Data Structures and Algorithms

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Chapter 5

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Data Structures and Algorithms
Chapter 5

- Dynamic Data Structures
  - Records, Pointers
  - Lists
- Abstract Data Types
  - Stack, Queue
  - Ordered Lists
  - Priority Queue

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Records

- Records are used to group a number of (different) fields.
- A *person* record may group *name*, *age*, *city*, *nationality*, *ssn*.
- The grouping of fields is a basic and often used technique.
- It is available in all programming languages.

Records in Java

- In java a *class* is used to group fields:

```
class rec { int a; int b; };

public class dummy {

  static rec r;

  public static void main(String args[]) {
    r = new rec();
    r.a = 15; r.b = 8;
    System.out.print(“Adding a and b yields “);
    System.out.println(r.a + r.b);
  }
}
```

Records in C

- In C a *struct* is used to group fields:

```
struct rec {
  int a;
  int b;
};

struct rec r;

int main() {
  r.a = 5; r.b = 8;
  printf("The sum of a and b is %d\n", r.a + r.b);
}

// gcc -o dummy dummy.c ; ./dummy
```

Recursive Data Structures

- The counterpart of recursive functions are recursively defined data structures.
- Example: list of integers

$$\text{list} = \left\{ \begin{array}{c} \text{integer} \\ \text{integer}, \text{list} \end{array} \right\}$$

- In C:

```
struct list {
  int value;
  Struct list * tail; };
```

Recursive Data Structures/2

- The storage space of recursive data structures is not known in advance.
  - It is determined by the number of elements that will be stored in the list.
  - This is only known during runtime (program execution).
  - The list can grow and shrink during program execution.

Recursive Data Structures/3

- There must be a mechanism to constrain the initial storage space of recursive data structures (it is potentially infinite).
- There must be a mechanism to grow and shrink the storage space of a recursive data structures during program execution.

Pointers

- A common technique is to allocate the storage space (memory) dynamically.
- That means the storage space is allocated when the program executes.
- The compiler only reserves space for an address to these dynamic parts.
- These addresses are called pointers.

Pointers/2

- integer i
- pointer p to an integer (55)
- record r with integer components a (17) and b (24)
- pointer s that points to r

Address	Variable	Memory
1af782	i	23
1af783	p	1af789
1af784	r	17
1af785		24
1af786	s	1af784
1af787		
1af788		
1af789		55
1af78a		

Pointers in C

1. To follow (chase, dereference) a pointer variable we write *p
   - `*p = 12`
2. To get the address of a variable i we write &i
   - `p = &i`
3. To allocate memory we use malloc(sizeof(Type)), which returns an address in the memory heap
   - `p = malloc(sizeof(int))`
4. To free storage space pointed to by a pointer p we use free
   - `free(p)`

Pointers in C/2

- To declare a pointer to type T we write T*
  - `int* p`
- Note that * is used for two purposes:
  - Declaring a pointer variable
    `int* p`
  - Following a pointer
    `*p = 15`
- In other languages these are syntactically different.

Pointers in C/3

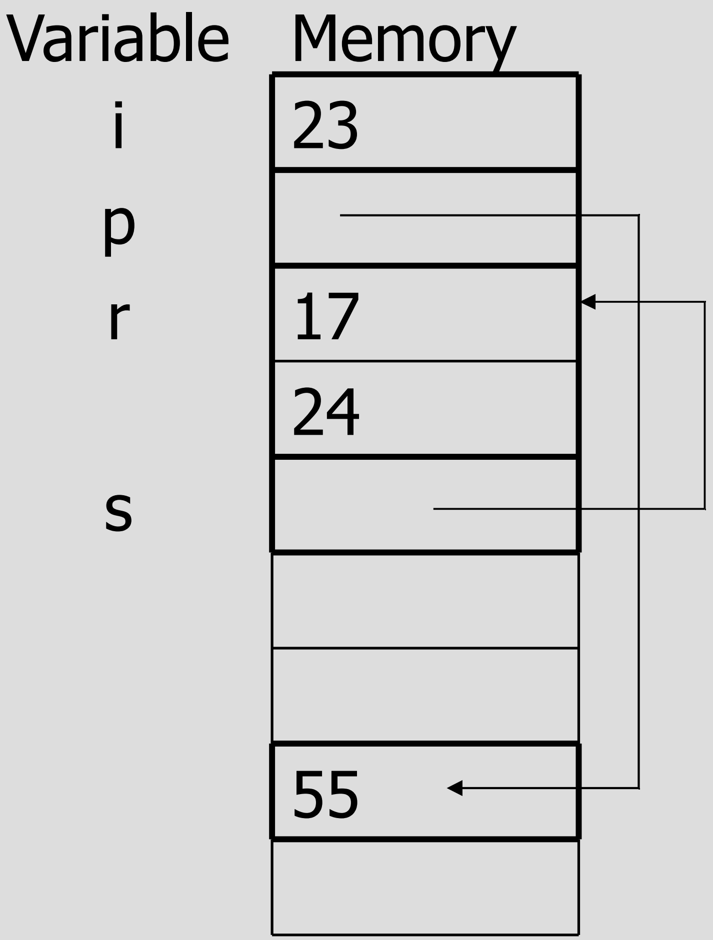
- int i
  i = 23
- int* p
  p = malloc(sizeof(int))
  *p = 55
- struct rec r
  rec.a = 17
  rec.b = 24
- struct rec* s;
  s = &r

Address	Variable	Memory
1af782	i	23
1af783	p	1af789
1af784	r	17
1af785		24
1af786	s	1af784
1af787		
1af788		
1af789		55
1af78a		

Pointers in C/4

Alternative notation:

Address	Variable	Memory
1af782	i	23
1af783	p	1af789
1af784	r	17
1af785		24
1af786	s	1af784
1af787		
1af788		
1af789		55
1af78a		



Pointers/3

- Pointers are only one mechanism
  to implement recursive data structures.
- The programmer does not have to be aware
  of their existence.
  The storage space can be managed automatically.
- In C the storage space has to be managed explicitly.
- In Java
  - an object is implemented as a pointer.
  - creation of objects (new)
    automatically allocates storage space.
  - accessing an object will automatically follow the pointer.
  - deallocation is done automatically (garbage collection).

