {$A$,$B$,$C$ }.

# Example

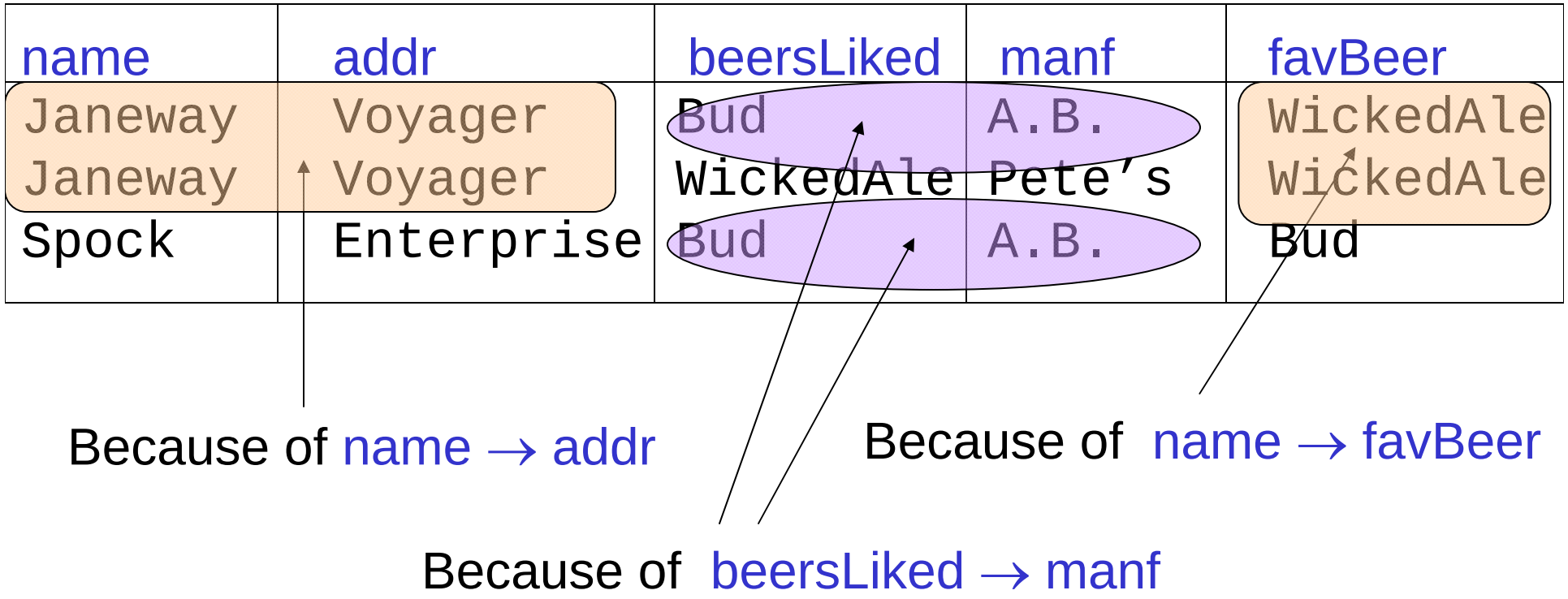
Table Drinkers(name, addr, beersLiked, manf, favBeer)

| name | addr | beersLiked | manf | favBeer |
|---|---|---|---|---|
| Janeway | Voyager | Bud | A.B. | WickedAle |
| Janeway | Voyager | WickedAle | Pete's | WickedAle |
| Spock | Enterprise | Bud | A.B. | Bud |

Reasonable FD's to assert:

1. name → addr
2. name → favBeer
3. beersLiked → manf

# Example FDs


name
addr
beersLiked
manf
favBeer
Janeway
Voyager
Bud
A.B.
WickedAle
Janeway
Voyager
WickedAle
Pete's
WickedAle
Spock
Enterprise
Bud
A.B.
Bud
Because of name → addr
Because of name → favBeer
Because of beersLiked → manf

# FD's With Multiple Attributes

FDs with more than one attribute on the right don't increase expressivity …

… but allow for convenient shorthands that combine FDs

**Example:** name $\rightarrow$ addr and name $\rightarrow$ favBeer
become name $\rightarrow$ addr favBeer

More than one attribute on the left may be essential.

