

## 2. Satisfiability of Comparisons

We consider *finite* sets of comparisons, that is, atoms with the predicates “=”, “≠”, “≤”, and “<” (also called built-in predicates). An example of such a set is the following:

$$x \leq y, x < z, y \leq 5, y \neq 4.$$

We interpret built-in atoms either over the rational numbers  $\mathbb{Q}$  or the integers  $\mathbb{Z}$ . We define as usual when an assignment that maps variables to numbers satisfies an atom. An assignment satisfies a set if it satisfies every atom in the set. A set  $A$  of atoms is *satisfiable over*  $\mathbb{Q}$  if there is an assignment that maps the variables in  $A$  to elements of  $\mathbb{Q}$  and satisfies  $A$ . The set  $A$  is *satisfiable over*  $\mathbb{Z}$  if there is an assignment that maps the variables in  $A$  to elements of  $\mathbb{Z}$  and satisfies  $A$ .

We are interested in coming up with methods to check whether such a set is satisfiable. For each of the following classes of conjunctions of comparisons, describe a method by which one can check satisfiability: Comparisons with

1. “=”
2. “=” and “≠”
3. “≤”, ranging over the rational numbers
4. “≤” and “≠”, ranging over the rational numbers
5. “≤” and “<”, ranging over the rational numbers
6. “≤”, ranging over the integers
7. “≤” and “<”, ranging over the integers.