Data Structures and Algorithms

Werner Nutt

Acknowledgments

- The course follows the book "Introduction to Algorithms", by **Cormen, Leiserson, Rivest and Stein**, MIT Press [CLRST]. Many examples displayed on these slides are taken from their book.
- These slides are based on those developed by Michael Böhlen for his course.

(See http://www.inf.unibz.it/dis/teaching/DSA/)

- The slides also include a number of additions made by Roberto Sebastiani and Kurt Ranalter when they taught later editions of this course

(See http://disi.unitn.it/~rseba/DIDATTICA/dsa2011_BZ/)

DSA, Chapter 1: Overview

- Introduction, syllabus, organisation
- Algorithms
- Recursion (principle, trace, factorial, Fibonacci)
- Sorting (bubble, insertion, selection)

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Learning Outcomes

The main things we will learn in this course:

- To *distinguish* between a *problem* and
  an *algorithm* that solves it
- To get to know a *toolbox* of *classical* algorithms
- To *think algorithmically* and
  get the spirit of how algorithms are designed
- To learn a number of algorithm design *techniques*
  (such as divide-and-conquer)
- To analyze (in a precise and formal way)
  the *efficiency* and the *correctness* of algorithms.

Syllabus

1. Introduction, recursion (chap 1 in CLRS)
2. Correctness and complexity of algorithms (2, 3)
3. Divide and conquer, recurrences (4)
4. Heapsort, Quicksort (6, 7)
5. Dynamic data structures, abstract data types (10)
6. Binary search trees (12)
7. Hash tables (11)
8. Graphs: Principles and graph traversal (22)
9. Shortest path (23)

Literature

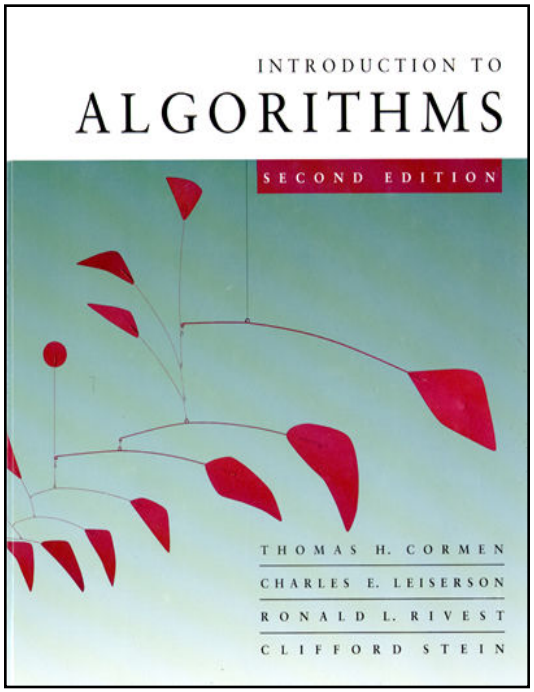
Cormen, Leiserson, Rivest and Stein (CLRS), *Introduction to Algorithms*, Second Edition, MIT Press and McGraw-Hill, 2001

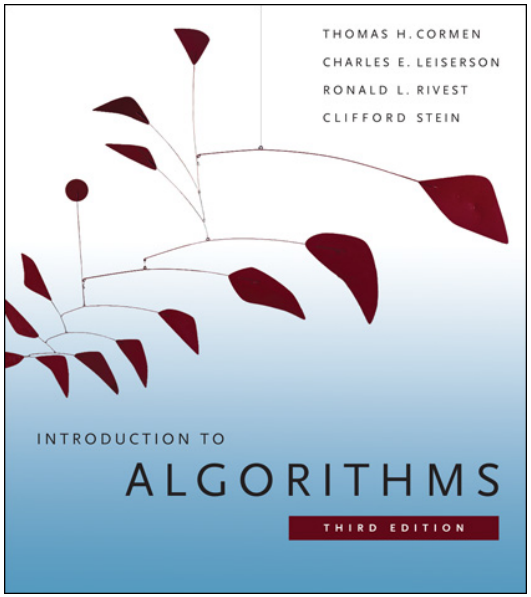
and

Third Edition, MIT Press, 2009

(See http://mitpress.mit.edu/algorithms/)

Course is based on this book





Other Literature

Kurt Mehlhorn and Peter Sanders

Algorithms and Data Structures - The Basic Toolbox

Offers alternate presentation of topics of the course

Free download from

http://www.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf

Course Organization

- Lectures: Tue 10-12, Fri 10-12
  Office hours: Tue 14:00-16:00 (but let me know if you want to come)
- Labs (starting this week): Fri 14:00-16:00
- Teaching Assistants
  - Julien Corman, JulienLouisMichel.Corman@unibz.it
  - Flavio Vella, Flavio.Vella@unibz.it

- Home page:
  http://www.inf.unibz.it/~nutt/Teaching/DSA1819/

Assignments

The assignments are a crucial part of the course

- Roughly each week an assignment has to be solved
- The schedule for the publication and the handing in of the assignments will be announced at the next lecture.
- A number of assignments include programming tasks. It is strongly recommended that you implement and run all programming exercises.
- Assignments will be marked. The assignment mark will count towards the course mark.
- Any attempt at plagiarism (copying from the web or copying from other students) leads to a 0 mark for all assignments.

Assignments, Midterm Exam, Final Exam, and Course Mark

- There will be
  - one written exam at the end of the course
  - one mock exam around the middle of the course
  - assignments
- To pass the course, one has to pass the written exam.
- Students who do not submit exercises and do not take part in the mock exam (or fail the mock exam) will be marked on the final exam alone.
- For students who submit all assignments, and take part in the midterm, the final mark will be a weighted average

  50% exam mark + 5% mock exam
  + 45% assignment mark

Assignments, Midterm Exam, Final Exam, and Course Mark

- If students submit fewer assignments, or do not take part in the midterm, the percentage will be lower.
- Assignments for which the mark is lower than the mark of the written exam will not be considered.
- Similarly, the mock exam will not be considered if the mark is lower than the mark of the final exam.
- The mock exam and assignment marks apply to all future exam sessions.

Organisation of Labs

- You will attend always the lab of the same teaching assistant (TA) during the course
- The TA will mark your assignments and be your first contact person for questions on the assignments
- To help us organize the labs, send an email by Wednesday evening to Werner.Nutt@unibz.it containing a group of students that would like to attend the same lab
- One mail per group is enough
- The mail should contain for each student of the group
  - name
  - email address
  - student number
- All members of the group to be registered must appear in the cc

General Remarks

- Algorithms are first designed on paper
  … and later keyed in on the computer.

- The most important thing is to be simple and precise.

- During lectures:
  - Interaction is welcome; ask questions
    (I will ask you anyway ☺)
  - Additional explanations and examples if desired
  - Speed up/slow down the progress

DSA, Chapter 1:

- Introduction, syllabus, organisation
- Algorithms
- Recursion (principle, trace, factorial, Fibonacci)
- Sorting (bubble, insertion, selection)

What are Algorithms About?