**Example:** bar beer $\rightarrow$ price

# Functional Dependencies

## 2. Keys and Superkeys

1. Meaning of FDs
2. **Keys and Superkeys**
3. Inferring FDs

# Keys of Relations

*R* relation, *K* a set of attributes of *R*

- *K* is a *superkey* for relation *R* if
  *K* functionally determines all of *R*

- *K* is a *key* for *R* if *K* is a superkey,
  but no proper subset of *K* is a superkey
  (that is, *K* is a *minimal superkey*)

*Sometimes we call "keys" also "candidate keys",*
*to indicate they are candidates for choosing the primary key*

# Example

Drinkers(name, addr, beersLiked, manf, favBeer)

We have

name $\rightarrow$ addr favBeer

beersLiked $\rightarrow$ manf

Therefore, {name, beersLiked} determine all the other attributes

Hence, {name, beersLiked} is a superkey

# Example (cntd.)

Neither {name} nor {beersLiked} is a superkey:

- name $\rightarrow$ manf doesn't hold
- beersLiked $\rightarrow$ addr doesn't hold

Hence: {name, beersLiked} is a key

There are no other keys, but lots of superkeys:

Any superset of {name, beersLiked} is a superkey

# ER and Relational Keys

- Keys in ER concern entities
- Keys in relations concern tuples
- Usually, one tuple corresponds to one entity,
  so the ideas are similar
- But — in poor relational designs,
  one tuple may represent several entities,
  … so ER keys and relational keys are different

# Example Data

| name | addr | beersLiked | manf | favBeer |
|---|---|---|---|---|
| Janeway | Voyager | Bud | A.B. | WickedAle |
| Janeway | Voyager | WickedAle | Pete's | WickedAle |
| Spock | Enterprise | Bud | A.B. | Bud |

The relational key is {name, beersLiked}

But in E/R,

- name is a key for Drinkers
- beersLiked is a key for Beers.

The relation contains

- 2 tuples for Janeway entity
- 2 tuples for Bud entity.

# Where Do Keys Come From?

1. Just assert one key *K*
   - The only FDs are $K \rightarrow A$ for all attributes *A*

2. Assert FDs and
   deduce the keys by systematic exploration
   - ER model gives us FDs from entity-set keys and from many-one relationships
   - Other FDs we may know from our domain knowledge

     ("no two courses take place in a room at the same time")

# Functional Dependencies

## 3. Inferring FDs

1. Meaning of FDs
2. Keys and Superkeys
3. **Inferring FDs**

# Inferring FDs

We are given FD's

$$X_1 \to A_1,\ X_2 \to A_2, \ldots,\ X_n \to A_n\ ,$$

and we want to know whether an FD

$$Y \to B$$

must hold in any relation that satisfies the given FDs

**Example:** If $A \to B$ and $B \to C$ hold,

then surely $A \to C$ holds

*Important for design of good relation schemas*

# Inference Test

We are given a set of FDs and want to know whether

$$Y \rightarrow B$$

follows from the given FDs.

Test: We consider two tuples
and assume they agree in all attributes of $Y$:

| $Y$ | |
|---|---|
| 0000000 | . . . 0 |
| 00000 | ?? . . .? |

# Inference Test (cntd.)

Apply the given FDs to infer that these tuples must also agree on certain other attributes

- If $B$ is one of these attributes, then $Y \rightarrow B$ is true
- Otherwise, the two tuples, with any forced equalities, form a two-tuple relation
  - that satisfies the given FDs
  - but does not satisfy $Y \rightarrow B$.

  This would show that $Y \rightarrow B$ does not follow from the given FDs.

# Inference Test: Example

Drinkers(name, addr, beersLiked, manf, favBeer)

with

1. name $\rightarrow$ addr
2. name $\rightarrow$ favBeer
3. beersLiked $\rightarrow$ manf

Which of the FDs below follow from the given FDs?

4. addr $\rightarrow$ favBeer
5. name beersLiked $\rightarrow$ favBeer

# Closure Test

An easier test is based on the concept of attribute closure

- Let R be a relation,
  $\mathcal{F}$ a set of FDs over R,
  $Y$ a set of attributes of R.

- The closure of $Y$ with respect to $\mathcal{F}$, written $Y^+$,
  consists of all attributes that are determined by $Y$, given $\mathcal{F}$.

