Data Structures and Algorithms

Chapter 1

Werner Nutt

Acknowledgments

- The course follows the book "Introduction to Algorithms '", by **Cormen, Leiserson, Rivest and Stein**, MIT Press [CLRST]. Many examples displayed in these slides are taken from their book.
- These slides are based on those developed by Michael Böhlen for this course.

(See http://www.inf.unibz.it/dis/teaching/DSA/)

- The slides also include a number of additions made by Roberto Sebastiani and Kurt Ranalter when they taught later editions of this course

(See http://disi.unitn.it/~rseba/DIDATTICA/dsa2011_BZ//)

DSA, Chapter 1: Overview

- Introduction, syllabus, organisation
- Algorithms
- Recursion (principle, trace, factorial, Fibonacci)
- Sorting (bubble, insertion, selection)

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Learning Outcomes

The main things we will learn in this course:

- To *think algorithmically* and
  get the spirit of how algorithms are designed
- To get to know a *toolbox* of *classical* algorithms
- To learn a number of algorithm design *techniques*
  (such as divide-and-conquer)
- To analyze (in a precise and formal way)
  the *efficiency* and the *correctness* of algorithms.

Syllabus

1. Introduction, recursion (chap 1 in CLRS)
2. Correctness and complexity of algorithms (2, 3)
3. Divide and conquer, recurrences (4)
4. Heapsort, Quicksort (1)
5. Dynamic data structures, abstract data types (10)
6. Binary search trees, red-black trees (12, 13)
7. Hash tables (11)
8. Dynamic programming (15)
9. Graphs: Principles and graph traversal (22)
10. Minimum spanning tree and shortest path (23, 24)

Literature

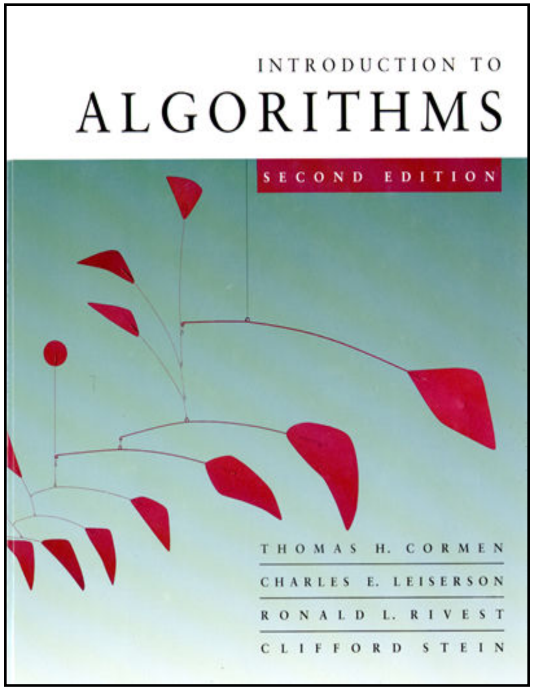
Cormen, Leiserson, Rivest and Stein (CLRS), *Introduction to Algorithms*, Second Edition, MIT Press and McGraw-Hill, 2001

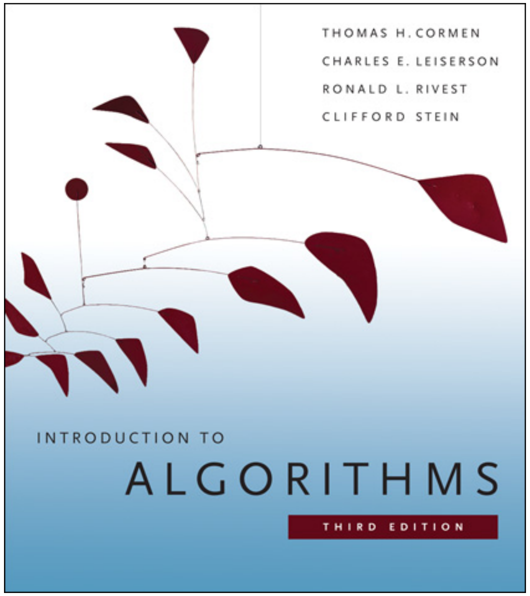
and

Third Edition, MIT Press, 2009

(See http://mitpress.mit.edu/algorithms/)

Course is based on this book





Other Literature

Kurt Mehlhorn and Peter Sanders
Algorithms and Data Structures - The Basic Toolbox

Offers alternate presentation of topics of the course

Free download from

http://www.mpi-inf.mpg.de/~mehlhorn/ftp/Mehlhorn-Sanders-Toolbox.pdf

Course Organization

- Lectures: Wed 10:30-12:30, Fri 10:30-12:30
- Labs (provisional, starting next week)
  - Mouna Kacimi, Mouna.Kacimi@unibz.it
    Tue 14:00-16:00
  - Camilo Thorne, cthorne@inf.unibz.it
    Tue 14:00-16:00
- Home page:
  http://www.inf.unibz.it/~nutt/Teaching/DSA1314/

Assignments

The assignments are a crucial part of the course

- Each week an assignment has to be solved
- The schedule for the publication and the handing in of the assignments will be announced at the next lecture.
- A number of assignments include programming tasks. It is strongly recommended that you implement and run all programming exercises.
- Assignments will be marked. The assignment mark will count towards the course mark.
- Any attempt at plagiarism (copying from the web or copying from other students) leads to a 0 mark for all assignments.

Assignments, Midterm Exam, Final Exam, and Course Mark

- There will be
  - one written exam at the end of the course
  - one midterm exam around the middle of the course
  - assignments
- To pass the course, one has to pass the written exam.
- Students who do not submit exercises or do not take part in the midterm (or fail the midterm) will be marked on the final exam alone.
- For students who submit all assignments, and take part in the midterm, the final mark will be a weighted average

  50% exam mark + 10% midterm
  \+ 40% assignment mark

Assignments, Midterm Exam, Final Exam, and Course Mark

- If students submit fewer assignments, or do not take part in the midterm, the percentage will be lower.
- Assignments for which the mark is lower than the mark of the written exam will not be considered.
- Similarly, the midterm will not be considered if the mark is lower than the mark of the final exam.
- The midterm and assignment marks apply to three exam sessions.

General Remarks

- Algorithms are first designed on paper … and later keyed in on the computer.
- The most important thing is to be simple and precise .
- During lectures:
  - Interaction is welcome; ask questions (I will ask you anyway ☺)
  - Additional explanations and examples if desired
  - Speed up/slow down the progress

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What are Algorithms About?

Solving problems in everyday life

- Travel from Bolzano to Munich
- Cook Spaghetti alla Bolognese *(I know, not in Italy,…)*
- Register for a Bachelor thesis at FUB

For all these problems, there are

- instructions
- recipes
- procedures,

which describe a complex operation in terms of

- elementary operations *("beat well …")*
- control structures and conditions *("… until fluffy")*

Algorithms

Problems involving numbers, strings, mathematical objects:

- for two numbers, determine their sum, product, …
- for two numbers, find their greatest common divisor
- for a sequence of strings,
  find an alphabetically sorted permutation of the sequence
- for two arithmetic expressions, find out if they are equivalent
- for a program in Java,
  find an equivalent program in byte code
- on a map, find for a given house the closest bus stop

We call instructions, recipes, for such problems *algorithms*

What have algorithms in common with recipes?
How are they different?