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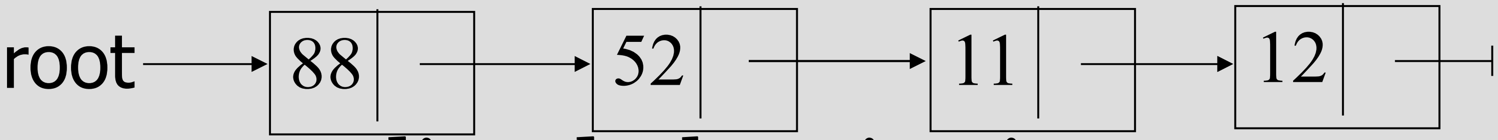
Lists

- A list of integers:



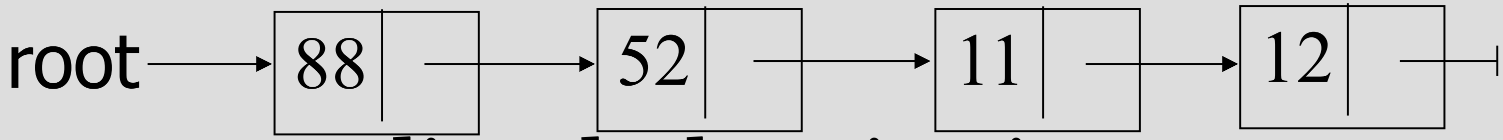
- Corresponding declaration in Java:

```
class node {
  int val;
  node next;
}

node root;
```

- Accessing a field: `p.a`

Lists/2

- A list of integers:



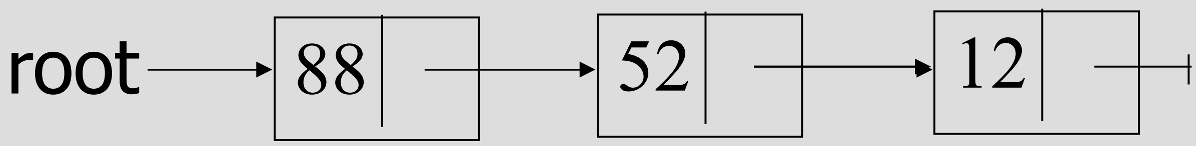
- Corresponding declaration in C:

```
struct node {
  int val;
  struct node* next;
}

struct node* root;
```

- Accessing a field: `(*p).a = p->a`

Lists/3

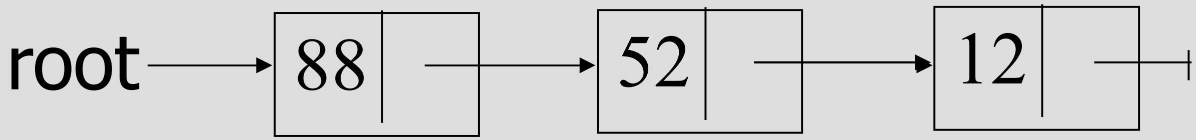
- Populating the list with integers (Java):




```
root = new node();

root.val = 88;
root.next = new node();

p = root.next;
p.val = 52;
p.next = new node();

p = p.next;
p.val = 12;
p.next = null;
```

Lists/4

- Populating the list with integers (C):

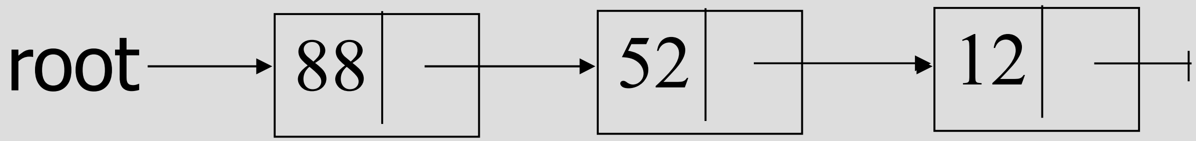



```
root = malloc(sizeof(struct node));
root->val = 88;
root->next = malloc(sizeof(struct node));

p = root->next;
p.val = 52;
p->next = malloc(sizeof(struct node));

p = p->next;
p->val = 12;
p->next = NULL;
```

List Traversal

6
5

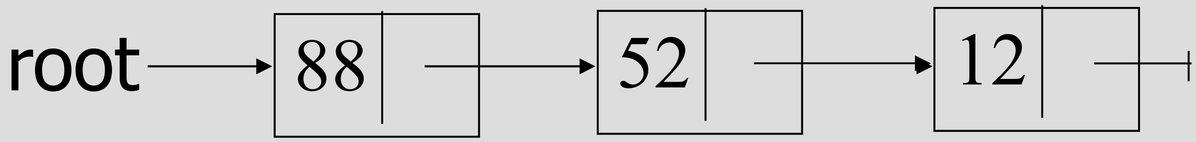
- Print all elements of a list (Java):




```
p = root;
while (p != null) {
  System.out.printf("%d,", p.val);
  p = p.next
}
System.out.printf("\n");
```

List Traversal

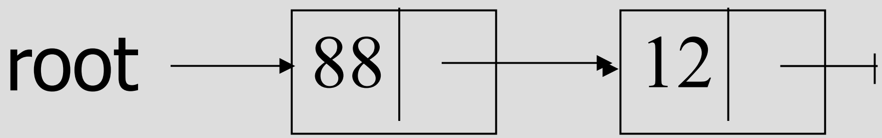
- Print all elements of a list (C):