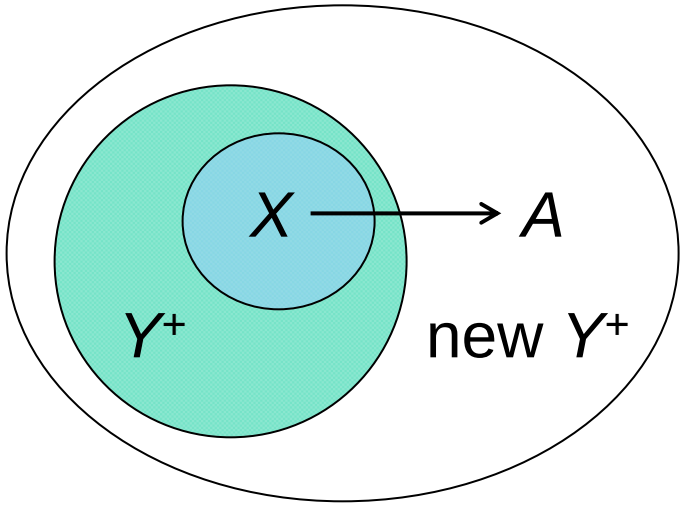
- Observation:
  $Y \rightarrow B$ follows from $\mathcal{F}$
  if and only if
  $B \in Y^+$

# The Closure Algorithm

- Initialization: $Y^+ = Y$
- Loop:
  - Look for an FD $X \to A$ in $\mathcal{F}$ such that $X \subseteq Y^+$ and $A \notin Y^+$
  - Add $A$ to $Y^+$
- Until: there is no applicable FD left in $\mathcal{F}$
- Output: $Y^+$

*This is a typical example of a fixpoint algorithm*

# Closure Algorithm: The Idea



# Example

Contracts(cno, supplno, projno, depno, partno, qty, value)

Short: *CSPrDPaQV*

A designer has found the following set of FDs:

- *C* is a key, i.e., $C \rightarrow SPrDPaQV$
- A project purchases each part using a single contract, $PrPa \rightarrow C$
- A department purchases at most one part from a supplier, $SD \rightarrow Pa$

His colleague has come up with a slightly different set:

- A project purchases each part using a single contract, $PrPa \rightarrow C$
- A contract determines project, supplier and department, $C \rightarrow PrSD$
- *SPrD* is a key, $SDPr \rightarrow CPaQV$

Are the findings of the second designer different from those of the first?

# Finding All Implied FDs

We know how we can determine whether one FD follows from a set of FDs $\mathcal{F} = \{ X_1 \rightarrow A_1, \ldots, X_n \rightarrow A_n \}$

Question: How can we find all such FDs?

Motivation: To get a better schema, we "normalize", i.e.,
we break one relation schema
into two or more schemas.

# Finding All Implied FD's: Example

Relation R with attributes *ABCD*, with FD's

$$AB \rightarrow C, \quad C \rightarrow D, \quad D \rightarrow A.$$

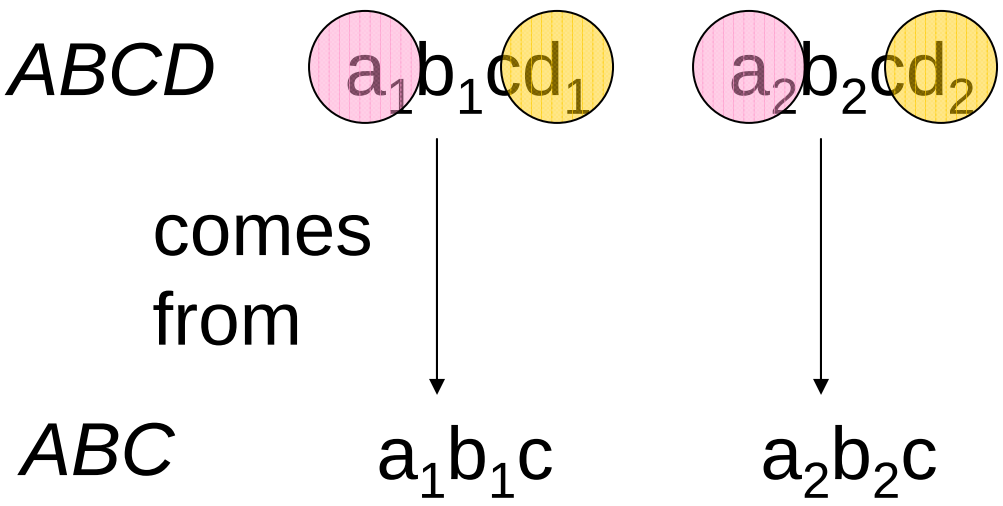
Decompose R into *ABC*, *AD*,

(i.e., project onto *ABC* and *AD*)

Question: What FDs hold in *ABC* ?