There are problems we solve in everyday life

- Travel from Bolzano to Berlin
- Cook Spaghetti alla Bolognese *(I know, not in Italy,…)*
- Register for a Bachelor thesis at FUB

For all these problems, there are

- instructions
- recipes
- procedures,

which describe a complex operation in terms of

- elementary operations *(“beat well …”)*
- control structures and conditions *(“… until fluffy”)*

Algorithms

Problems involving numbers, strings, mathematical objects:

- for two numbers, determine their sum, product, …
- for two numbers, compute their greatest common divisor
- for a sequence of strings,
  find an alphabetically sorted permutation of the sequence
- for two arithmetic expressions, find out if they are equivalent
- for a program in Java,
  create an equivalent program in byte code
- on a map, find for a given house the closest bus stop

We call instructions, recipes, for such problems *algorithms*

What have algorithms in common with recipes?
How are they different?

History

- *First algorithm:* Euclidean Algorithm,
  greatest common divisor, 400-300 B.C.
- *Name:* Persian mathematician Mohammed al-Khowarizmi, in Latin became “Algorismus”

  كتاب الجمع و التفريق بحساب الهند

  = *Kitāb al-Dscham‘ wa-l-tafrīq bi-ḥisāb al-Hind*

  = *Book on connecting and taking apart in the calculation of India*
- *19th century*
  - Charles Babbage: Difference and Analytical Engine
- *20th century*
  - Alan Turing, Alonzo Church: formal models computation
  - John von Neumann: architecture of modern computers

Data Structures, Algorithms, and Programs

- Data structure
  - Organization of data to solve the problem at hand
- Algorithm
  - Outline, the essence of a computational procedure, step-by-step instructions
  - Specify transitions of the data structures
- Program
  - implementation of an algorithm in some programming language

Overall Picture

Using a computer to help solve problems:

- Precisely specifying the problem
- Designing programs
  - architecture
  - algorithms
- Writing programs
- Verifying (testing) programs

Data Structure and Algorithm Design Goals

Correctness

Efficiency

Implementation Goals

Robustness

Reusability

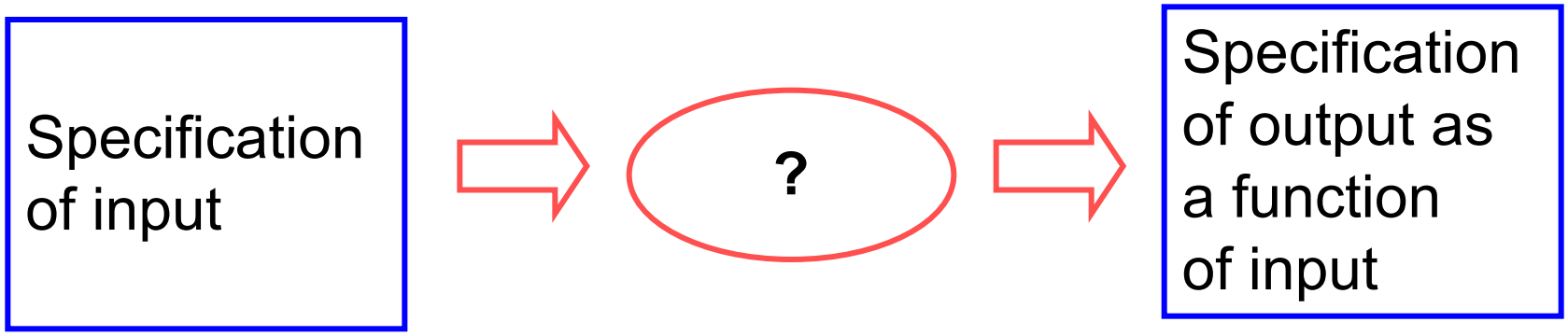
Adaptability

This course is not about:

- Programming languages
- Computer architecture
- Software architecture
- SW design and implementation principles

We will only touch upon
the theory of complexity
and computability.

Algorithmic Problem



There is an infinite number of possible input *instances* satisfying the specification.

For example: An array of distinct integer numbers:

[-909, -1, -20, 908, 1000000000, 100000]

Question from a Google Interview

You are given an array of distinct numbers.

You need to return an index to a "local minimum" element, which is defined as an element that is smaller than both its adjacent elements.

In the case of the array edges, the condition is reduced to one adjacent element.

If there are multiple "local minima",
returning any one of them is fine.

Question from a Google Interview/2

You are given an unsorted sequence of integers *A*.

Find the longest subsequence *B* such that
elements of this subsequence are strictly increasing numbers.

Elements in the subsequence *B* must appear
in the same relative order as in the sequence *A*.

Example:

input: *A* = [-1, 2, 100, 100, 101, 3, 4, 5, -7]

output: *B* = [-1, 2, 3, 4, 5]

Question from a Google Interview/3

You have a sorted array containing
the age of every person on Earth

[0, 0, 0, 0, ..., 1, 1, ..., 28, 28, ..., 110, ...].

Find out how many people have each age.

Question from a Google Interview/4

You are given a text file that has list of dependencies between (any) two projects in a source code repository.

Write an algorithm to determine the build order, i.e., which project needs to be built first, followed by which project, based on the dependencies.

You get a bonus point, if you can detect any circular dependencies and throw an exception if found.

Example: `ProjectDependencies.txt`

a → *b* (means "*a* depends on *b*", so *b* needs to be built first and then *a*)

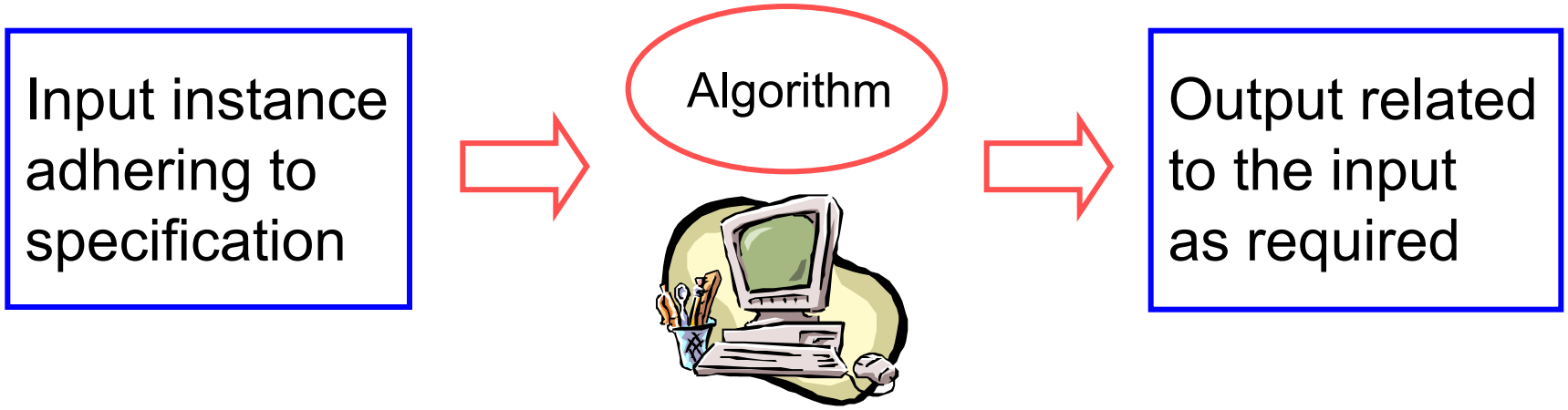
b → *c*

b → *d*

c → *d*

Then the build order can be *d*, *c*, *b*, *a*, in that order.

Algorithmic Solution



- Algorithm describes actions on the input instance
- There may be many correct algorithms for the same algorithmic problem.