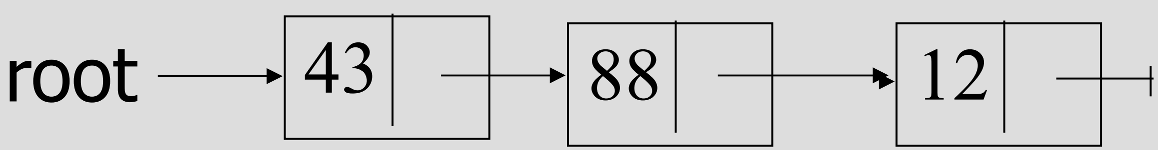

```
p = root;
while (p != null) {
  printf("%d,", p->val);
  p = p->next
}
printf("\n");
```

List Insertion

- Insert 43 at the beginning (Java):

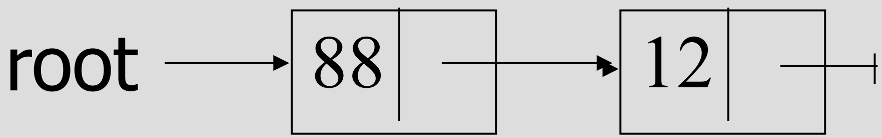



```
p = new node();
p.val = 43
p.next = root;
root = p;
```

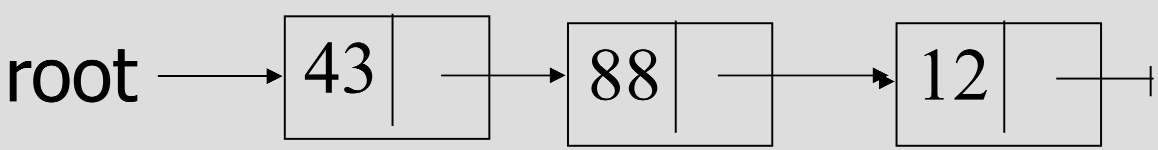


List Insertion

- Insert 43 at the beginning (C):

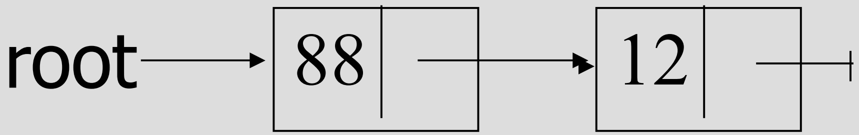



```
p = malloc(sizeof(struct node));
p->val = 43
p->next = root;
root = p;
```



List Insertion/2

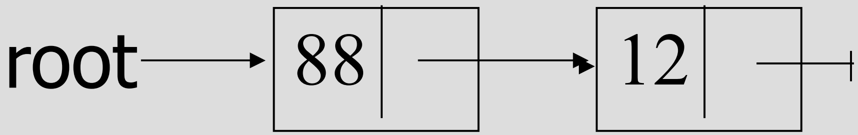
Insert 43 at end (Java):




```
if (root == null) {
  root = new node();
  root.val = 43;
  root.next = null;
} else {
  q = root;
  while (q.next != null) { q = q.next; }
  q.next = new node();
  q.next.val = 43;
  q.next.next = null;
}
```

List Insertion/2

Insert 43 at end (C):




```
if (root == null) {
  root = malloc(sizeof(struct node));
  root->val = 43;
  root->next = null;
} else {
  q = root;
  while (q->next != null) { q = q->next; }
  q->next = malloc(sizeof(struct node));
  q->next->val = 43;
  q->next->next = null;
}
```

List Deletion

Delete element x from a non-empty list (Java):

```
p = root;
if (p.val == x) {
  root = p.next;
} // no need of freeing in java
else {
  while (p.next != null && p.next.val != x) {
    p = p.next;
  }
  tmp = p.next;
  p.next = tmp.next;
}
```

List Deletion

Delete element x from a non-empty list (C):

```
p = root;
if (p->val == x) {
  root = p->next;
  free(p);
} else {
  while (p->next != null && p->next->val != x) {
    p = p->next;
  }
  tmp = p->next;
  p->next = tmp->next
  free(tmp);
}
```

List

- Cost of operations:
  - Insertion at beginning: O(1)
  - Insert at end: O(n)
  - Check isEmpty: O(1)
  - Delete from the beginning: O(1)
  - Search: O(n)
  - Delete: O(n)
  - Print: O(n)