Answer: Not only $AB \rightarrow C$, but also $C \rightarrow A$ !

# Why?




$d_1=d_2$ because $C \rightarrow D$

$a_1=a_2$ because $D \rightarrow A$

Thus, tuples in the projection with equal *C*s have equal *A*s: $C \rightarrow A$.

# Projecting FDs

How can we find the FDs that hold on the projection of R?

Basic Idea:

1. Start with the given FDs

2. Find all *nontrivial* FDs that follow from the given FDs
   *(nontrivial = left and right sides disjoint)*

3. Restrict to those FDs that involve only attributes of the projected schema

# A Simple, Exponential Algorithm

1. For each set of attributes $X$, compute $X^+$

2. Add $X \rightarrow A$ for all $A \in X^+ - X$

3. However, drop $XY \rightarrow A$ whenever we discover $X \rightarrow A$

   (Because $XY \rightarrow A$ follows from $X \rightarrow A$ in any projection)

4. Finally, return only FDs involving projected attributes

# Optimizations

Suppose that $Z$ is the set of all attributes of R. Then:

- $\varnothing^+ = \varnothing$
- $Z^+ = Z$
- If $X \subseteq Y$ and $X^+ = Z$ then $Y^+ = Z$

*This can be used for optimizations!*

# Example

Relation $ABC$ with FDs $A \rightarrow B$ and $B \rightarrow C$

Project onto $AC$

- $A^+ = ABC \Rightarrow A \rightarrow B, A \rightarrow C$

  (Optimization: We do not need to compute $AB^+$ or $AC^+$ )
- $B^+ = BC \Rightarrow B \rightarrow C$
- $C^+ = C \Rightarrow$ nothing
- $BC^+ = BC \Rightarrow$ nothing

Resulting FDs: $A \rightarrow B$, $A \rightarrow C$, and $B \rightarrow C$.

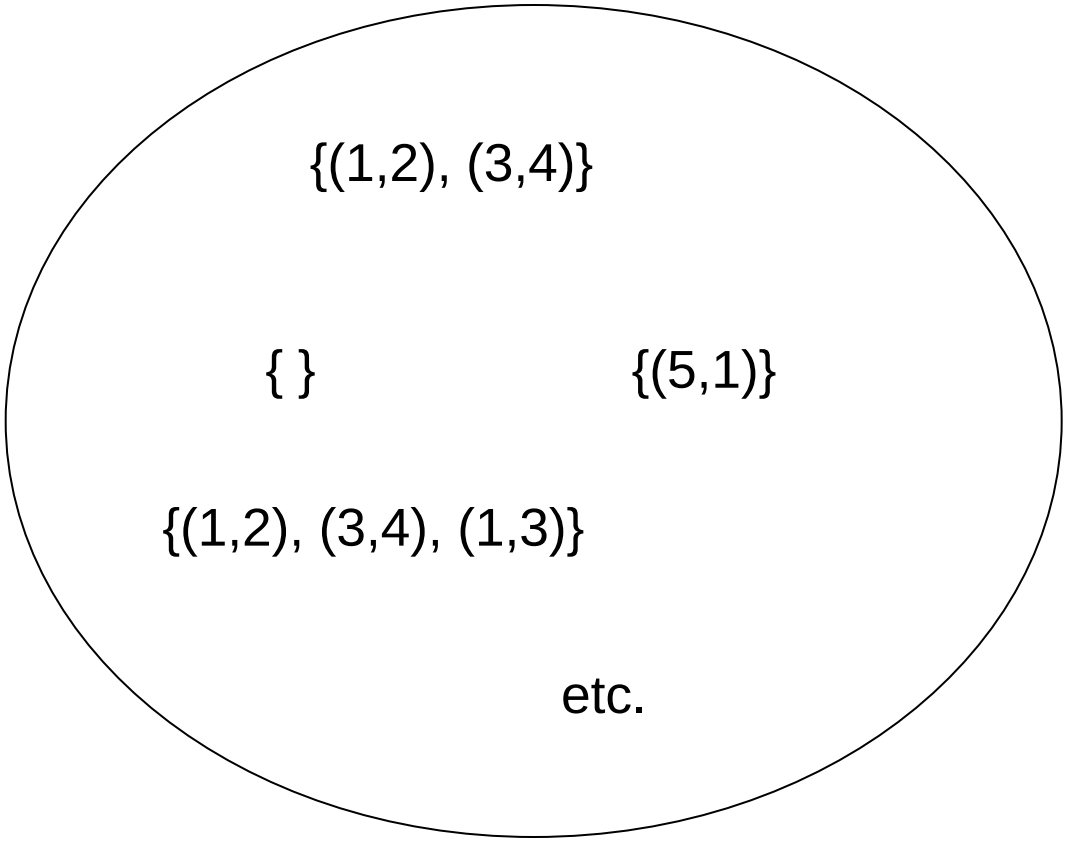
Projection onto $AC$: $A \rightarrow C$.