History

- *First algorithm:* Euclidean Algorithm, greatest common divisor, 400-300 B.C.
- *Name:* Persian mathematician Mohammed al-Khowarizmi, in Latin became “Algorismus”

  كتاب الجمع و التفريق بحساب الهند

  Kitāb al-Dschamʿ wa-l-tafrīq bi-ḥisāb al-Hind =

  = Book on connecting and taking apart in the calculation of India
- *19th century*
  - Charles Babbage: Difference and Analytical Engine
  - Ada Lovelace: Program for Bernoulli numbers
- *20th century*
  - Alan Turing, Alonzo Church: formal models computation
  - John von Neumann: architecture of modern computers

Data Structures, Algorithms, and Programs

- Data structure
  - Organization of data to solve the problem at hand
- Algorithm
  - Outline, the essence of a computational procedure, step-by-step instructions
- Program
  - implementation of an algorithm in some programming language

Overall Picture

Using a computer to help solve problems:

- Precisely specifying the problem
- Designing programs
  - architecture
  - algorithms
- Writing programs
- Verifying (testing) programs

Data Structure and Algorithm Design Goals

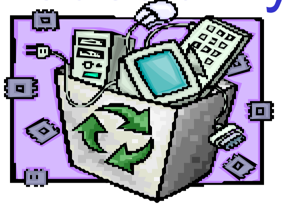
Correctness

Efficiency

Implementation Goals

Robustness

Reusability



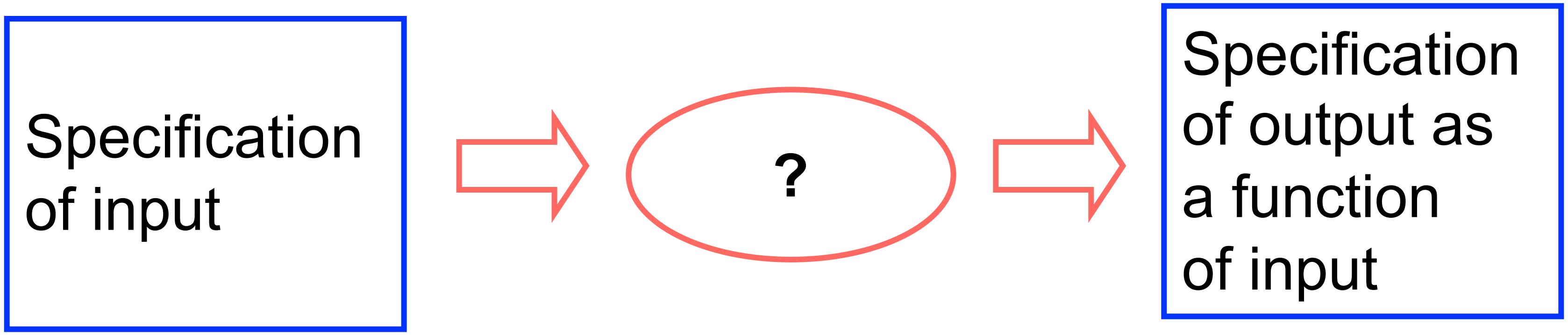
Adaptability

This course is not about:

- Programming languages
- Computer architecture
- Software architecture
- SW design and implementation principles

We will only touch upon
the theory of complexity
and computability.

Algorithmic Problem



Specification of input

?

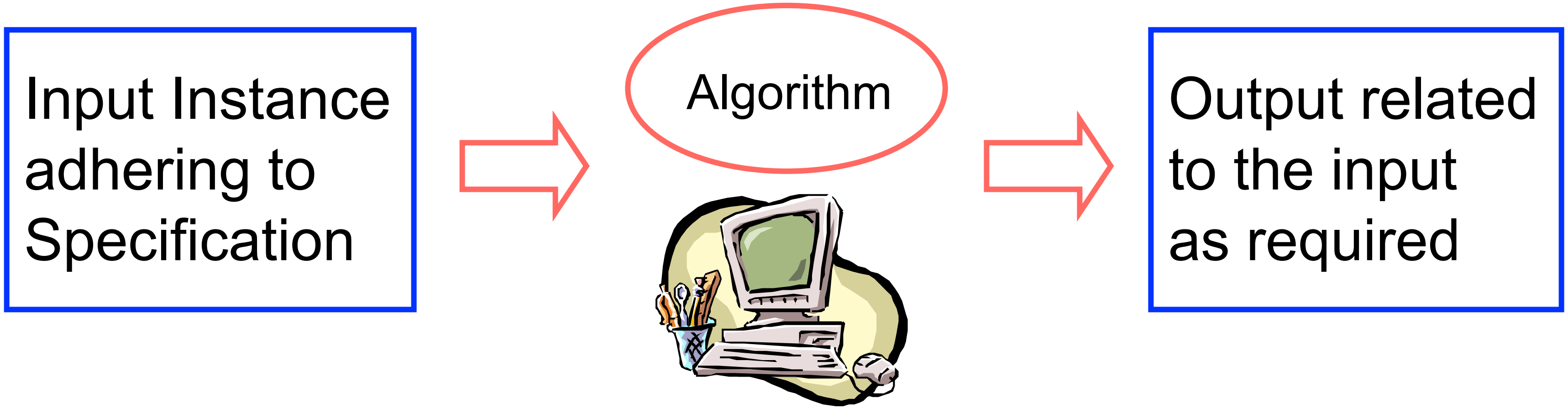
Specification of output as a function of input

There is an infinite number of possible input *instances* satisfying the specification.

For example: A sorted, non-decreasing sequence of natural numbers, on nonzero, finite length:

1, 20, 908, 909, 100000, 1000000000.

Algorithmic Solution



Input Instance adhering to Specification

Algorithm

Output related to the input as required

- Algorithm describes actions on the input instance
- There may be many correct algorithms for the same algorithmic problem.

Definition

An algorithm is a sequence of *unambiguous* instructions for solving a problem, i.e.,

- for obtaining a *required output*
- for any *legitimate input*

in a finite amount of time.

➔ This presumes a mechanism to execute the algorithm

Properties of algorithms:

- Correctness, Termination, (Non-)Determinism,
  Run Time, …

How to Develop an Algorithm

- Precisely define the problem.
  Precisely specify the input and output.
  Consider all cases.

- Come up with a simple plan
  to solve the problem at hand.
  - The plan is independent of a (programming) language
  - The precise problem specification influences the plan.

- Turn the plan into an implementation
  - The problem representation (data structure) influences the implementation

Example 1: Searching

INPUT

- A - (un)sorted sequence of *n* (*n* > 0) numbers
- q - a single number

$a_1, a_2, a_3, \ldots, a_n;\ q$

$\longrightarrow$

2 5 6 10 11; 5

2 5 6 10 11; 9

OUTPUT

- index of number q in sequence A, or *NIL*

j

$\longrightarrow$

2

NIL

Searching/2, search1

search1
INPUT: A[1..n] (un)sorted array of integers, *q* an integer.
OUTPUT: index *j* such that A[*j*]=*q* or *NIL* if A[*j*] ≠ *q* for all *j* ($1 \le j \le n$)

```
j := 1
while j ≤ n and A[j] ≠ q do j++
if j ≤ n then return j
else return NIL
```

- The code is written in *pseudo-code* and INPUT and OUTPUT of the algorithm are specified.
- The algorithm uses a *brute-force* technique, i.e., scans the input sequentially.

