Ontology and Database Systems: Foundations of Database Systems

Part 1: Databases and Queries

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Relational Databases: Principles

A database has two parts: **schema** and **instance**

The schema describes *how data is organized:*

- relations with their names, arity, and names and types of attributes
- integrity constraints like key and foreign key constraints, functional dependencies, inclusion dependencies, domain constraints

The instance contains the *actual data:*

- for every relation, there is a relation instance
- the relation instance is a set (multiset?) of tuples of the right arity and type

Often, we ignore types and integrity constraints
Sometimes, we ignore also the attribute names

Example Schema: Students and Courses

Relation schemas

`Student(sid: INTEGER, sname: STRING, city: STRING, age: INTEGER)`

`Course(cid: INTEGER, cname: STRING, faculty: STRING)`

`Enrolled(sid: INTEGER, cid: INTEGER, aY: STRING, mark: STRING)`

Integrity constraints

- Primary keys

  `Student(sid)`
  `Course(cid)`
  `Enrolled(sid, cid, aY)`

- Foreign keys:

  `Enrolled(sid)` references `Student(sid)`
  `Enrolled(cid)` references `Course(cid)`

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Schemas: Formalization

A **relation schema** consists of

- a relation name
- an ordered list of attributes, possibly with types

Abstract notation $R(A_1, \ldots, A_n)$, or $R(A_1 : \tau_1, \ldots, A_n : \tau_n)$

The *arity* of R, written *ary*(R), is the number of arguments of R

A **database schema** $\mathcal{S}$ consists of

- a *signature* Σ, which is a set of relation schemas
- a set Γ of *integrity constraints* over Σ,
  which may be expressed as formulas in first-order logic (FOL)

Simplified notation: $\mathcal{S} = \{R_1, \ldots, R_m\}$, or $\mathcal{S} = \{R_1/n_1, \ldots R_m/n_m\}$,
(i.e., we only mention the names, or the names with their arity)

Exercise: Express the primary and foreign key constraints in the Students and Courses schema by FOL formulas

Domain: Formalization

We assume there is an infinite set of constants **dom**, called the **domain**

When we ignore types, we do not make any assumptions about the constants in **dom**

Otherwise, **dom** $= \bigcup_{i=1}^{k} \tau_i$, where $\tau_1, \ldots, \tau_k$ are the types

Definition

A type τ with an order "$<$" is an *ordered type.* The order "$<$" is

- *dense* if for every $a, b \in \tau$ with $a < b$, there is a $c \in \tau$ such that $a < c < b$
- *discrete* if for every a, $b \in \tau$ with $a < b$, there are at most finitely many c such that $a < c < b$

Example

Consider integers, reals, strings, and booleans.
Which type has a dense and which a discrete ordering?

Relation Instances

Relation R with arity n:

- an instance of R is a finite set of n-tuples over **dom**

Relation R with schema $R(A_1 \colon \tau_1, \ldots, A_n \colon \tau_n)$:

- as before, plus the components of the n-tuples in an instance have to be of the right type

Schema Instances

An **instance of the signature** Σ is a function $\mathbf{I}$ that

- maps every $R \in \Sigma$ to an instance of R, denoted $\mathbf{I}(R)$

Every instance $\mathbf{I}$ of Σ can be seen as a **first-order interpretation/structure** (also denoted $\mathbf{I}$):

- domain of $\mathbf{I}$ is $\Delta^{\mathbf{I}} =$ **dom**
- $c^{\mathbf{I}} = c$, for every $c \in$ **dom**
  (proper names, i.e., every constant is interpreted as itself)
- $R^{\mathbf{I}} = \mathbf{I}(R)$

A function $\mathbf{I}$ is an **instance of the schema** $\mathcal{S} = (\Sigma, \Gamma)$ if

- $\mathbf{I}$ is an instance of Σ
- $\mathbf{I}$ satisfies every integrity constraint $\gamma \in \Gamma$ in the sense of first-order logic (FOL)

Logic Programming Perspective

Often an alternate definition of instances is helpful

Definition

- A *fact* over a relation R with arity n is an expression $R(a_1, \ldots, a_n)$, where $a_1, \ldots, a_n \in$ **dom**
- A *relation instance* is a finite set of facts over R
- A *signature instance* $\mathbf{I}$ of Σ is a finite set of facts over the relations in Σ

Example

$\mathbf{I}_{\text{univ}} = \{$ `Student`(123, Egger, Bozen, 25), `Student`(777, Hussein, Dresden, 23),
`Course`(104, Programming, CS), `Course`(106, Databases, CS),
`Course`(217, Optics, PHYS)
`Enrolled`(123, 104, 14/16, fail), `Enrolled`(123, 104, 15/16, fail),
`Enrolled`(123, 104, 16/17, pass), `Enrolled`(123, 106, 15/16, pass),
`Enrolled`(777, 217, 15/16, pass) $\}$

Relational Database Queries

A **query** over a schema $\mathcal{S}$ is

- a **function** that maps every instance of $\mathcal{S}$ to a set of tuples such that
  - all tuples have the same length ($=$ arity of the query)
  - tuple values at the same position have the same type
- a **piece of syntax** that defines such a function

Query languages are/should be **declarative**:

- you express what you want to know, not how to compute it
  (a query engine analyzes the query and creates an execution plan)

Relational Query Languages

- Theoretical languages
  - Relational Algebra (that's how Codd started it)
  - Relational Calculus (= FOL in essence)
  - Datalog (drops negation, adds recursion)
- Commercial language: SQL
  - = Relational Calculus (at its core)
  - + Relational Algebra
  - + a bit of Datalog (implemented in IBM DB2, Microsoft SQL Server)
  - + aggregates, arithmetic, nulls, . . . , functions, procedures

Relational Calculus Queries

Definition

A **query** in (domain) relational calculus (RelCalc) has the form

$$Q = \{(x_1, \ldots, x_n) \mid \phi\}$$

where

- ϕ is a predicate logic formula
- $x_1, \ldots, x_n$ are the free variables of ϕ

We say that

- ϕ is the **body** of the query,
- $x_1, \ldots, x_n$ are the **output variables**, and
- n is the **arity** of the query.