Definition

An algorithm is a sequence of *unambiguous* instructions for solving a problem, i.e.,

- for obtaining a *required output*
- for any *legitimate input*

in a finite amount of time.

➔This presumes a mechanism to execute the algorithm

Properties of algorithms:

- Termination, Correctness, (Non-)Determinism, Running Time, …

How to Develop an Algorithm

- Precisely define the problem.
  Precisely specify the input and output.
  Consider all cases.

- Come up with a simple (= abstract ?) plan
  to solve the problem at hand.
  - The plan is independent of a (programming) language
  - The precise problem specification influences the plan.

- Turn the plan into an implementation
  - The problem representation (data structure)
    influences the implementation

Example 1: Searching

INPUT

- A: (un)sorted sequence/array of *n* numbers, ($n > 0$)
- q: a single number

$a_1, a_2, a_3, \ldots, a_n;\ q$

2 5 6 10 11; 5

2 5 6 10 11; 9

OUTPUT

- index of number q in sequence/array A, or *-1*

j

2

-1

Example 1: Searching

INPUT

- A: (un)sorted sequence/array of n numbers, ($0 \leq n$)
- q: a single number

$a_1, a_2, a_3, \ldots, a_n;\ q$

2 5 6 10 11; 5

2 5 6 10 11; 9

OUTPUT

- index j of A such that A[j] = q, or *-1 if no such j exists*

j

2

-1

How Can One Come Up with an Idea?

- What is the simplest case? How can we solve it?
- What could be a partial solution?
- How can we extend a partial solution of one fragment to a partial solution of a (slightly) larger fragment?
- Can we extend some partial solution to a full solution?

Searching/2, search1

search1
INPUT: A[1..n] (un)sorted array of integers, q an integer.
OUTPUT: index j such that A[j] = q, or -1 if A[j] $\neq q$ for all j ($1 \leq j \leq n$)

```
int search1(int[] A, int q)
j := 1
while j ≤ n and A[j] ≠ q do
    j++
if j ≤ n
    then return j
    else return -1
```

- The code is written in *pseudo-code* and INPUT and OUTPUT of the algorithm are specified.
- The algorithm uses a *brute-force* technique, i.e., scans the input sequentially.

Preconditions, Postconditions

Precondition:

- what does the algorithm get as input?

Postcondition:

- what does the algorithm produce as output?
- … how does this relate to the input?

Make sure you have considered the special cases:

- empty set, number 0, empty reference *NULL*, …

Pseudo-code

Like Java, Pascal, C, or any other imperative language

- Control structures:

  (if then else, while, and for loops)

- Assignment: :=
- Array element access: A[i]
- Access to element of composite type (record or object):

  A.b

CLRS uses b[A]

Control Structures in Pseudo-code: Examples

- for-to:

```
for i := 1 to n do
    A[i] := A[i] + 1
```

- for-downto:

```
for i := n downto 1 do
    A[i] := A[i] + 2
```

- while:

```
while A[i] > 0 do
    A[i] := A[i] - i
```

- if-then-else:

```
if A[i] > 0
  then pos := true
  else pos := false
```

Searching, Java Solution

```
import  java.io.*;

class search {
  static final int n = 5;
  static int q = 22;
  static int a[] = { 11, 1, 4, -3, 22 };

  public static void main(String args[]) {
    int j = 0;
    while (j < n && a[j] != q) { j++; }
    if (j < n) { System.out.println(j); }
    else { System.out.println(-1); }
  }
}
```

Searching/3, search2

Another idea:

Run through the array
and set a pointer if the value is found.

search2
INPUT: A[1..n] (un)sorted array of integers, *q* an integer.
OUTPUT: index *j* such that A[*j*] = *q*, or *-1* if A[*j*] ≠ *q* for all *j* ($1 \leq j \leq n$)

```
int search2(int[] A, int q)
ptr := -1;
for j := 1 to n do
  if a[j] = q then ptr := j
return ptr;
```

Does it work?

search1 vs search2

Are the solutions equivalent?

Can one construct an example such that, say,

- search1 returns 3
- search2 returns 7 ?

But both solutions satisfy the specification (or don't they?)

Searching/4, search3

A third idea:

Run through the array and
return the index of the value in the array.

```
search3
INPUT:     A[1..n] (un)sorted array of integers, q an integer
OUTPUT: index j such that A[j]=q or -1 if A[j] ≠ q for all j (1 ≤ j ≤ n)

int search3(int[] A, int q)
for j := 1 to n do
  if a[j] = q then return j
return -1
```

Comparison of Solutions

Metaphor: shopping behavior when buying a beer:

- **`search1`**: scan products;
  stop as soon as a beer is found and go to the exit.
- **`search2`**: scan products until you get to the exit;
  if during the process you find a beer,
  put it into the basket
  (instead of the previous one, if any).
- **`search3`**: scan products;
  stop as soon as a beer is found
  and exit through next window.

Comparison of Solutions/2

- search1 and search3 return *the same result*
  (index of the first occurrence of the search value)
- search2 returns the index of the last occurrence
  of the search value
- search3 does not finish the loop
  (as a general rule, you better avoid this)

Iteration in Pseudocode and in Java/C/C++

- In pseudo-code, array indexes range from 1 to length
- In Java/C/C++, array indexes range from 0 to length-1
- Examples:
  - Pseudo-code

```
for j := 1 to n do
```

    Java:

```
for (j=0; j < a.length; j++) { …
```

  - Pseudo-code

```
for j := n downto 2 do
```

    Java:

```
for (j=a.length-1; j >= 1; j--) { …
```

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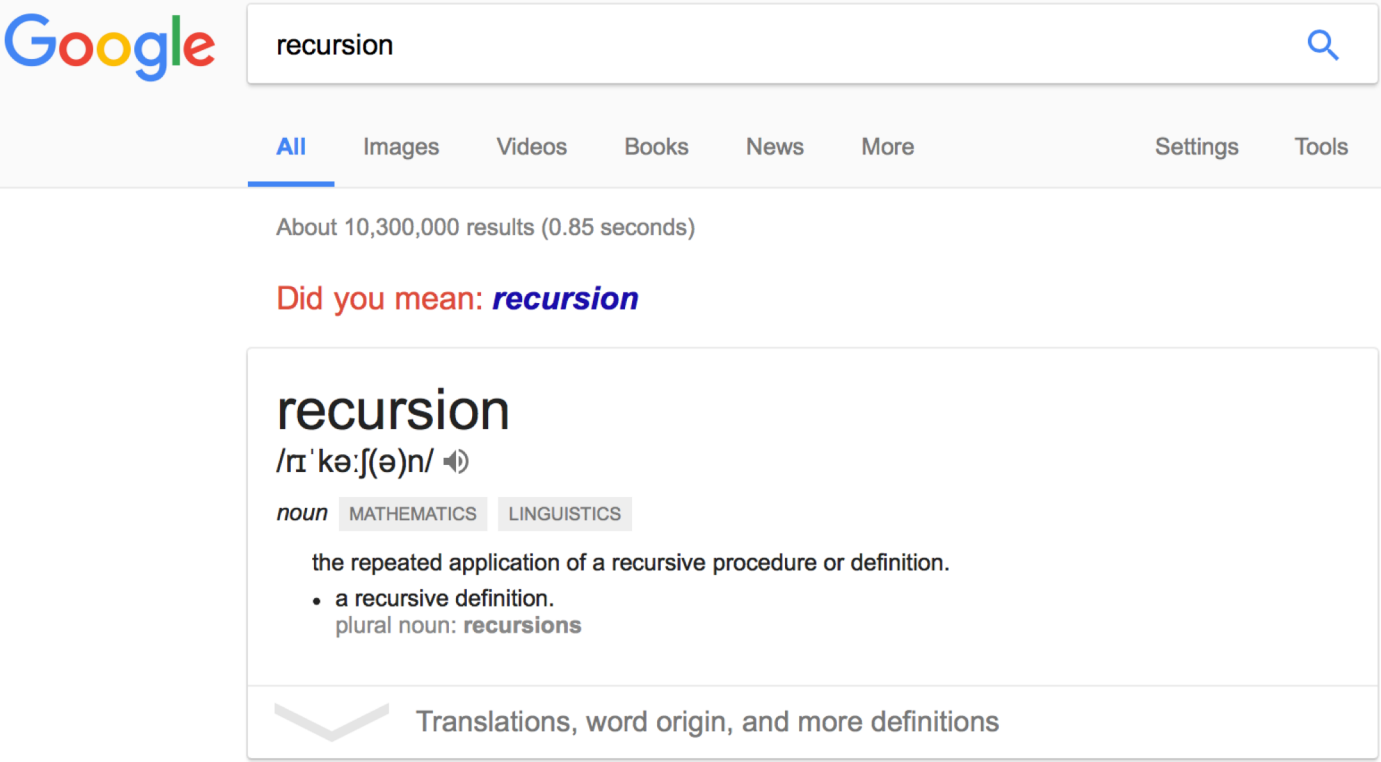
- Sorting (bubble, insertion, selection)