Preconditions, Postconditions

Specify preconditions and postconditions of algorithms:

Precondition:

- what does the algorithm get as input?

Postcondition:

- what does the algorithm produce as output?
- … how does this relate to the input?

Make sure you have considered the special cases:

- empty set, number 0, pointer nil, …

Pseudo-code

Like Java, Pascal, C, or any other imperative language

- Control structures:

  (if then else , while, and for loops)

- Assignment: :=
- Array element access: A[i]
- Access to element of composite type (record or object):

  A.b

CLRS uses b[A]

Searching, Java Solution

```
import  java.io.*;

class search {
  static final int n = 5;
  static int j, q;
  static int a[] = { 11, 1, 4, -3, 22 };

  public static void main(String args[]) {
    j = 0; q = 22;
    while (j < n && a[j] != q) { j++; }
    if (j < n) { System.out.println(j); }
    else { System.out.println("NIL"); }
  }
}
```

Searching, C Solution

```
#include <stdio.h>
#define n 5


int j, q;
int a[n] = { 11, 1, 4, -3, 22 };
int main() {
  j = 0;  q = -2;
  while (j < n && a[j] != q) { j++; }
  if (j < n) { printf("%d\n", j); }
  else { printf("NIL\n"); }
}

// compilation: gcc -o search search.c
// execution: ./search
```

Searching/3, search2

Another idea:

Run through the array
and set a pointer if the value is found.

search2
INPUT: A[1..n] (un)sorted array of integers, *q* an integer.
OUTPUT: index *j* such that A[*j*]=*q* or *NIL* if A[*j*] ≠ *q* for all *j* ($1 \le j \le n$)

```
ptr := NIL;
for j := 1 to n do
   if a[j] = q then ptr := j
return ptr;
```

Does it work?

search1 vs search2

Are the solutions equivalent?

- No!

Can one construct an example such that, say,

- search1 returns 3
- search2 returns 7 ?

But both solutions satisfy the specification (or don’t they?)

Searching/4, search3

An third idea:

Run through the array and
return the index of the value in the array.

search3
INPUT: A[1..n] (un)sorted array of integers, *q* an integer.
OUTPUT: index *j* such that A[*j*]=*q* or *NIL* if A[*j*] ≠ *q* for all *j* ($1 \leq j \leq n$)

```
for j := 1 to n do
  if a[j] = q then return j
return NIL
```

Comparison of Solutions

Metaphor: shopping behavior when buying a beer:

- **search1**: scan products;
  stop as soon as a beer is found and go to the exit.
- **search2**: scan products until you get to the exit;
  if during the process you find a beer,
  put it into the basket
  (instead of the previous one, if any).
- **search3**: scan products;
  stop as soon as a beer is found
  and exit through next window.

Comparison of Solutions/2

- search1 and search3 return *the same result* (index of the first occurrence of the search value)
- search2 returns the index of the last occurrence of the search value
- search3 does not finish the loop
  (as a general rule, you better avoid this)

Beware: Array Indexes in Java/C/C++

- In pseudo-code, array indexes range from 1 to length
- In Java/C/C++, array indexes range from 0 to length-1
- Examples:
  - Pseudo-code

```
for j := 1 to n do
```

    Java:

```
for (j=0; j < a.length; j++) { …
```

  - Pseudo-code

```
for j := n to 2 do
```

    Java:

```
for (j=a.length-1; j >= 1; j--) { …
```

Suggested Exercises

- Implement the three variants of search (with input and output of arrays)
  - Create random arrays for different lengths
  - Compare the results
  - Add a counter for the number of cycles and return it, compare the result
- Implement them to scan the array in reverse order

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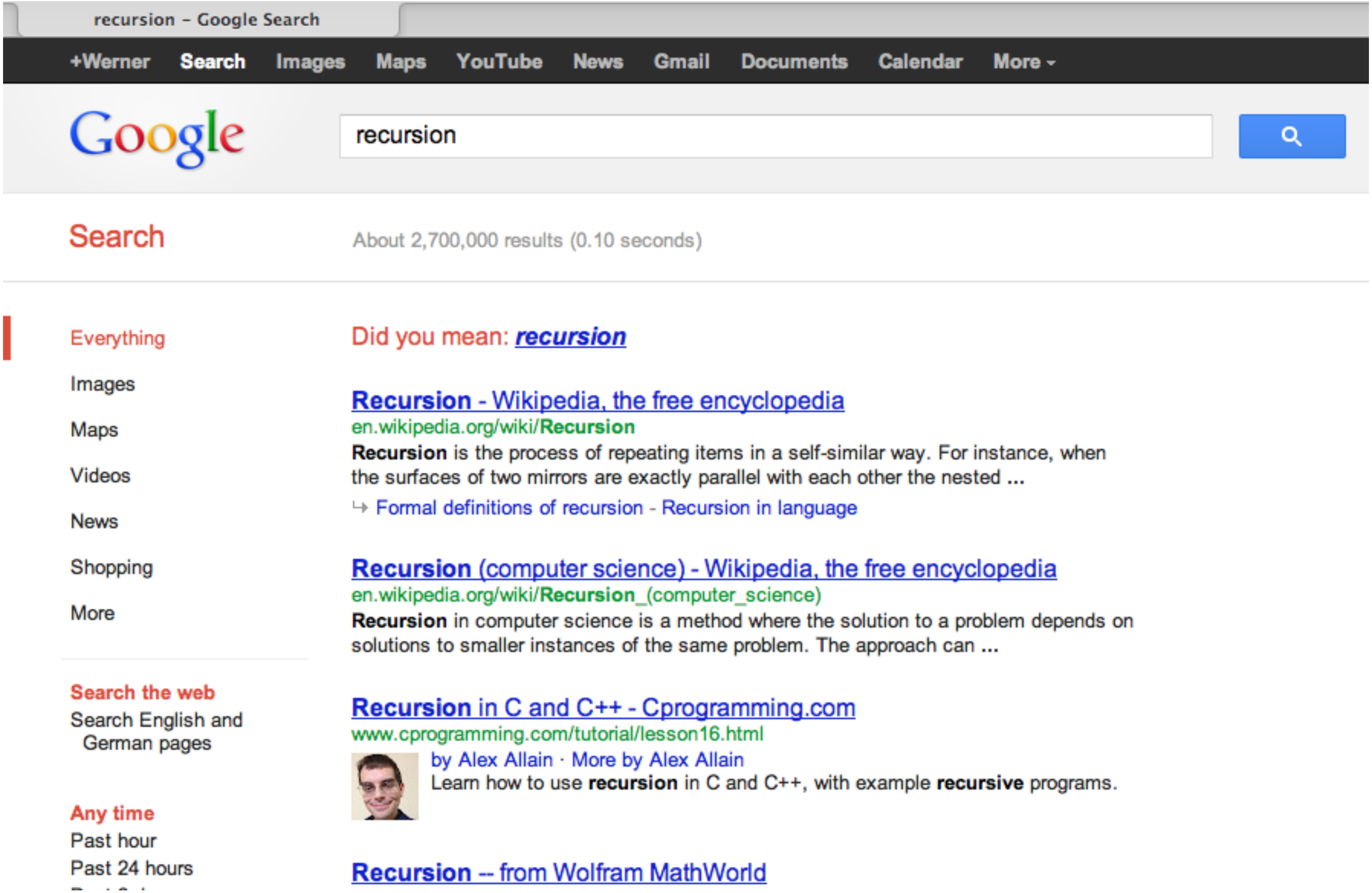
Recursion

An object is recursive if

- a part of the object refers to the entire object, or
- one part refers to another part and vice versa

(mutual recursion)



About 2,700,000 results (0.10 seconds)

Did you mean: ***recursion***

Recursion - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Recursion**
Recursion is the process of repeating items in a self-similar way. For instance, when the surfaces of two mirrors are exactly parallel with each other the nested ...
↳ Formal definitions of recursion - Recursion in language

Recursion (computer science) - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/**Recursion**_(computer_science)
Recursion in computer science is a method where the solution to a problem depends on solutions to smaller instances of the same problem. The approach can ...