If the arity is not important, we write $\bar{x}$ instead of $x_1, \ldots, x_n$

We sometimes write Q_ϕ to denote the query defined by ϕ

Reminder on Predicate Logic Formulas

A *term* is a constant or a variable

An *atom* is an expression $R(t_1, \ldots, t_n)$ where R is a relation symbol of arity n and $t_1, \ldots, t_n$ are terms

A *formula* F is an atom or has the form

- $(F_1 \land F_2)$, $(F_1 \lor F_2)$, or $(F_1 \rightarrow F_2)$
- $\neg F$
- $(\exists\, x\, F)$, $(\forall\, x\, F)$

where F, F_1, F_2 are formulas.

(Operators have the usual precedences.
We drop parentheses that are not needed for the structure of a formula.)

Exercise (once the semantics has been defined):
Show that the logical symbols $\land$, $\exists$, $\neg$ suffice to express all other symbols

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Equality and Built-in Predicates

Sometimes we use also the predicate symbols

$$\text{“}{=}\text{”}, \quad \text{“}{<}\text{”}, \quad \text{“}{\leq}\text{”}, \quad \text{“}{\neq}\text{”}$$

Atoms with these symbols are called

- equalities (“$=$”)
- comparisons (“$<$”, “$\leq$”)
- disequalities (“$\neq$”)

Clearly, they can only be applied to terms of the same type

Comparisons can only be used for terms of a type that is ordered

Bound and Free Variables

Definition

- An occurrence of a variable x in formula ϕ is *bound* if it is within the scope of a quantifier $\exists x$ or $\forall x$
- An occurrence of a variable in ϕ is *free* iff it is not bound
- A variable of formula ϕ is *free* if it has a free occurrence

Free variables specify the output of a query

Relational Calculus Queries: Semantics

In FOL, the semantics of a formula is defined in terms of *interpretations* and *assignments*. Recall:

- every instance $\mathbf{I}$ defines a first-order interpretation $\mathbf{I}$
- an assignment is a mapping $\alpha\colon$ **var** $\rightarrow$ **dom**

There is a classical recursive definition of when
an interpretation $\mathbf{I}$ and an assignment α *satisfy* a formula ϕ, written

$$\mathbf{I}, \alpha \models \phi,$$

which we take for granted

Definition

Let $Q = \{(x_1, \ldots, x_n) \mid \phi\}$ be a query. We define the **answer** of Q over $\mathbf{I}$ as

$$Q(\mathbf{I}) = \{\alpha(\bar{x}) \mid \mathbf{I}, \alpha \models \phi\}$$

Exercise

Express the following queries over our university schema in Relational Calculus

- Which are the names of students that have passed an exam in CS?
- Which students (given by their id) have never failed an exam in CS?
- Which students (given by their id) have passed the exams for all courses in CS?

Evaluate the expressions over the instance $\mathbf{I}_{\text{univ}}$

Relational Algebra

Expressions E are built up from

- relation symbols R

using the operators

- *union* $(E_1 \cup E_2)$, *intersection* $(E_1 \cap E_2)$, *set difference* $(E_1 \setminus E_2)$, called boolean operators
- *selection* $\sigma_C(E)$
- *projection* $\pi_X(E)$
- *cartesian product* $E_1 \times E_2$
- *join* $E_1 \bowtie_C E_2$
- *attribute renaming* $(\rho_{A \leftarrow B}(E))$

where C is a condition involving equalities and comparisons between attributes and constants, and X is a set of attributes of E

For an instance $\mathbf{I}$, an expression E is evaluated as a set of tuples $E(\mathbf{I})$

A **query** is an **expression**

Relational Algebra: Remarks

- An operator not only returns a set of tuples as the result, but also a schema for the result.
- Operators that mention attributes can only be applied to expressions that have that attribute in their schema.
- Boolean operators can only be applied to expressions with the same schema.

Relational Algebra: Examples

What is the meaning of the following queries?

- $\sigma_{\mathsf{city}='\mathsf{Bozen}' \wedge \mathsf{age} > 21}(\mathtt{Student})$
- $\pi_{\mathsf{cname,faculty}}(\mathtt{Course})$
- $\pi_{\mathsf{cname}}(\mathtt{Course} \bowtie_{\mathtt{Course}.\mathsf{cid}=\mathtt{Enrolled}.\mathsf{cid}} \mathtt{Enrolled})$
- $\pi_{\mathsf{sid}}(\mathtt{Student}) \setminus \pi_{\mathsf{sid}}(\mathtt{Enrolled})$

Relational Algebra: Exercise

Express the following queries over our university schema in Relational Algebra

- What are the names of the courses for which student Egger has failed an exam?
- Which students have failed an exam for the same course at least twice?
- Which students have never failed an exam in Physics?

Evaluate the expressions over the instance $\mathbf{I}_{\text{univ}}$