Recursion

An object is recursive if

- a part of the object refers to the entire object, or
- one part refers to another part and vice versa

(mutual recursion)


Google
recursion
All
Images
Videos
Books
News
More
Settings
Tools
About 10,300,000 results (0.85 seconds)
Did you mean: recursion
recursion
/rɪˈkəːʃ(ə)n/
noun
MATHEMATICS
LINGUISTICS
the repeated application of a recursive procedure or definition.
a recursive definition.
plural noun: recursions
Translations, word origin, and more definitions
Feedback
Recursion - Wikipedia
https://en.wikipedia.org/wiki/Recursion
Recursion occurs when a thing is defined in terms of itself or of its type. Recursion is used in a variety of disciplines ranging from linguistics to logic. The most common application of recursion is in mathematics and computer science, where a function being defined is applied within its own definition.
Disambiguation · Recursion (computer science) · Corecursion · Recursion
Recursion (computer science) - Wikipedia
https://en.wikipedia.org/wiki/Recursion_(computer_science)
Most basic examples of recursion, and most of the examples presented here, demonstrate direct recursion, in which a function calls itself. Indirect recursion occurs when a function is called not by itself but by another function that it called (either directly or indirectly).
Recursive functions and algorithms · Recursive data types · Types of recursion

Source: http://bluehawk.monmouth.edu/~rclayton/web-pages/s11-503/recursion.jpg

Recursion/2

- A recursive definition: a concept is defined by referring to itself.

  E.g., arithmetic expressions (like (3 * 7) – (9 / 3)):

  EXPR := VALUE | (EXPR OPERATOR EXPR)

- A recursive procedure: a procedure that calls itself

  Classical example: factorial, that is n! = 1 * 2 * 3 *...* n

  n! = n * (n-1)!

  … or is there something missing?

The Factorial Function

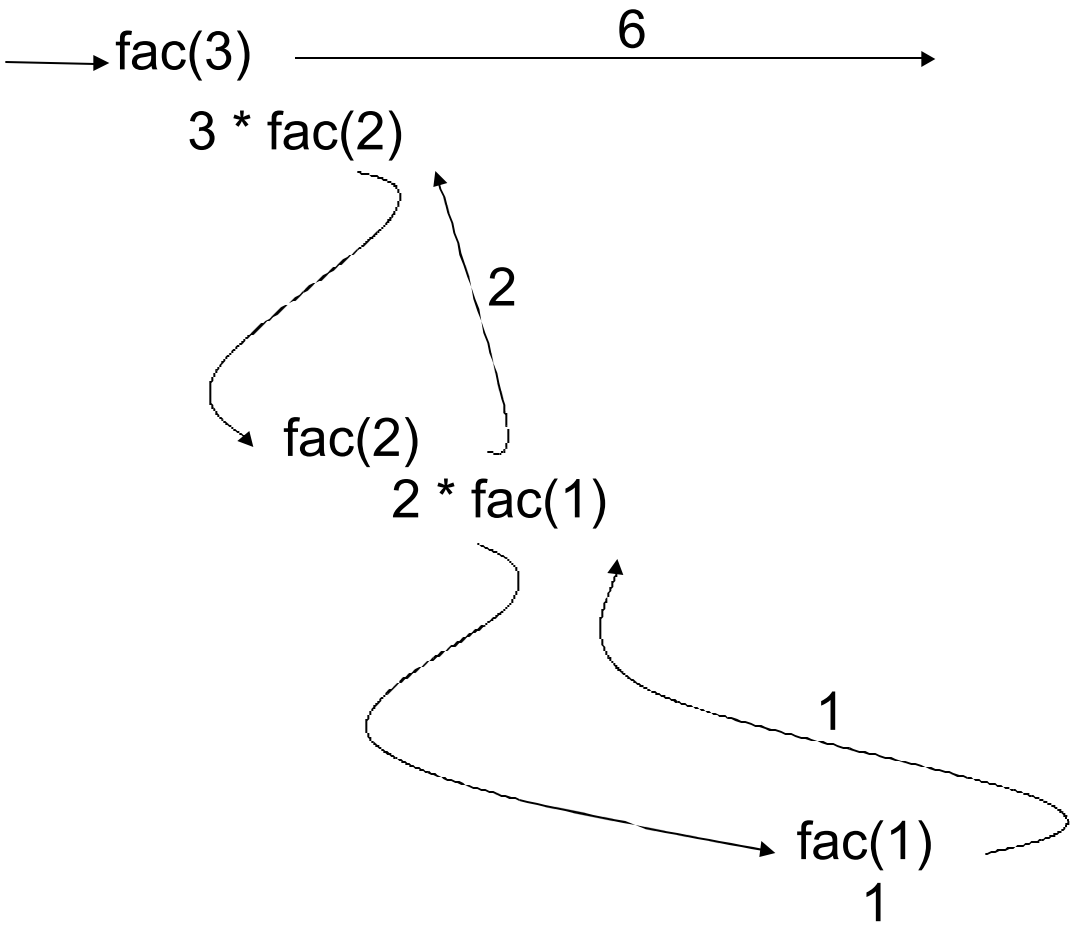
Pseudocode of factorial:

```
fac1
INPUT:    n, a natural number.
OUTPUT:   n! (factorial of n)

int fac1(int n)
  if n < 2 then return 1
  else return n * fac1(n-1)
```