Recursion in C and C++ - Cprogramming.com
www.cprogramming.com/tutorial/lesson16.html
by Alex Allain · More by Alex Allain
Learn how to use **recursion** in C and C++, with example **recursive** programs.

Recursion -- from Wolfram MathWorld

Source: http://bluehawk.monmouth.edu/~rclayton/web-pages/s11-503/recursion.jpg

Recursion/2

- A recursive definition: a concept is defined by referring to itself.

  E.g., arithmetical expressions (like (3 * 7) - (9 / 3)):

  EXPR := VALUE | (EXPR OPERATOR EXPR)

- A recursive procedure: a procedure that calls itself

  Classical example: factorial, that is n! = 1 * 2 * 3 *...* n

  n! = n * (n-1)!

  … or is there something missing?

The Factorial Function

Pseudocode of factorial:

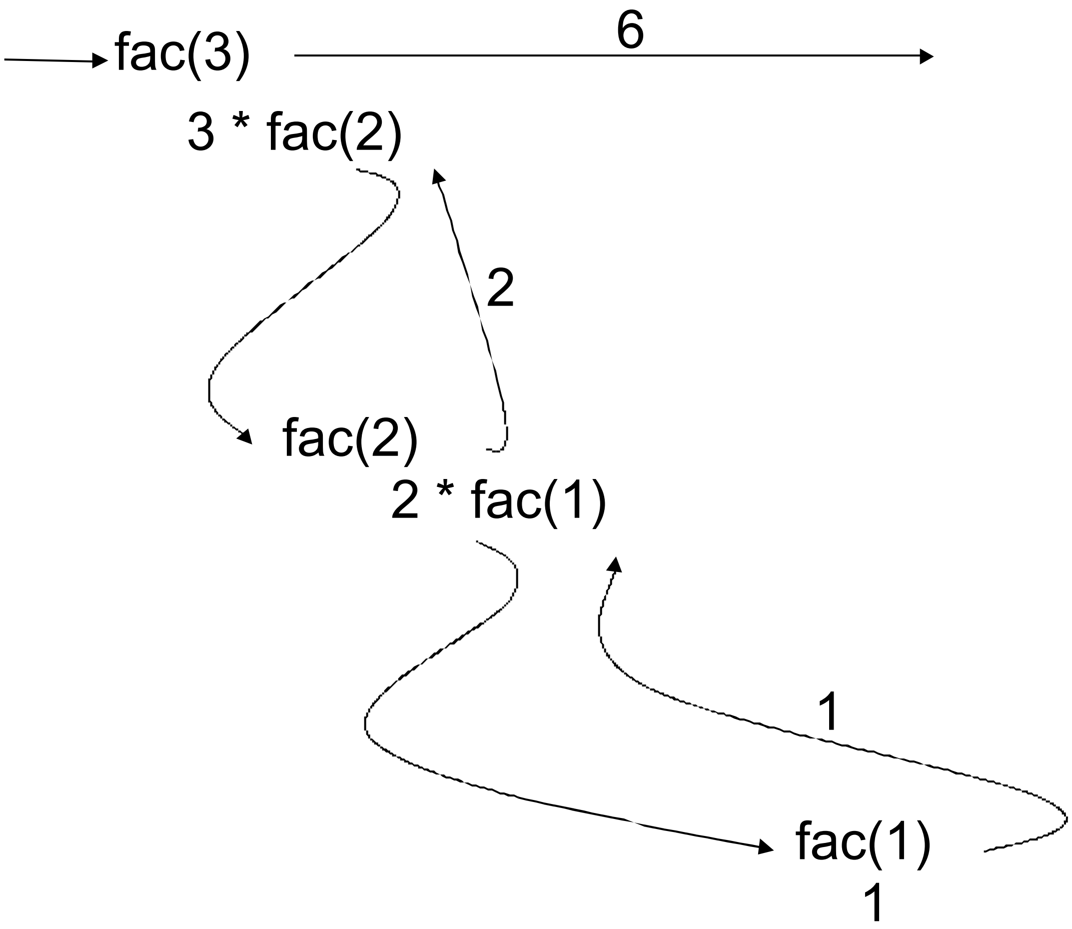
```
fac1
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac1(n)
  if n < 2 then return 1
  else return n * fac1(n-1)
```

This is a recursive procedure. A recursive procedure has

- a termination condition
  (determines when and how to stop the recursion).
- one (or more) recursive calls.

Tracing the Execution



fac(3)

6

3 * fac(2)

2

fac(2)

2 * fac(1)

1

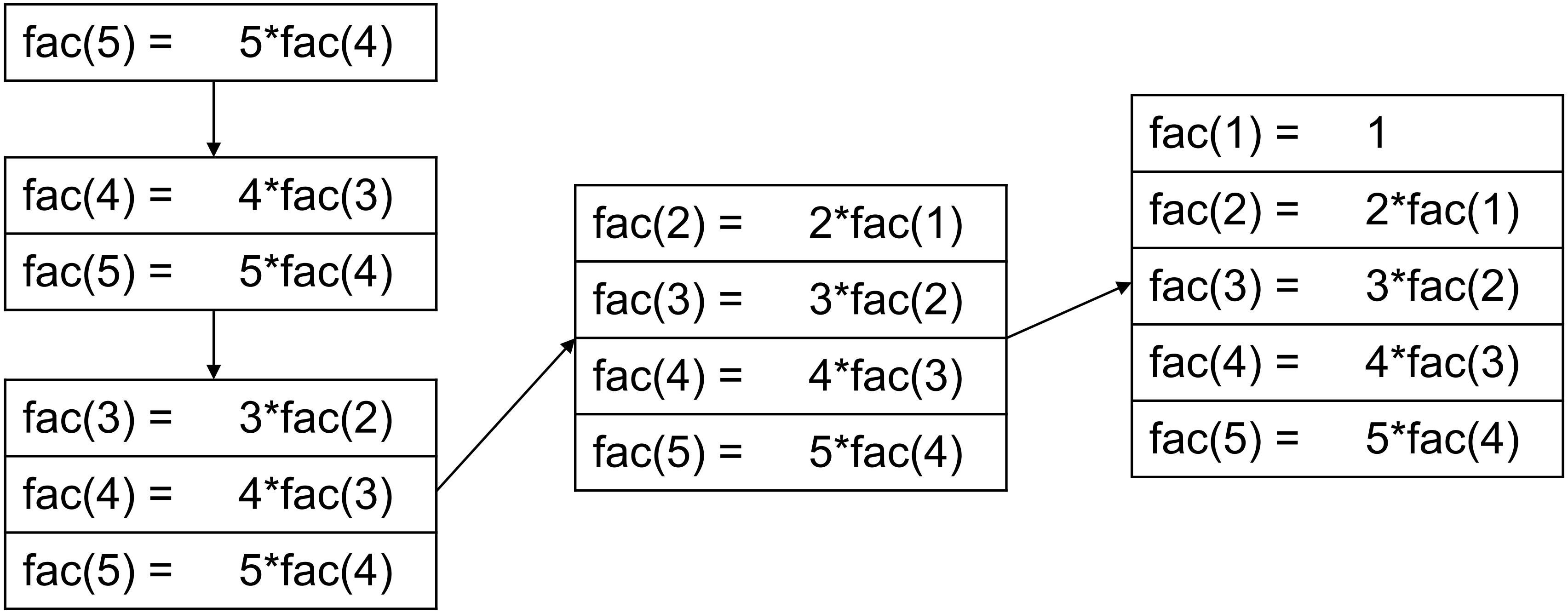
fac(1)