(The only FD that involves a subset of { $A$,$C$ })

# A Geometric View of FDs

- We consider the set of all *instances* of a particular relation R
- That is,
  - all finite sets of tuples
  - that have the proper number of components.
- Each instance is a point in this space

# Example: R(A,B)



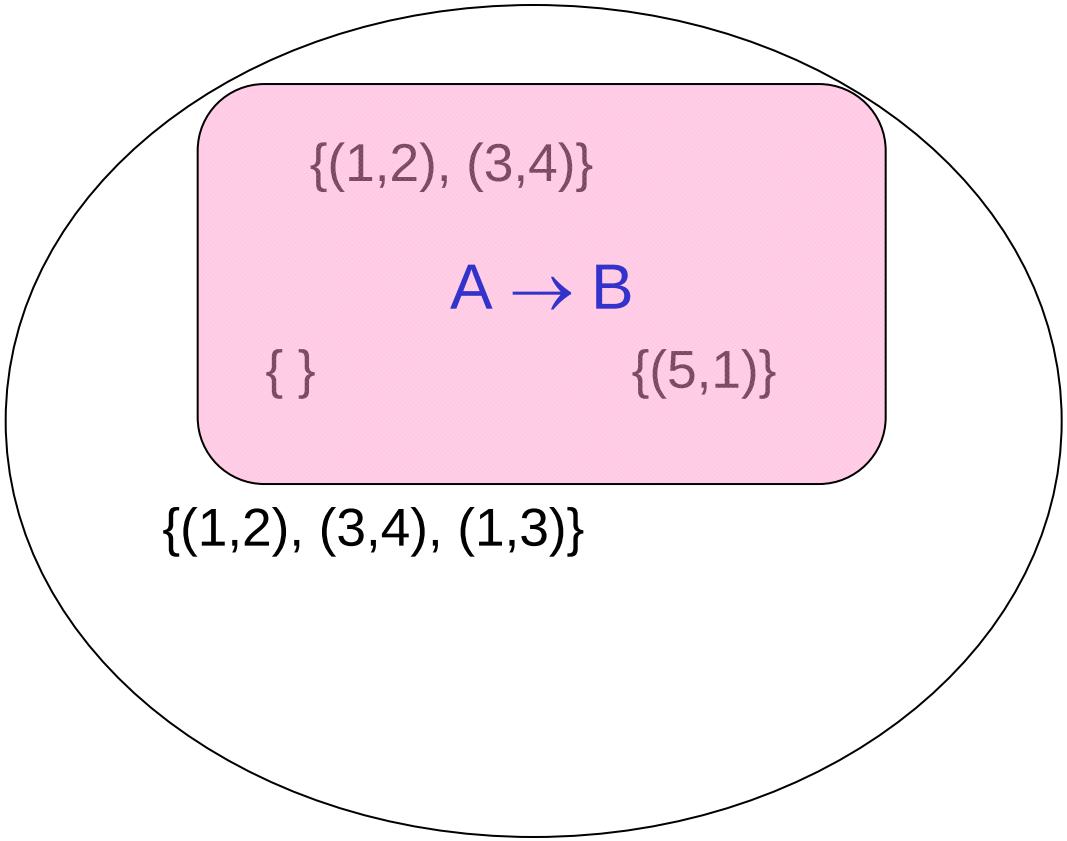
# An FD is a Subset of Instances

- For each FD $X \rightarrow A$, there is the subset of all instances that satisfy the FD

- Thus, we can identify an FD
  with a region in the space
- An FD is trivial
  if and only if
  it is represented by the entire space

  Example: $A \rightarrow A$.