Suggested exercises

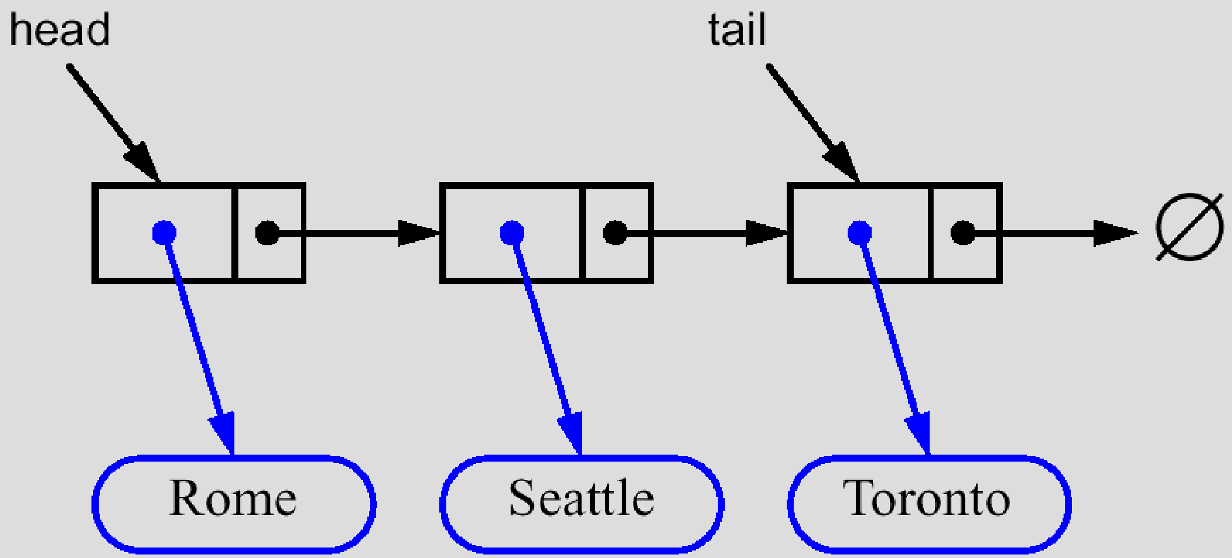
- Implement a linked list with the following functionalities: isEmpty, insertFirst, insertLast, search, deleteFirst, delete, print
- As before, with a recursive version of: insertLast, search, delete, print
  - are recursive versions simpler?
- Implement an efficient version of print which prints the list in reverse order

Variants of linked lists

- Linked lists with explicit head/tail
- Doubly linked lists

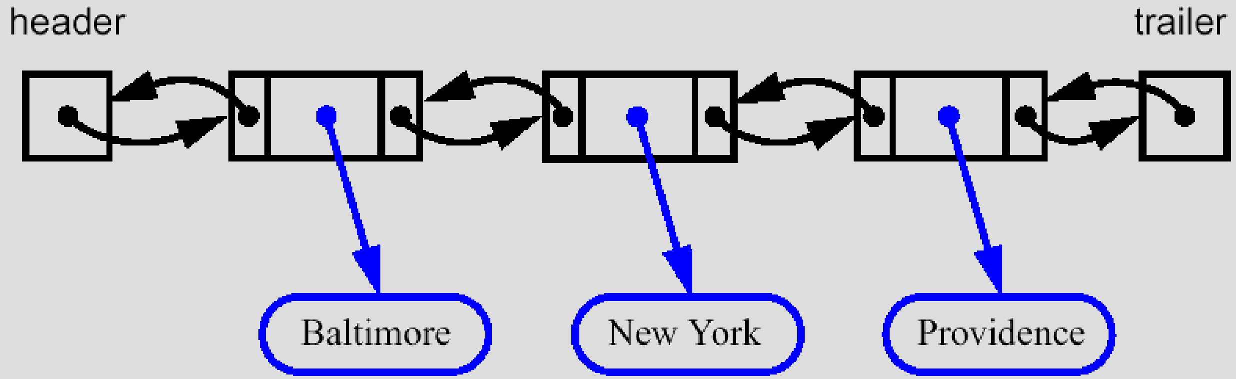
List with Explicit Head/Tail

- Instead of a *root* we can have a *head* and *tail*:

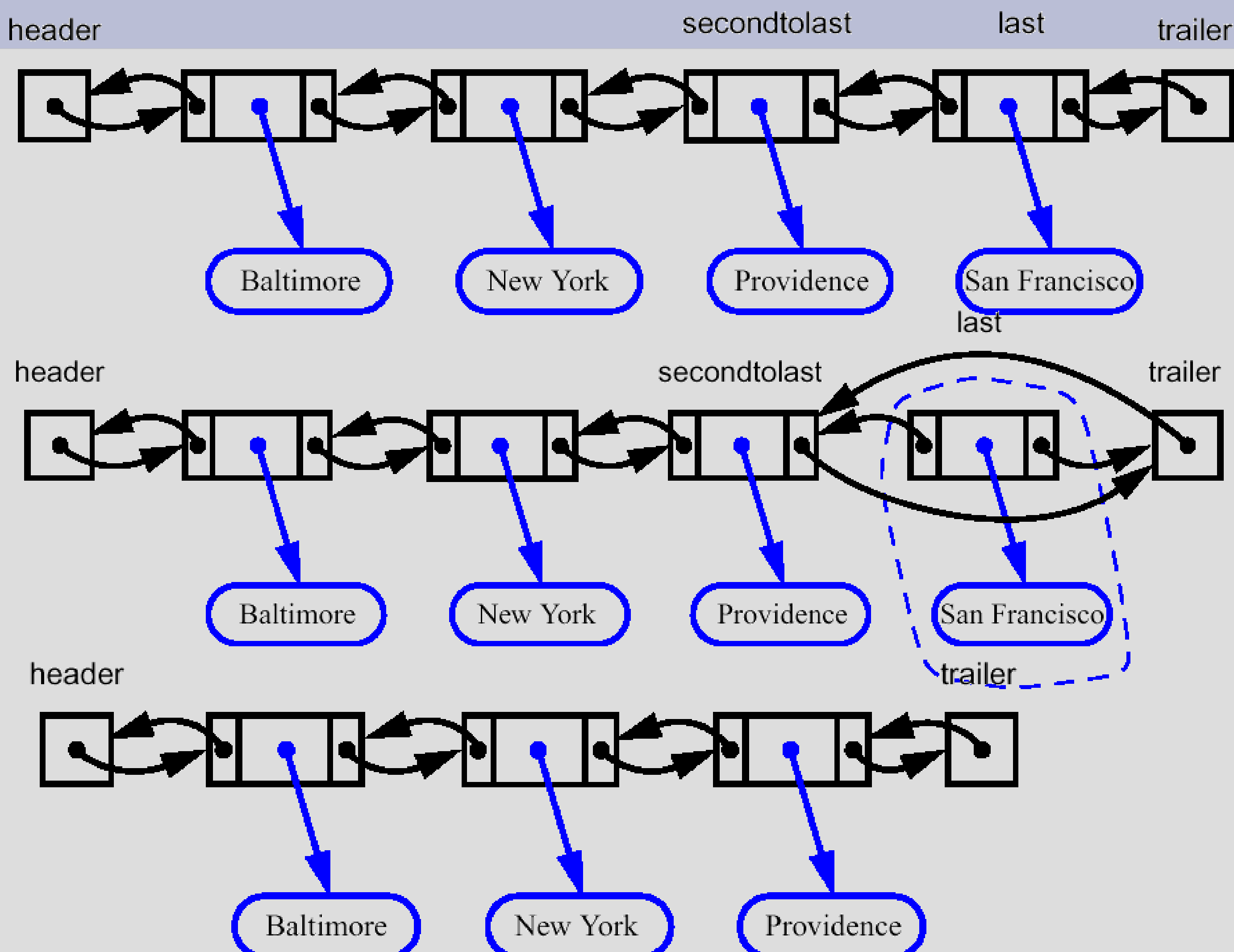


Doubly Linked Lists

- To be able to quickly navigate back and forth in a list we use **doubly linked lists**.



- A node of a doubly linked list has a **next** and a **prev** link.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Abstract Data Types (ADTs)

- An *ADT* is a mathematically specified entity that defines a set of its *instances*, with:
  - a specific *interface* – a collection of signatures of operations that can be invoked on an instance.
  - a set of *axioms* (*preconditions* and *post-conditions)* that define the semantics of the operations (i.e., what the operations do to instances of the ADT, but not how).

ADTs/2

- Why ADTs?
  - ADTs allows to break work into pieces that can be worked on independently – without compromising correctness.
    - They serve as *specifications* of *requirements* for the building blocks of solutions to algorithmic problems.
  - ADTs encapsulate *data structures* and algorithms that *implement* them.

ADTs/3

- Provides a language to talk on a higher level of abstraction.
- Allows to separate the concerns of *correctness* and the *performance analysis*
  1. Design the algorithm using an ADT
  2. Count how often different ADT operations are used
  3. Choose implementations of ADT operations

- ADT = Instance variables + procedures (Class = Instance variables + methods)

ADTs/4

- We discuss a number of popular ADTs:
  - Stacks, Queues
  - Ordered Lists
  - Priority Queues
  - Trees (next chapter)
- They illustrate the use of lists and arrays.

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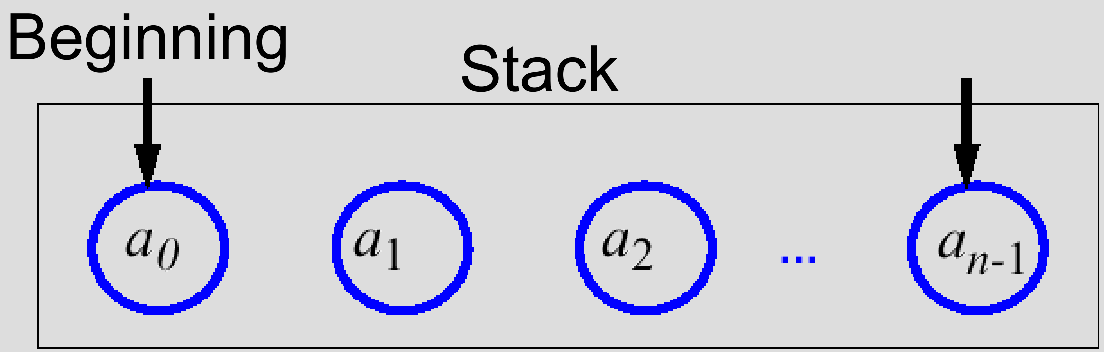
Stacks

- In a stack, insertions and deletions follow the last**-in-first-out** (LIFO) principle.
- Thus, the element that has been in the queue for the shortest time are deleted.
  - Example: OS stack, ...
- Solution: Elements are inserted at the **beginning** (push) and removed from the **beginning** (pop).



Stacks/2

- Appropriate data structure:
  - Linked list, one root: good
  - Array: fastest, limited in size
  - Doubly linked list: unnecessary

An Array Implementation

- Create a stack using an array
- A maximum size *N* is specified.
- The stack consists of an N-element array S and one integer variable *count*:
  - *count:* index of the front element (head)
  - *count* represents the position where to insert next element, and the number of elements in the stack

An Array Implementation/2

```
int size()
   return count
```

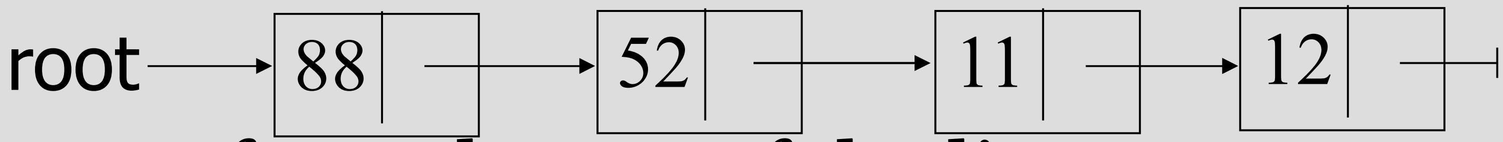
```
int isEmpty()
   return (count == 0)
```

```
Element pop()
   if isEmpty() then Error
   x = S[count-1]
   count--;
   return x
```

```
void push(element x)
   if count==N then Error;
   S[count] = x;
   count++;
```

A Linked-List implementation

- A list of integers:



- Insert from the top of the list

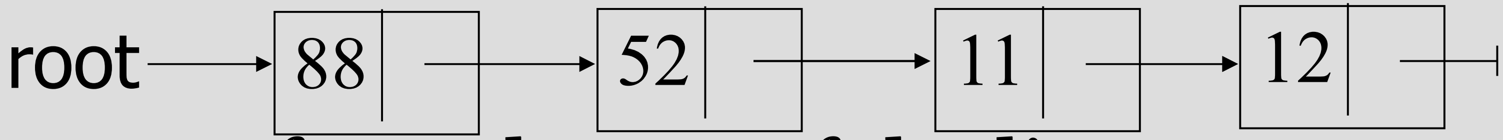
```
push(element x):

node p = new node();
p.val = x;
p.next = root;
root = p;
```

- Constant-time operation!