1

Bookkeeping

The computer maintains an activation stack for active procedure calls (→ compiler construction).

Example for fac(5). The stack is built up.



fac(5) = 5*fac(4)

fac(4) = 4*fac(3)
fac(5) = 5*fac(4)

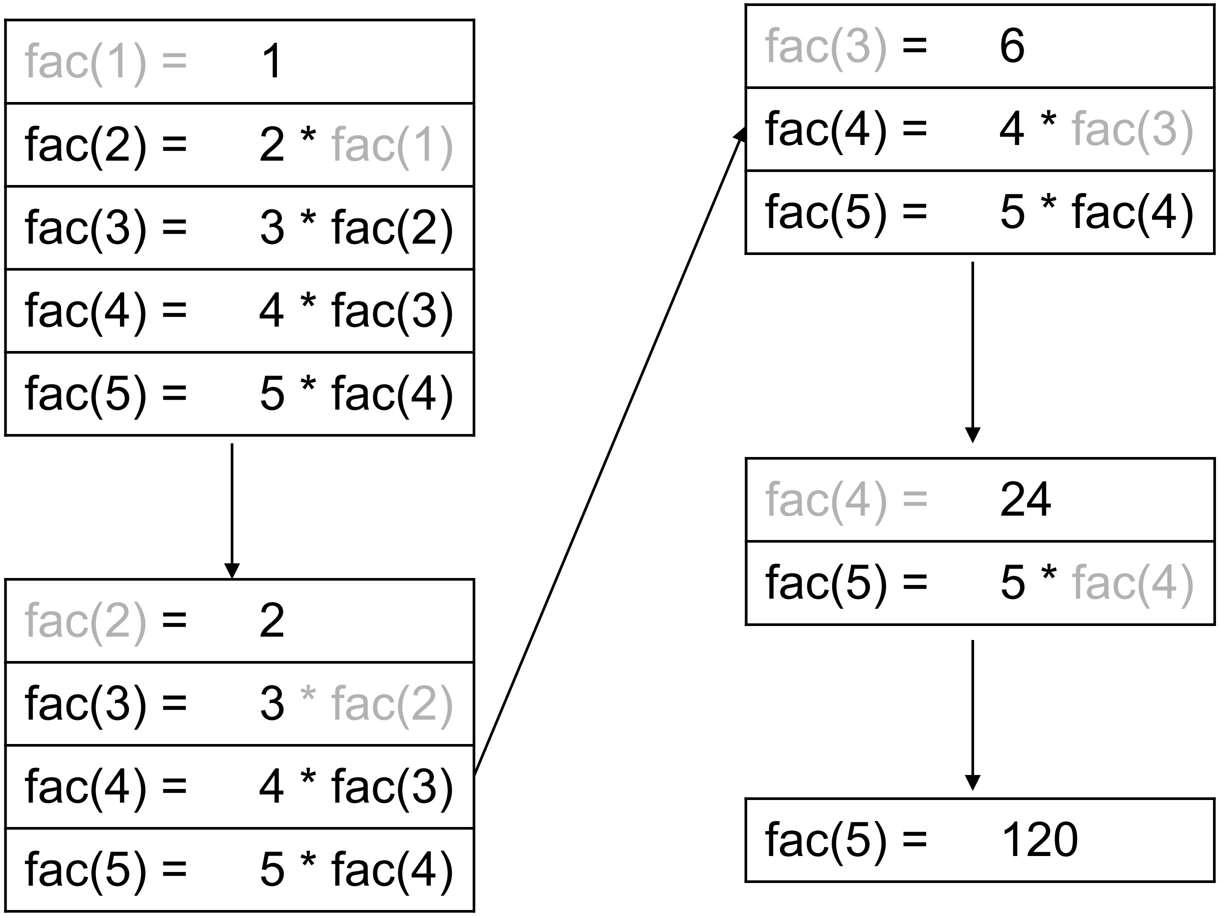
fac(3) = 3*fac(2)
fac(4) = 4*fac(3)
fac(5) = 5*fac(4)

fac(2) = 2*fac(1)
fac(3) = 3*fac(2)
fac(4) = 4*fac(3)
fac(5) = 5*fac(4)

fac(1) = 1
fac(2) = 2*fac(1)
fac(3) = 3*fac(2)
fac(4) = 4*fac(3)
fac(5) = 5*fac(4)

Bookkeeping/2

Then the activation stack is reduced



fac(1) = 1
fac(2) = 2 * fac(1)
fac(3) = 3 * fac(2)
fac(4) = 4 * fac(3)
fac(5) = 5 * fac(4)

fac(2) = 2
fac(3) = 3 * fac(2)
fac(4) = 4 * fac(3)
fac(5) = 5 * fac(4)

fac(3) = 6
fac(4) = 4 * fac(3)
fac(5) = 5 * fac(4)

fac(4) = 24
fac(5) = 5 * fac(4)

fac(5) = 120

Variants of Factorial

```
fac2
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac2(n)
  if n = 0 then return 1
  return n * fac2(n-1)
```

```
fac3
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac3(n)
  if n = 0 then return 1
  return n * (n-1) * fac3(n-2)
```

Analysis of the Variants

`fac2` is correct

- The return statement in the if clause terminates the function and, thus, the entire recursion.

`fac3` is incorrect

- Infinite recursion.
  The termination condition is never reached if n is odd:

```
fact(3)
= 3*2*fact(1)
= 3*2*1*0*fact(-1)
= ...
```

Variants of Factorial/2

```
fac4
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac4(n)
   if n <= 1 then return 1
   return n*(n-1)*fac4(n-2)
```

```
fac5
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac5(n)
   return n * fac5(n-1)
   if <= 1 then return 1
```

Analysis of the Variants/2

`fac4` is correct

- The return statement in the if clause terminates the function and, thus, the entire recursion.