# Example: $A \rightarrow B$ for R(A,B)

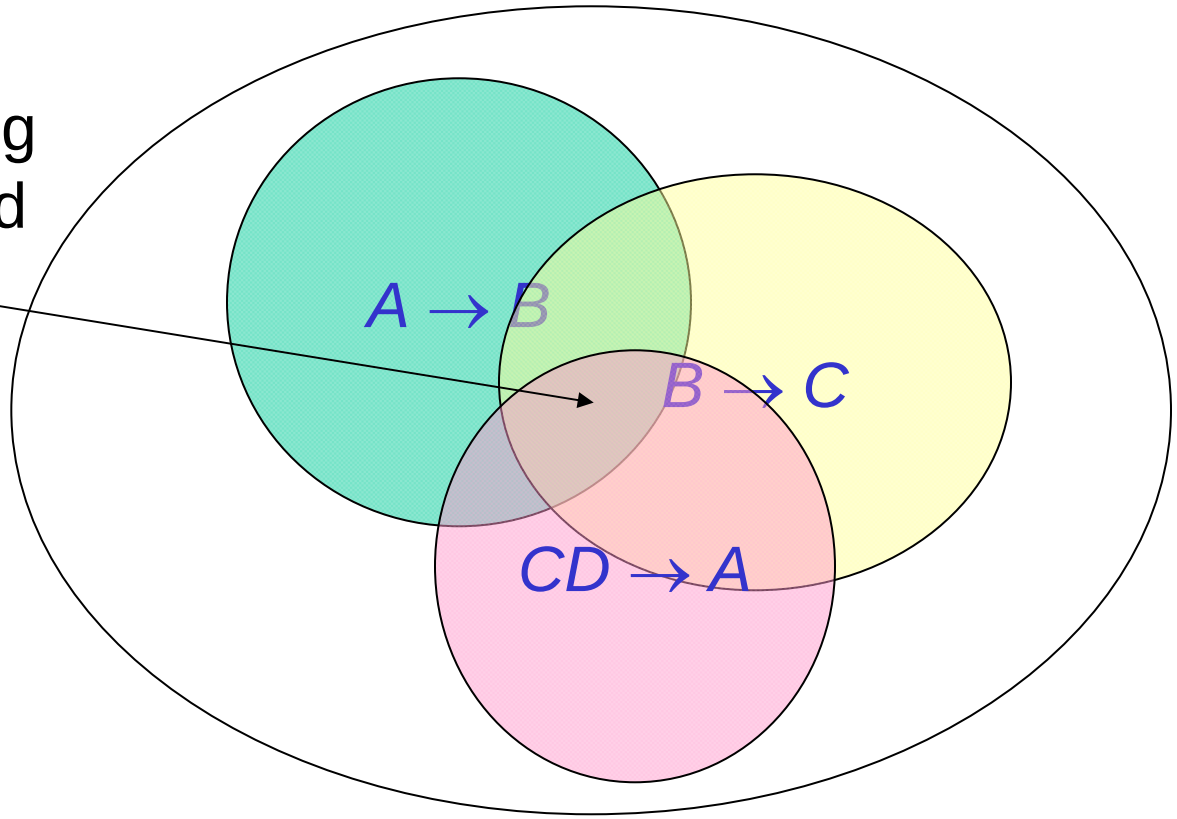


# Representing Sets of FDs

If each FD is a set of relation instances, then a collection of FDs corresponds to the intersection of those sets

*(Intersection = all instances that satisfy all of the FDs)*

Instances satisfying $A \rightarrow B$, $B \rightarrow C$, and $CD \rightarrow A$



# Entailment of FDs

$\mathcal{F} = \{ X_1 \rightarrow A_1, \ldots, X_n \rightarrow A_n \}$ set of FDs