This is a recursive procedure. A recursive procedure has

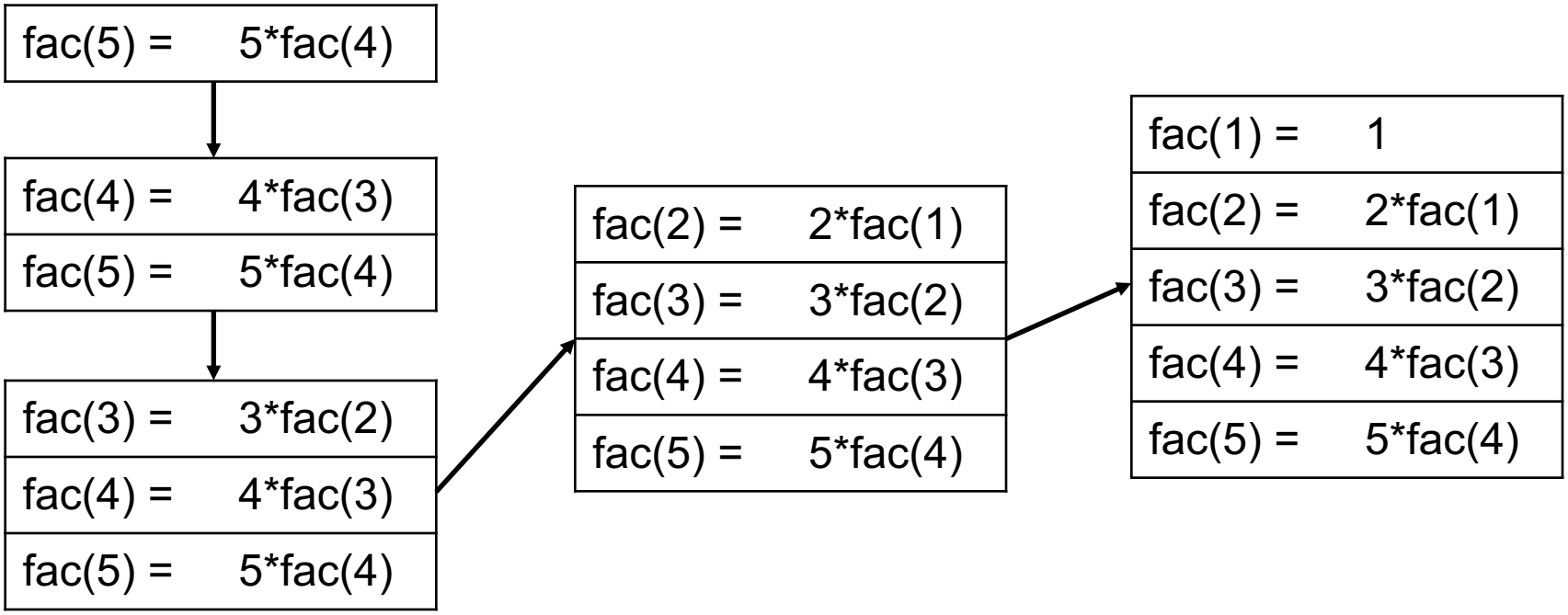
- a termination condition
  (determines when and how to stop the recursion).
- one (or more) recursive calls.

Tracing the Execution



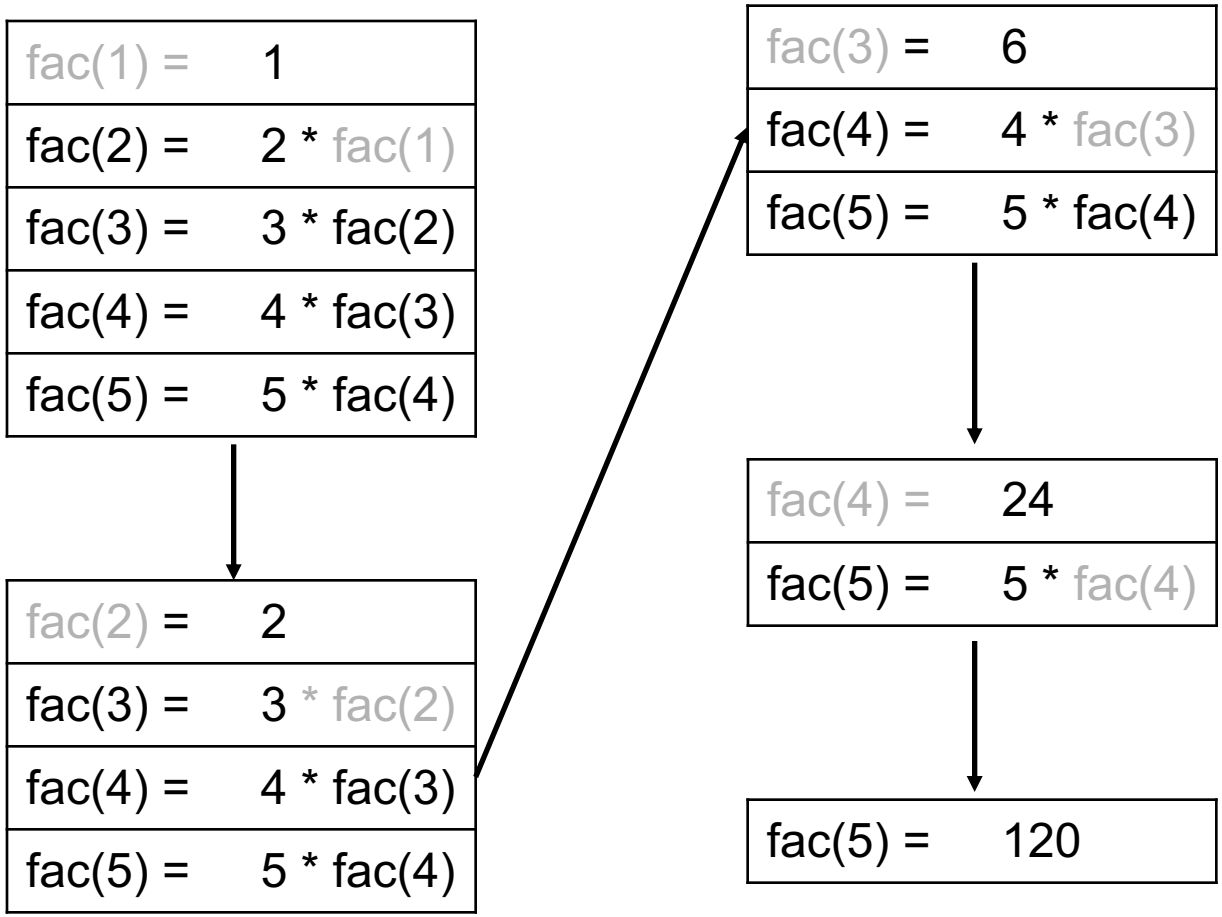
Bookkeeping

The computer maintains an activation stack
for active procedure calls (→ compiler construction).
Example for fac(5). The stack is built up.



Bookkeeping/2

Then the activation stack is reduced



Variants of Factorial

fac2
INPUT: n, a natural number.
OUTPUT: n! (factorial of n)

```
int fac2(int n)
  if n = 0 then return 1
  return n * fac2(n-1)
```

fac3
INPUT: *n, a natural number.*
OUTPUT: *n! (factorial of n)*

```
int fac3(int n)
  if n = 0 then return 1
  return n * (n-1) * fac3(n-2)
```

Analysis of the Variants

`fac2` is correct

- The return statement in the if clause terminates the function and, thus, the entire recursion.