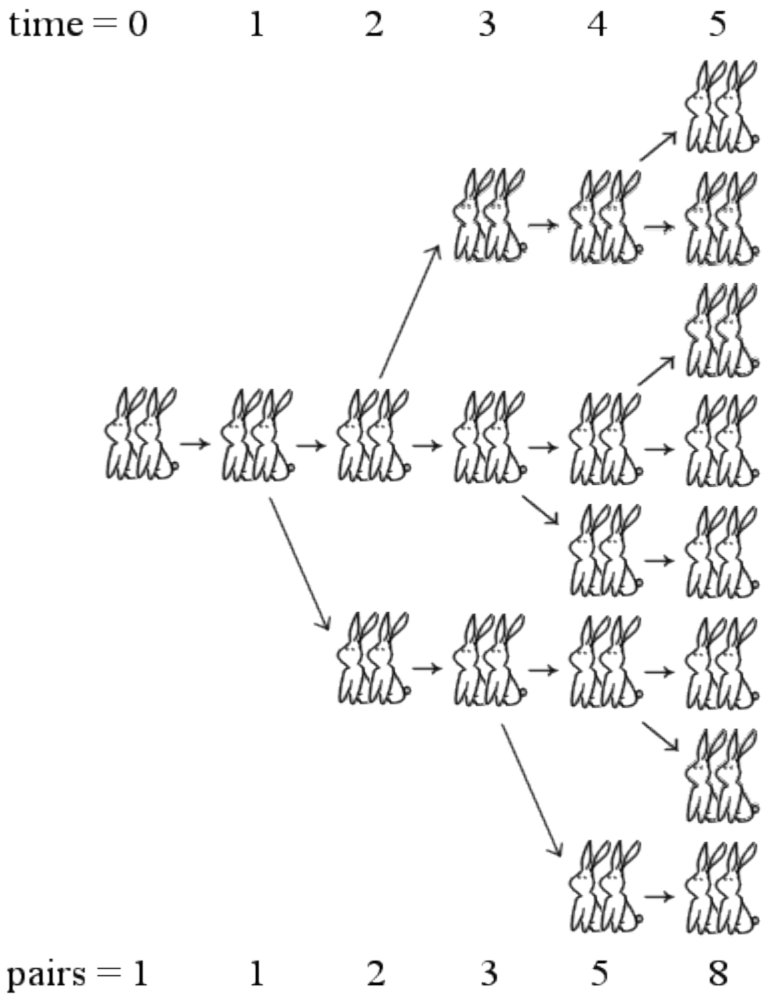
`fac5` is incorrect

- Infinite recursion.
  The termination condition is never reached.

Counting Rabbits

> *Someone placed a pair of rabbits*
> *in a certain place,*
> *enclosed on all sides by a wall,*
> *so as to find out*
> *how many pairs of rabbits*
> *will be born there in the course of one year,*
> *it being assumed*
> *that every month*
> *a pair of rabbits produces another pair,*
> *and that rabbits begin to bear*
> *young two months after their own birth.*
>
> *Leonardo di Pisa ("Fibonacci"),*
> *Liber abacci, 1202*

Counting Rabbits/2



Source: http://www.jimloy.com/algebra/fibo.htm

Fibonacci Numbers

Definition

- fib(1) = 1
- fib(2) = 1
- fib(n) = fib(n-1) + fib(n-2), n>2

Numbers in the series:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

Fibonacci Procedure

fib
INPUT: n – a natural number larger than 0.
OUTPUT: fib(n), the nth Fibonacci number.

```
fib(n)
  if n ≤ 2 then return 1
  else return fib(n-1) + fib(n-2)
```

A procedure with multiple recursive calls

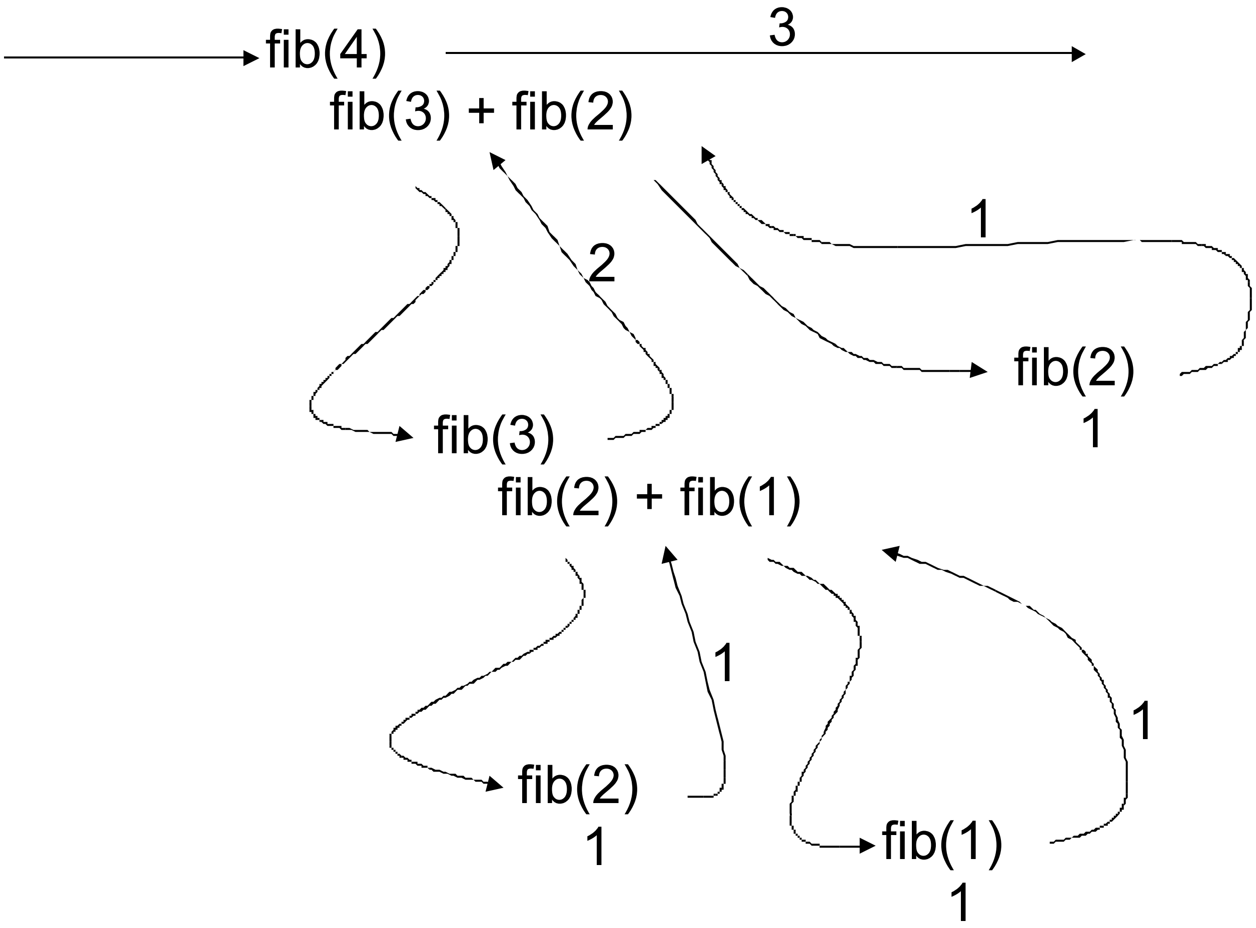
Fibonacci Procedure/2

```
public class fibclassic {

  static int fib(int n) {
    if (n <= 2) {return 1;}
    else {return fib(n - 1) + fib(n - 2);}
  }

  public static void main(String args[]) {
    System.out.println("Fibonacci of 5 is "
                       + fib(5));
  }
}
```