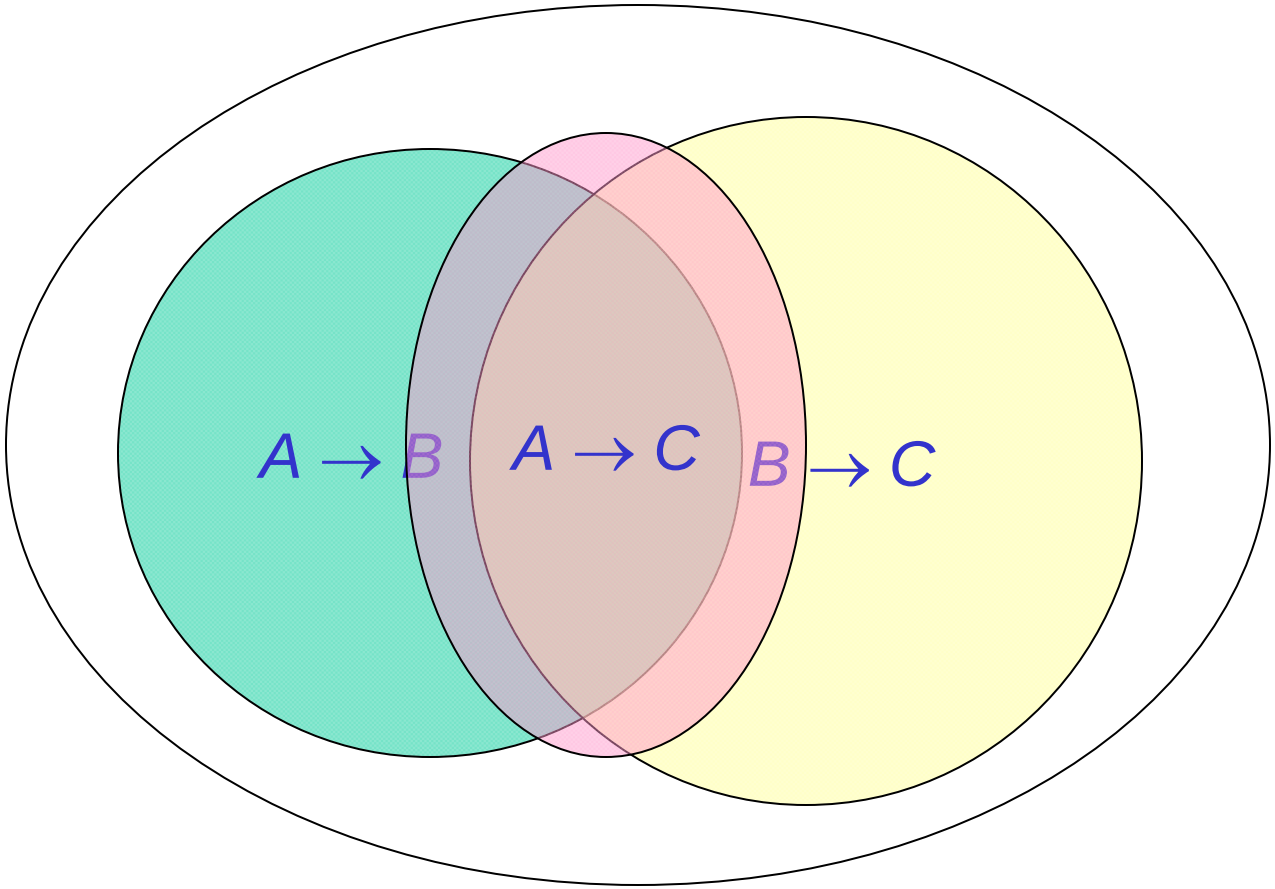
- An FD $Y \rightarrow B$ follows from $\mathcal{F}$ or is entailed by $\mathcal{F}$ if every instance that satisfies all FDs in $\mathcal{F}$ also satisfies $Y \rightarrow B$

This can be visualized:

- If $Y \rightarrow B$ follows from the set $\mathcal{F} = \{ X_1 \rightarrow A_1, \ldots, X_n \rightarrow A_n \}$, then in the space of instances the *region for* $Y \rightarrow B$ must include the *intersection of the regions for the FDs* $X_i \rightarrow A_i$ .

  That is:
  - Every instance satisfying all the $X_i \rightarrow A_i$ surely satisfies $Y \rightarrow B$.
  - But an instance could satisfy $Y \rightarrow B$, yet not be in this intersection.

# Example



# References

In preparing the lectures I have used several sources.
The main ones are the following:

Books:

- A First Course in Database Systems, by J. Ullman and J. Widom
- Fundamentals of Database Systems, by R. Elmasri and S. Navathe

Slides:

- The slides of this chapter are based on slides prepared by Jeff Ullman for his introductory course on database systems at Stanford University