A Linked-List implementation

- A list of integers:



- Extract from the top of the list

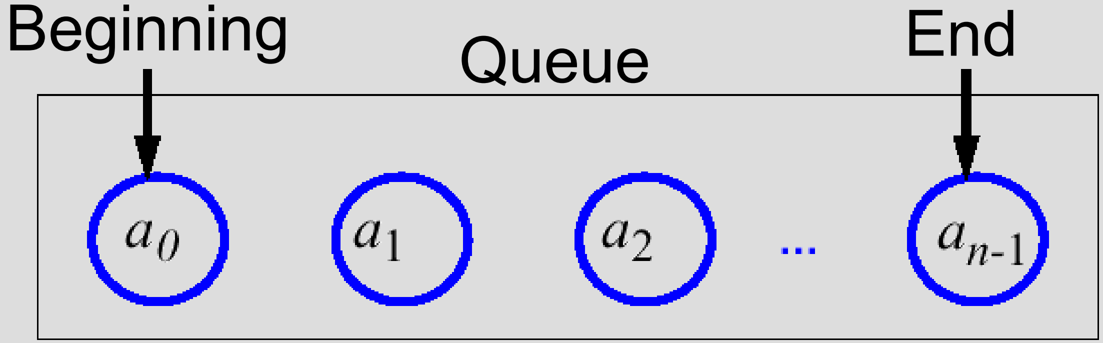
```
Element pop():

x = root.val;
root = root.next;
Return x;
```

- Constant-time operation!

Queues

- In a queue insertions and deletions follow the **first-in-first-out** (FIFO) principle.
- Thus, the element that has been in the queue for the longest time are deleted.
  - Example: Printer queue, …
- Solution: Elements are inserted at the **end** (enqueue) and removed from the **beginning** (dequeue).



Queues/2

- Appropriate data structure:
  - Linked list, root: inefficient insertions
  - Linked list, head/tail: good
  - Array: fastest, limited in size
  - Doubly linked list: unnecessary

An Array Implementation

- Create a queue using an array in a circular fashion
- A maximum size *N* is specified.
- The queue consists of an N-element array *Q* and two integer variables:
  - *f*, index of the front element (head, for dequeue)
  - *r*, index of the element after the last one (tail, for enqueue)

An Array Implementation/2

- “wrapped around” configuration

- what does f=r mean?

An Array Implementation/3

```
int size()
   return (N-f+r) mod N
```

```
int isEmpty()
   return size() == 0
```

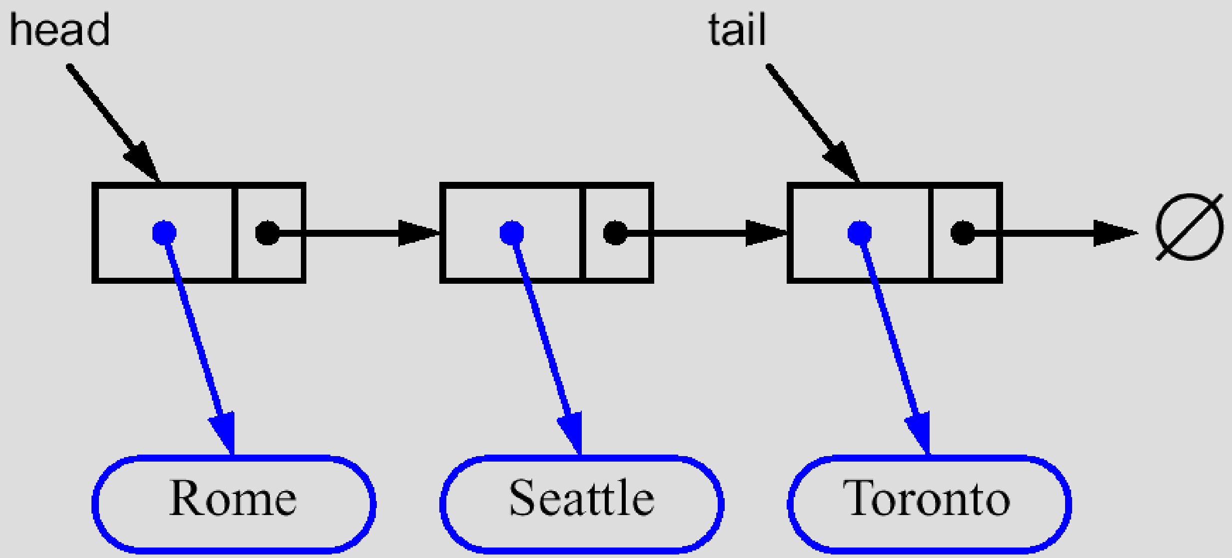
```
Element dequeue()
   if isEmpty() then Error
   x = Q[f]
   f = (f+1) mod N
   return x
```

```
void enqueue()
   if size()==N-1 then Error
   Q[r] = x
   r = (r+1) mod N
```