Tracing fib(4)



fib(4)

fib(3) + fib(2)

3

2

1

fib(3)

fib(2) + fib(1)

fib(2)

1

1

1

fib(2)

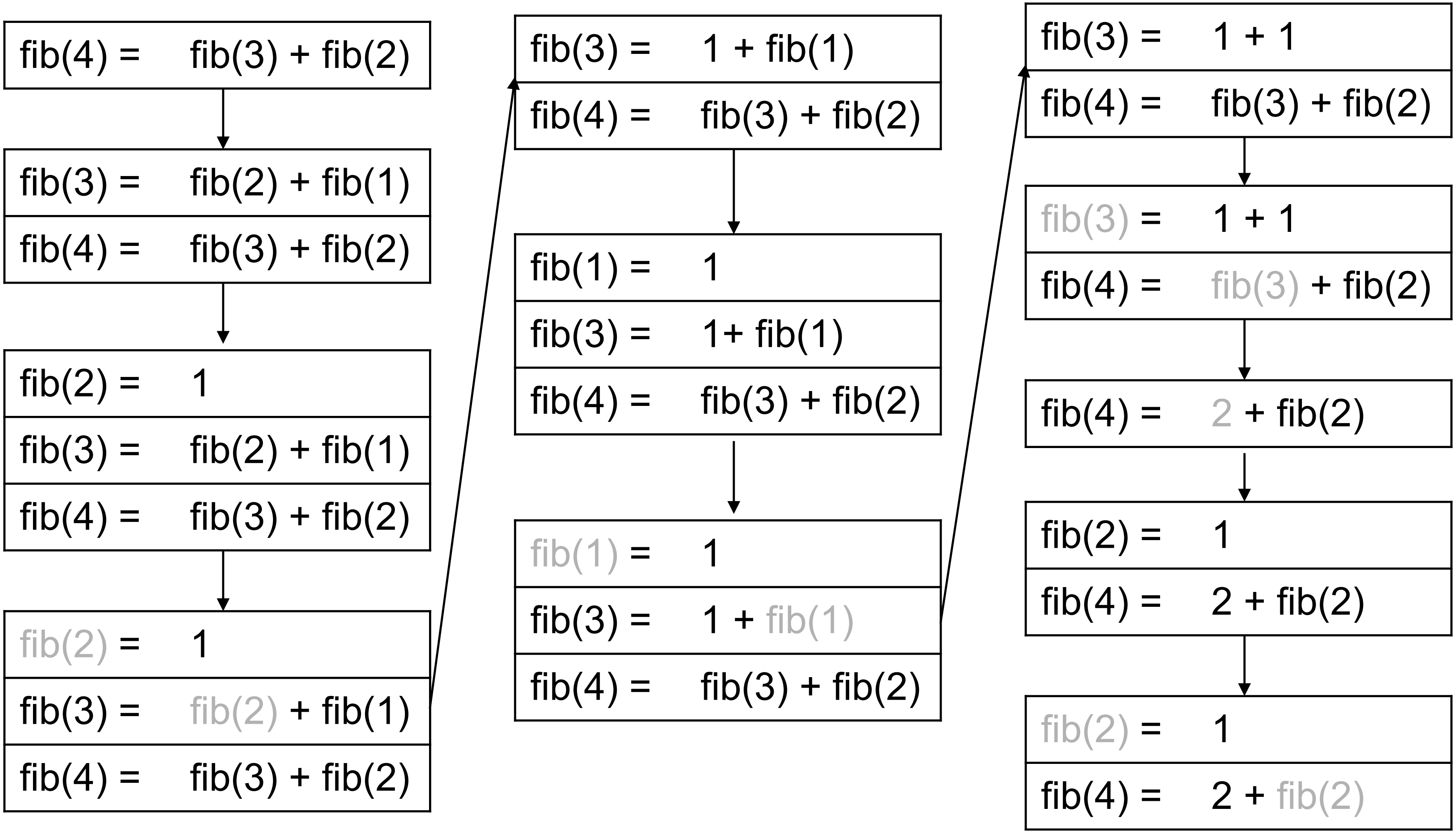
1

fib(1)

1

1

Bookkeeping



fib(4) = fib(3) + fib(2)

fib(3) = fib(2) + fib(1)
fib(4) = fib(3) + fib(2)

fib(2) = 1
fib(3) = fib(2) + fib(1)
fib(4) = fib(3) + fib(2)

fib(2) = 1
fib(3) = fib(2) + fib(1)
fib(4) = fib(3) + fib(2)

fib(3) = 1 + fib(1)
fib(4) = fib(3) + fib(2)

fib(1) = 1
fib(3) = 1+ fib(1)
fib(4) = fib(3) + fib(2)

fib(1) = 1
fib(3) = 1 + fib(1)
fib(4) = fib(3) + fib(2)

fib(3) = 1 + 1
fib(4) = fib(3) + fib(2)

fib(3) = 1 + 1
fib(4) = fib(3) + fib(2)

fib(4) = 2 + fib(2)