`fac3` is incorrect

- Infinite recursion.
  The termination condition is never reached if n is odd:

```
fact(3)
= 3*2*fact(1)
= 3*2*1*0*fact(-1)
= ...
```

Variants of Factorial/2

fac4
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

```
int fac4(int n)
  if n <= 1 then return 1
  return n*(n-1)*fac4(n-2)
```

fac5
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

```
int fac5(int n)
  return n * fac5(n-1)
  if n <= 1 then return 1
```

Analysis of the Variants/2

`fac4` is correct

- The return statement in the if clause terminates the function and, thus, the entire recursion.

`fac5` is incorrect

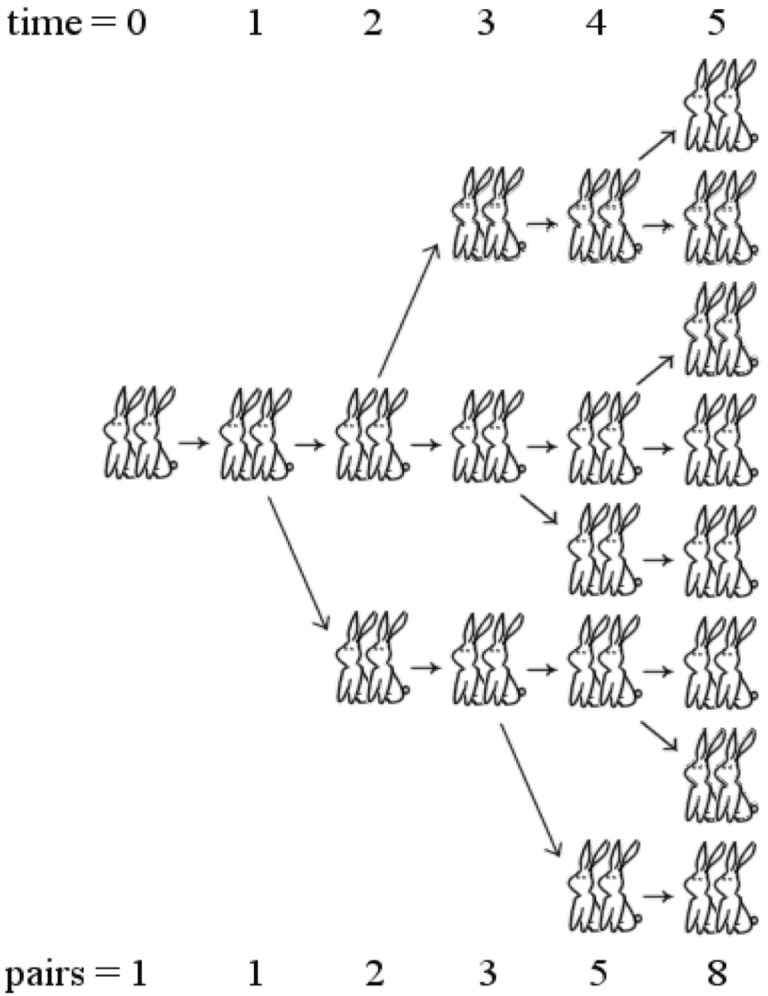
- Infinite recursion.
  The termination condition is never reached.

Counting Rabbits

Someone placed a pair of rabbits
in a certain place,
enclosed on all sides by a wall,
so as to find out
how many pairs of rabbits
will be born there in the course of one year,
it being assumed
that every month
a pair of rabbits produces another pair,
and that rabbits begin to bear
young two months after their own birth.

Leonardo di Pisa ("Fibonacci"),
Liber abacci, 1202

Counting Rabbits/2



Source: http://www.jimloy.com/algebra/fibo.htm

Fibonacci Numbers

Definition

- fib(0) = 1
- fib(1) = 1
- fib(n) = fib(n-1) + fib(n-2), n>1

Numbers in the series:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

Fibonacci Procedure

fib
INPUT: n – a natural number greater or equal than 0.
OUTPUT: fib(n), the nth Fibonacci number.

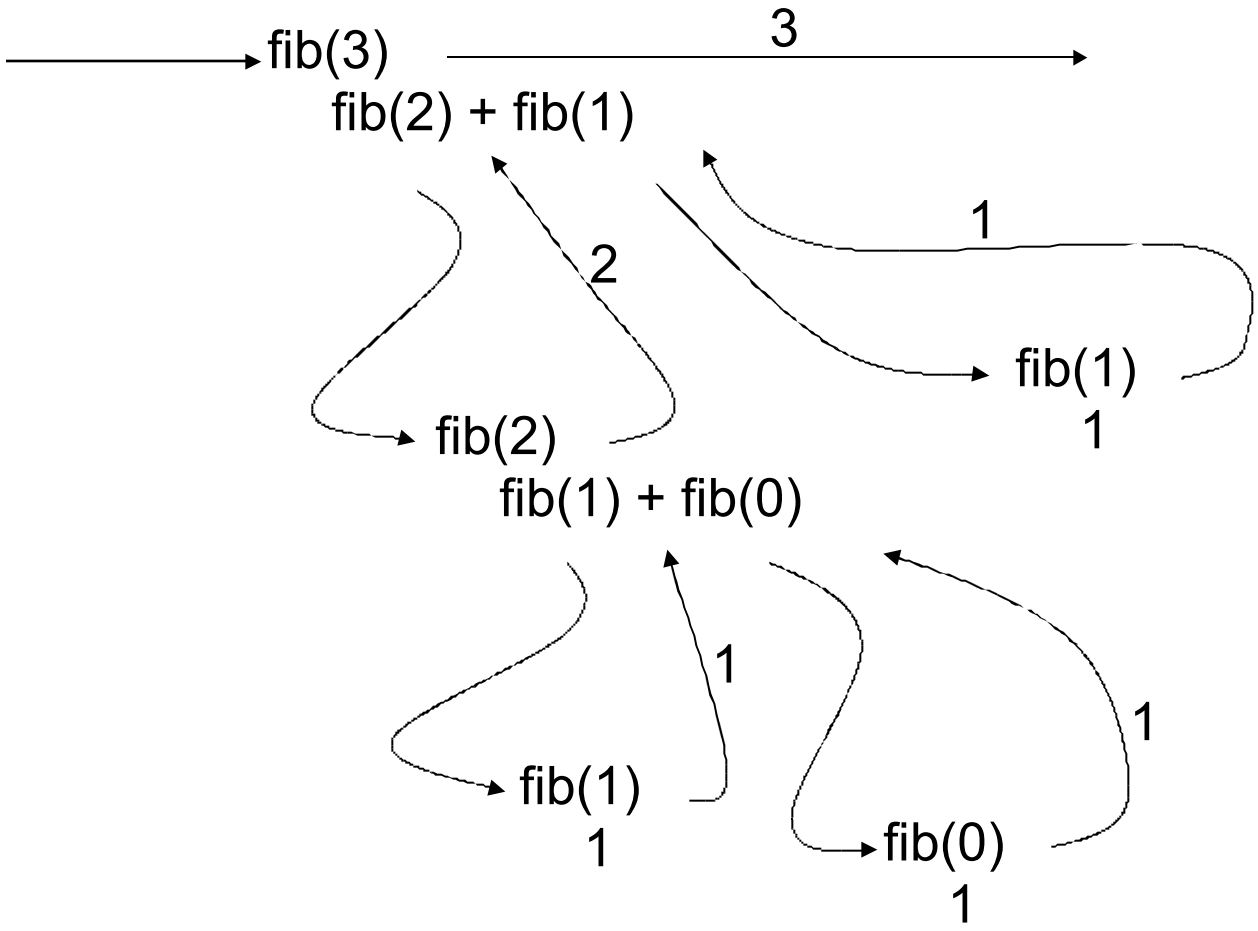
```
int fib(int n)
  if n ≤ 1 then return 1
  else return fib(n-1) + fib(n-2)
```