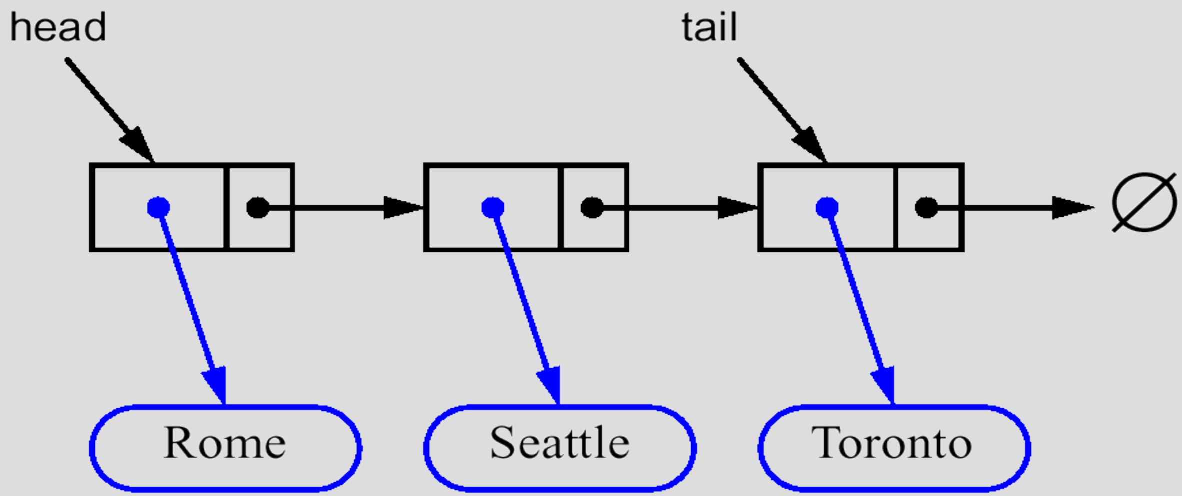
We assume arrays as in Java, with indexes from 0 to n-1

A Linked-List Implementation

- Use linked-list with head and tail
- Insert in tail, extract from head



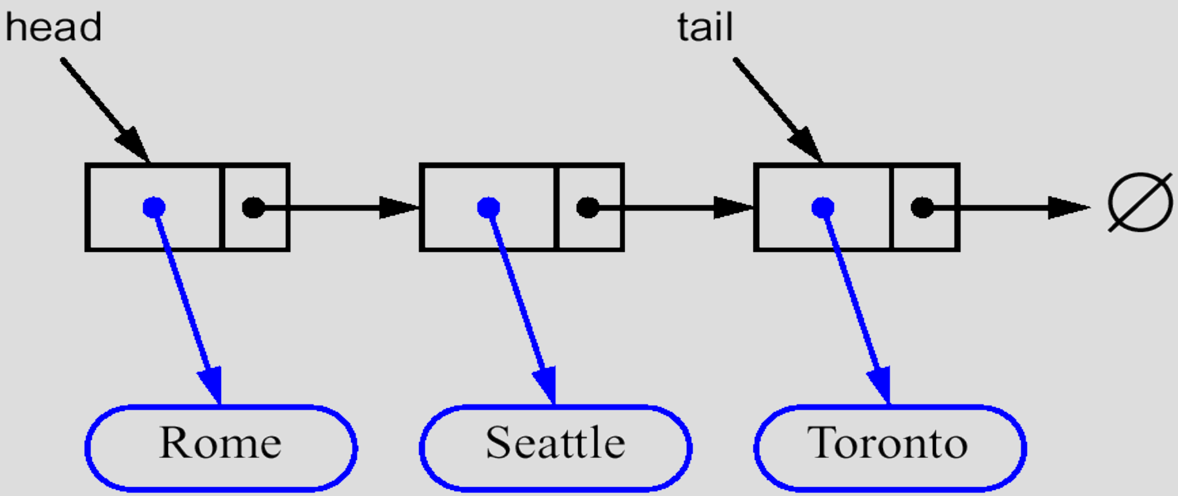
A Linked-List implementation/2



- Insert at the end of the list: O(1)

```
enqueue(element x):
node p = new node();
p.info = x; p.next = null;
tail.next=p;
tail=tail.next;
```

A Linked-List implementation/3



- Extract from the top of the list: O(1)

```
Element dequeue():
x = root.info;
root = root.next;
Return x;
```

Suggested exercises

- Implement stack and queue as arrays
- Implement stack and queue as linked lists, with the same interface as the array implementation

Suggested exercises/2

- Suppose a queue of integers is implemented with an array of 8 elements: draw the outputs and status of such array after the following operations:
  - enqueue 2,4,3,1,7,6,9
  - dequeue 3 times
  - Enqueue 2,3,4

  Can we enqueue any more element?
- Try the same with a stack
- Try similar examples (also with a stack)

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Ordered List

- In an ordered list elements are ordered according to a key.
- Example functions on ordered list:
  - `init()`
  - `isEmpty()`
  - `Search(element x)`
  - `delete(element x)`
  - `print()`
  - `insert(element x)`

Ordered List/2

- Declaration of an ordered list identical to unordered list
- Some operations (search, and hence insert and delete) are slightly different

Ordered List/3

- Insertion into an ordered list (java):

```
void insert(node l, int x) {
  node p;
  node q;

  if (root == NULL || root.val > x) {
    p = new node();
    p.val = x;
    p.next = root;
    root = p;
  } else {
…
```

Ordered List/4

- Insertion into an ordered list (java):

```
void insert(node l, int x) {
  ...
  } else {
    p = root;
    while (p.next != NULL && p.next.val < x)
      p = p.next;
    q = new node();
    q.val = x;
    q.next = p.next;
    p.next = q;
  }
}
```