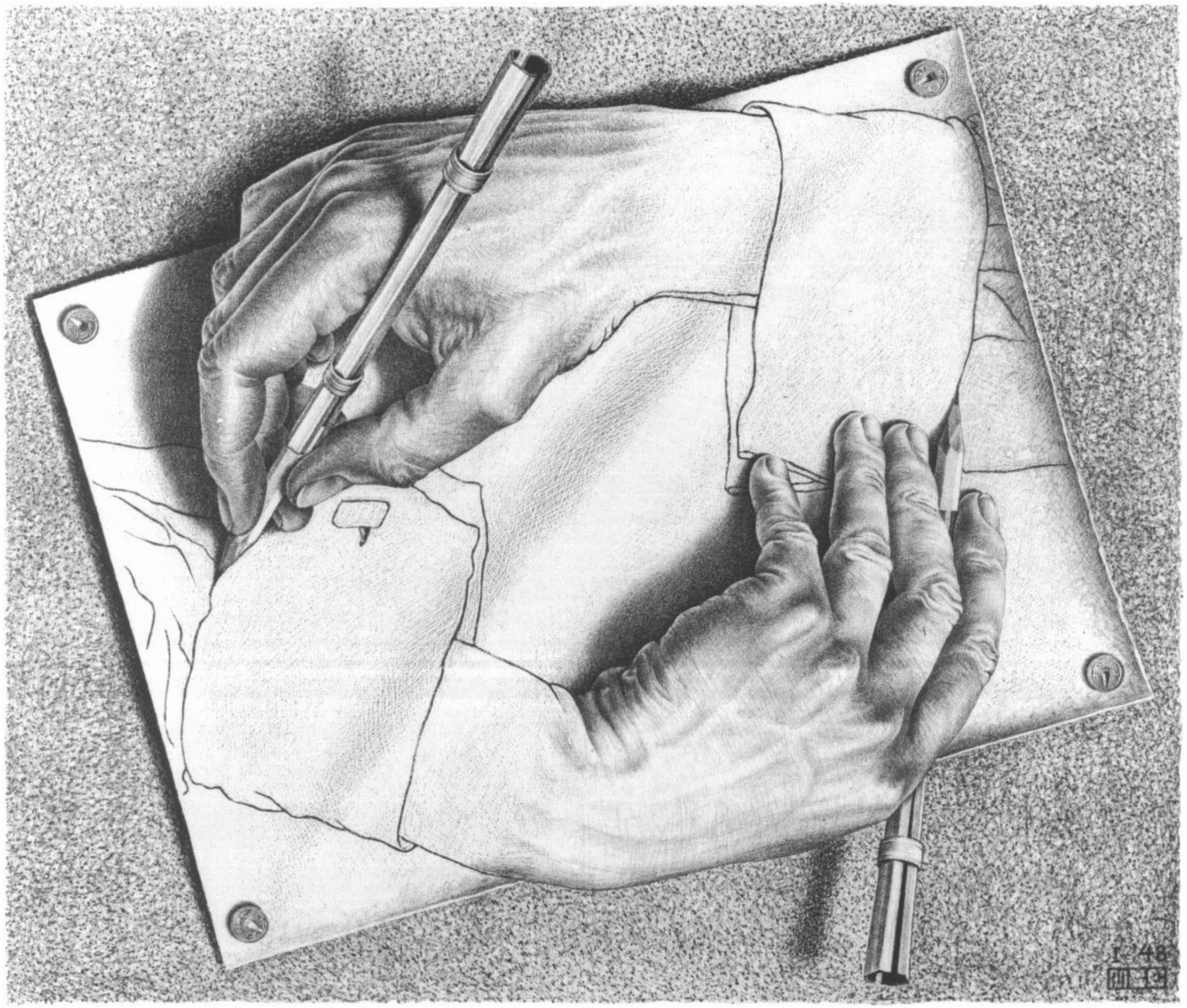
fib(2) = 1
fib(4) = 2 + fib(2)

fib(2) = 1
fib(4) = 2 + fib(2)

Questions

- How many recursive calls are made to compute *fib(n)*?
- What is the maximal height of the recursion stack during the computation?
- How are number of calls and height of the stack related to the size of the input?
- Can there be a procedure for *fib* with fewer operations?
- How is the size of the result *fib(n)* related to the size of the input *n*?

Mutual Recursion



Source: http://britton.disted.camosun.bc.ca/escher/drawing_hands.jpg

Mutual Recursion Example

- Problem: Determine whether a natural number is even
- Definition of even:
  - 0 is even
  - N is even if N - 1 is odd
  - N is odd if N - 1 is even

Implementation of `even`

`even`
INPUT: n – a natural number.
OUTPUT: true if n is even; false otherwise

```
odd(n)
   if n = 0 then return FALSE
   return even(n-1)

even(n)
   if n = 0 then return TRUE
   else return odd(n-1)
```

- How can we determine whether N is odd?

Is Recursion Necessary?

- Theory: You can always resort to iteration and explicitly maintain a recursion stack.
- Practice: Recursion is elegant and in some cases the best solution by far.
- In the previous examples recursion was never appropriate since there exist simple iterative solutions.
- Recursion is more expensive than corresponding iterative solutions since bookkeeping is necessary.

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- Sorting (bubble, insertion, selection)

Sorting

- Sorting is a classical and important algorithmic problem.
  - For which operations is sorting needed?
  - Which systems implement sorting?
- We look at sorting arrays
  (in contrast to files, which restrict random access)
- A key constraint are the restrictions on the space:
  in-place sorting algorithms (no extra RAM).
- The run-time comparison is based on
  - the number of comparisons (C) and
  - the number of movements (M).

Sorting

- Simple sorting methods use roughly ***n* * *n*** comparisons
  - Insertion sort
  - Selection sort
  - Bubble sort

- Fast sorting methods use roughly ***n* * *log n*** comparisons
  - Merge sort
  - Heap sort
  - Quicksort

What's the point of studying those simple methods?

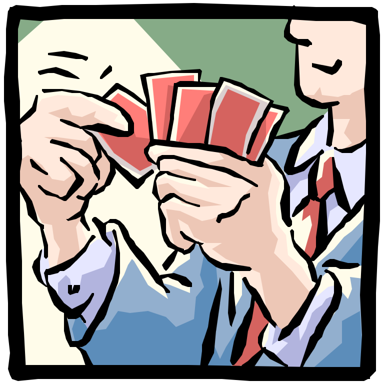
Example 2: Sorting

INPUT

sequence of *n* numbers

$a_1, a_2, a_3, \ldots, a_n$ $\longrightarrow$

2 5 4 10 7



OUTPUT

a permutation of the input sequence of numbers

$b_1, b_2, b_3, \ldots, b_n$

$\longrightarrow$

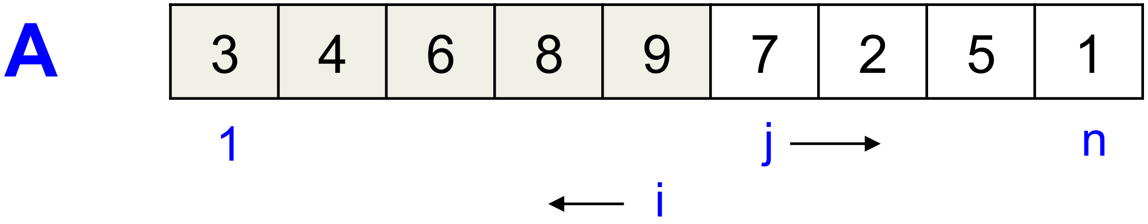
2 4 5 7 10

Correctness (requirements for the output)

For any given input the algorithm halts with the output:

- $b_1 \leq b_2 \leq b_3 \leq \ldots \leq b_n$
- $b_1, b_2, b_3, \ldots, b_n$ is a permutation of $a_1, a_2, a_3, \ldots, a_n$

Insertion Sort



Strategy

- Start with one sorted card.
- Insert an unsorted card at the correct position in the sorted part.
- Continue until all unsorted cards are inserted/sorted.

44	**55**	**12**	**42**	**94**	**18**	**06**	**67**
44	55	**12**	**42**	**94**	**18**	**06**	**67**
12	44	55	**42**	**94**	**18**	**06**	**67**
12	42	44	55	**94**	**18**	**06**	**67**
12	42	44	55	94	**18**	**06**	**67**
12	18	42	44	55	94	**06**	**67**
06	12	18	42	44	55	94	**67**
06	12	18	42	44	55	67	94

Insertion Sort/2

INPUT: A[1..n] – an array of integers
OUTPUT: permutation of A s.t. A[1]≤A[2]≤...≤A[n]

```
for j := 2 to n do // A[1..j-1] sorted
  key := A[j]; i := j-1;
  while i > 0 and A[i] > key do
    A[i+1] := A[i];  i--;
  A[i+1] := key
```

The number of comparisons during the *j* th iteration is

- at least 1: Cmin = $\sum_{j=2}^{n} 1$ = *n - 1*
- at most j-1: Cmax = $\sum_{j=2}^{n} (j-1)$ = *(n*n - n)/2*