A procedure with multiple recursive calls

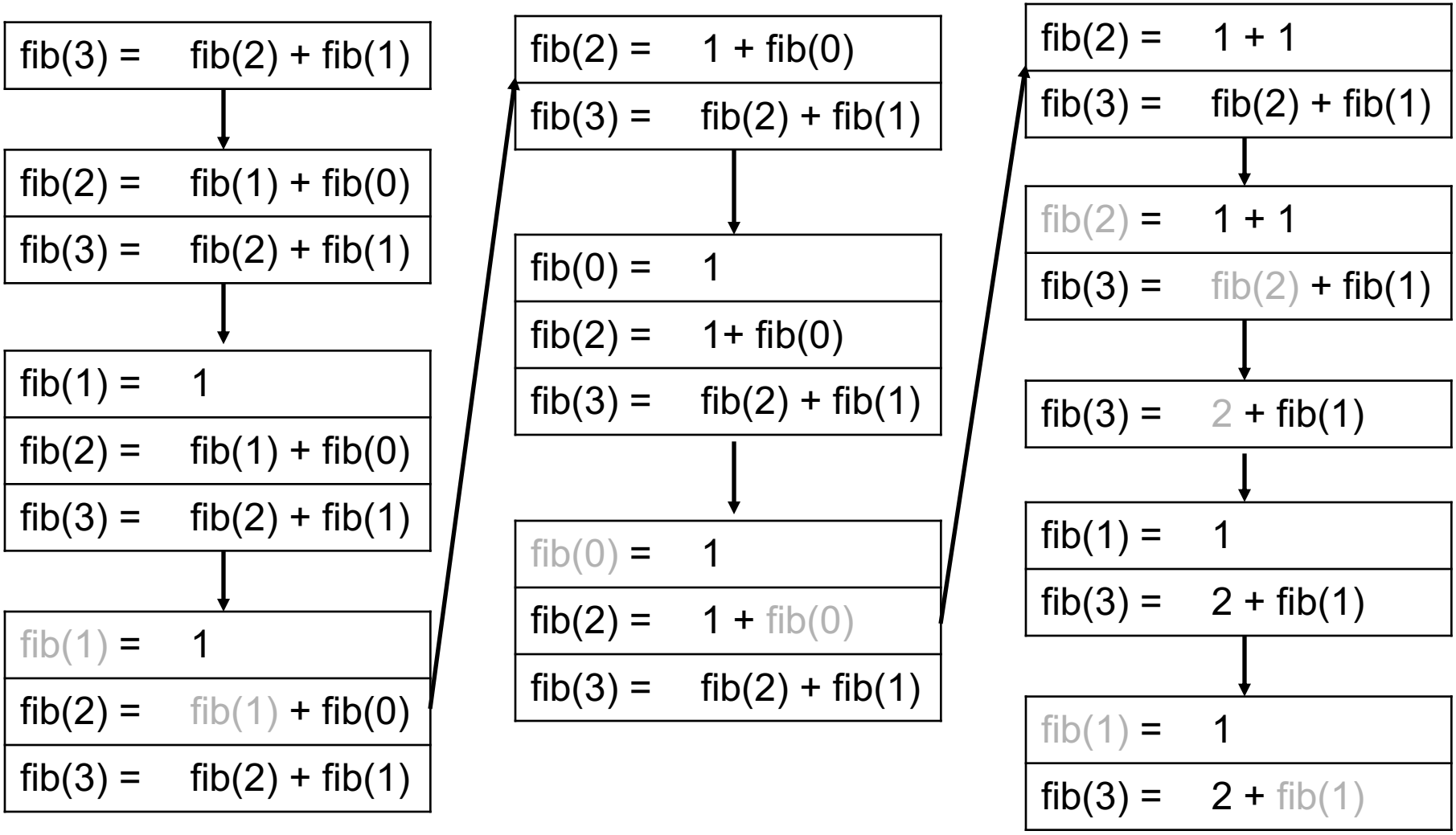
Fibonacci Procedure/2

```
public class fibclassic {

  static int fib(int n) {
    if (n <= 1) {return 1;}
    else {return fib(n-1) + fib(n-2);}
  }

  public static void main(String args[]) {
    System.out.println("Fibonacci of 5 is "
                       + fib(5));
  }
}
```

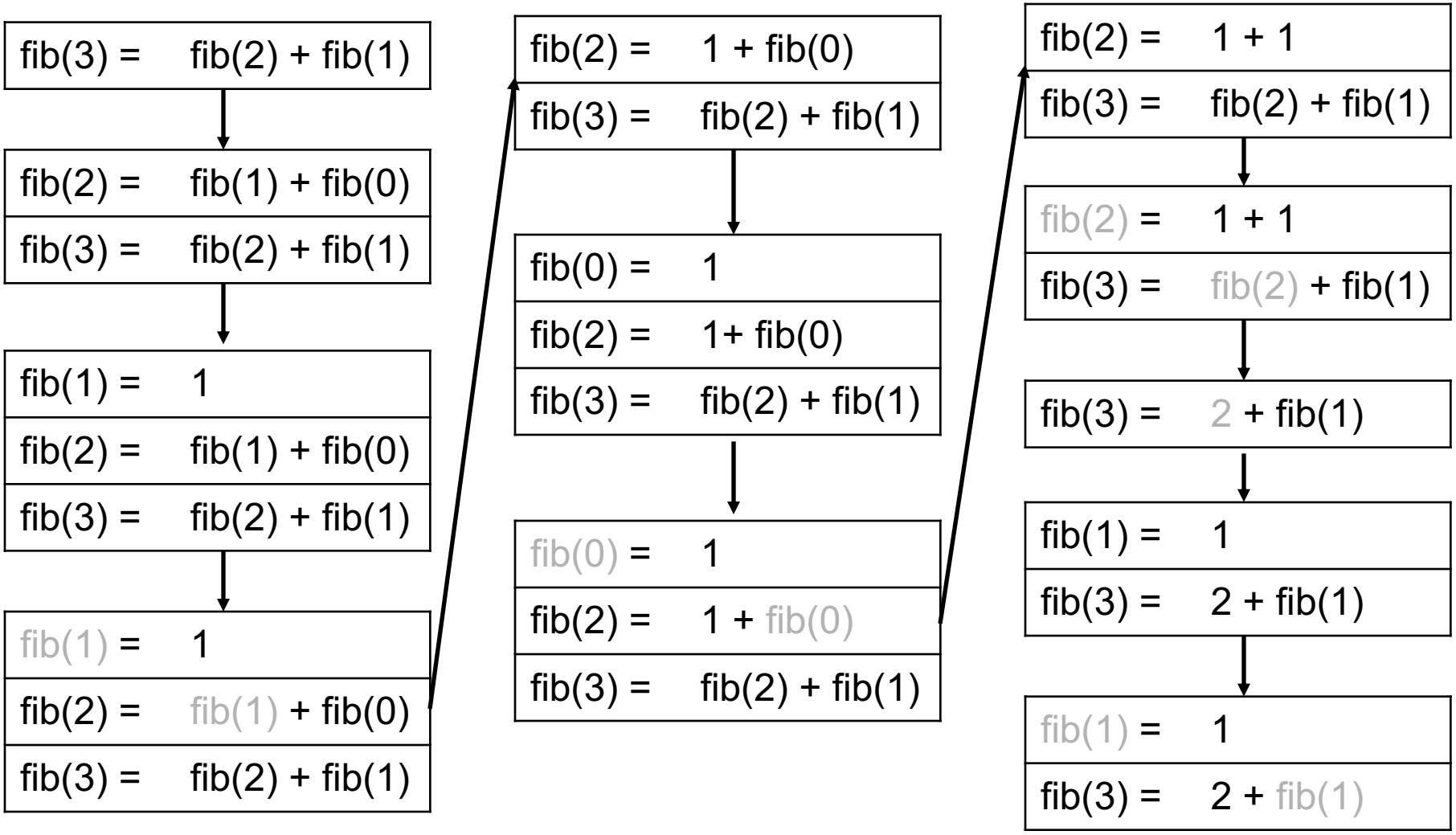
Tracing fib(3)



Bookkeeping



Bookkeeping



Questions

- What is the maximal height of the recursion stack during the computation of *fib(n)*?
- How many recursive calls are made to compute *fib(n)*?
- What does the tree of recursive calls (recursion tree) look like?
- Can we derive a lower bound for the number of calls from that?
- Can there be a procedure for *fib* with fewer operations?
- How is the size of the result *fib(n)* related to the size of the input *n*?

Saving Intermediate Values of fib(n)

```
int fib(int n)
```
INPUT: n – a natural number greater or equal than 0.
OUTPUT: fib(n), the nth Fibonacci number.

```
int fib(int n)
  if n ≤ 1 then return 1
  int[] fibVal = new int[n+1]
     // let's assume in this case that the array
     // boundaries start with 0 :-(
  fibVal[0] := 1; fibVal[1] := 1;
  for j := 2 to n do
     fibVal[j] := fibVal[j-1]+fibVal[j-2]
  return fibVal[n];
```

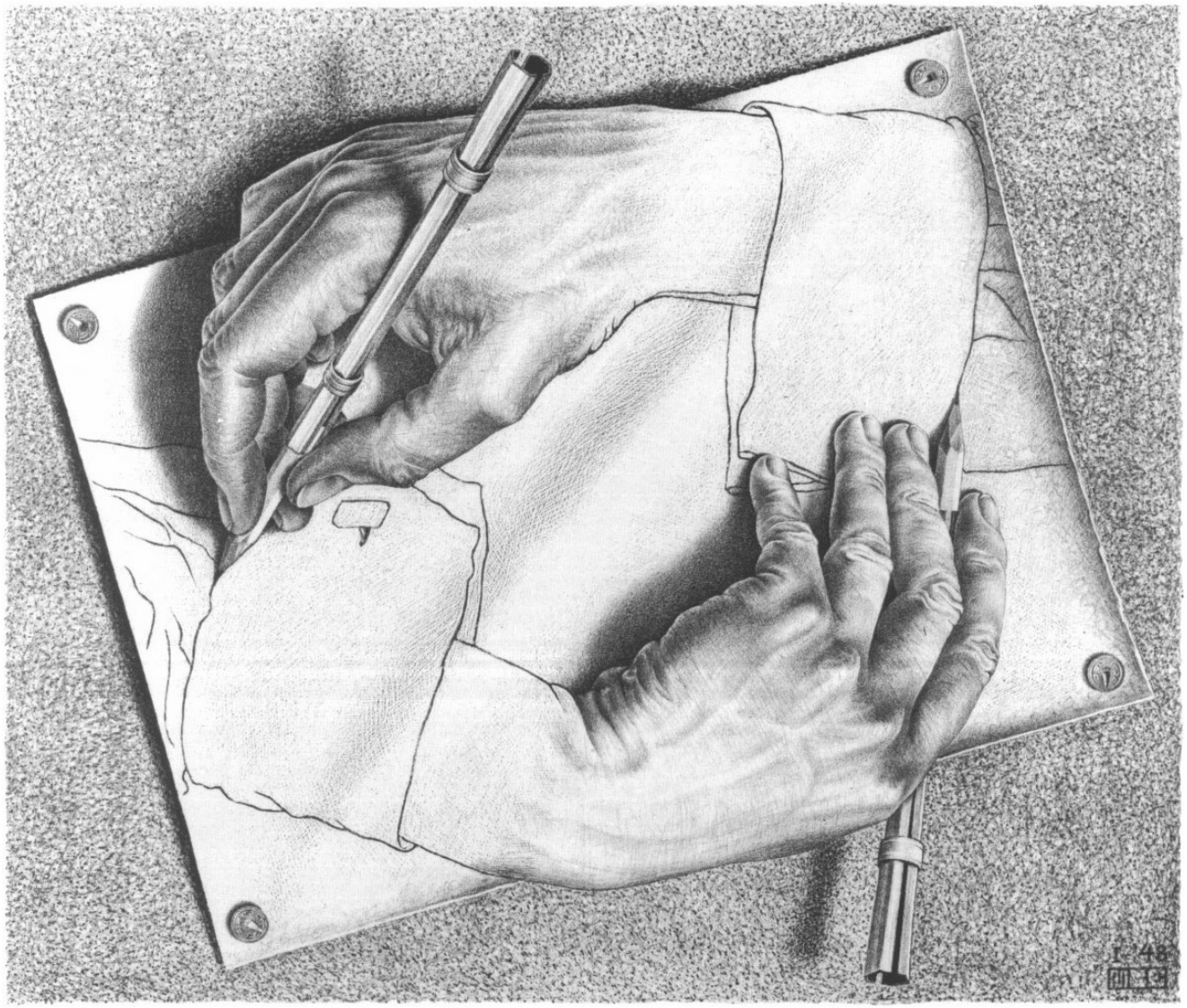
… but we only need the last two values of fib to compute the next one

Iterative Computation of fib(n)

```
int fib(int n)
```
INPUT: *n* – a natural number greater or equal than 0.
OUTPUT: fib(*n*), the *n*th Fibonacci number.

```
Int fib(int n)
  if n ≤ 1 then return 1
  int f, f1, f2;
  fib_j := 1; fib_jMinus1 := 1; fib_j_cached := fib_j

  for j:=2 to n do
            fib_j := fib_j + fib_jMinus1
     fib_jMinus1 := fib_j_cached
    fib_j_cached := fib_j
  return fib_j;
```

Mutual Recursion



Source: http://britton.disted.camosun.bc.ca/escher/drawing_hands.jpg

Mutual Recursion Example

- Problem: Determine whether a natural number is even
- Definition of *even*:
  - 0 is even
  - n is even if n - 1 is odd
  - n is odd if n - 1 is even

Implementation of even

```
even
INPUT:    n – a natural number.
OUTPUT:   true if n is even; false otherwise

boolean even(int n)
  if n = 0 then return TRUE
  else return odd(n-1)

boolean odd(int n)
  if n = 0 then return FALSE
  return even(n-1)
```

How can we determine whether n is odd?

Is Recursion Necessary?

- Theory: You can always resort to iteration and explicitly maintain a recursion stack.
- Practice: Recursion is elegant and in some cases the best solution by far.
- In the previous examples recursion was never appropriate since there exist simple iterative solutions.
- Recursion is more expensive than corresponding iterative solutions since bookkeeping is necessary.