Ordered List/5

- Insertion into an ordered list (C):

```
void insert(struct node* l, int x) {
  struct node* p;
  struct node* q;

  if (root == NULL || root->val > x) {
    p = malloc(sizeof(struct node));
    p->val = x;
    p->next = root;
    root = p;
  } else {
...
```

Ordered List/6

- Insertion into an ordered list (C):

```
void insert(struct node* l, int x) {
  …
  } else {
    p = root;
    while (p->next != NULL && p->next->val < x)
      p = p->next;
    q = malloc(sizeof(struct node));
    q->val = x;
    q->next = p->next;
    p->next = q;
  }
}
```

Ordered List

- Cost of operations:
  - Insertion: O(n)
  - Check isEmpty: O(1)
  - Search: O(n)
  - Delete: O(n)
  - Print: O(n)

Suggested exercises

- Implement an ordered list with the following functionalities: isEmpty, insert search, delete, print
- Implement also deleteAllOccurrences()
- As before, with a recursive version of: insert, search, delete, print
  - are recursive versions simpler?
- Implement an efficient version of print which prints the list in reverse order

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Priority Queues

- A priority queue is an *ADT* for maintaining a set S of elements, each with an associated value called key.
- A PQ supports the following operations
  - **Insert**(S,x) insert element x in set S ($S := S \cup \{x\}$)
  - **ExtractMax**(S) returns and removes the element of S with the largest key
- One way of implementing it: a heap

Priority Queues/2

- Removal of max takes constant time on top of Heapify $\Theta(log\ n)$

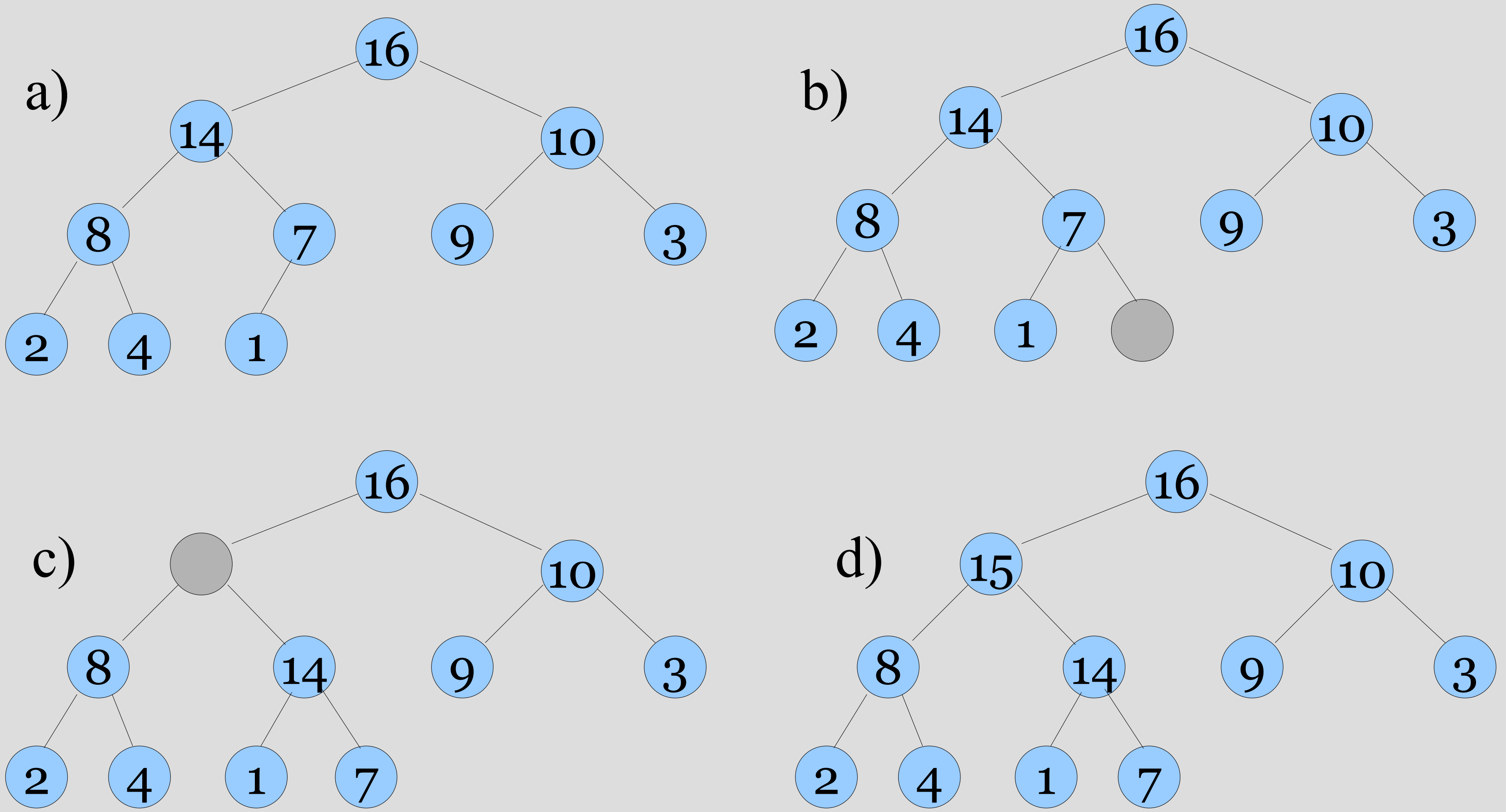
```
ExtractMax(A)
  // removes & returns largest elem of A
  max := A[1]
  A[1] := A[n]
  n := n-1
  Heapify(A, 1, n)
  return max
```

Priority Queues/3

- Insertion of a new element
  - enlarge the PQ and propagate the new element from last place ”up” the PQ
  - tree is of height *log n*, running time: Θ(log n)

```
Insert(A,key)
  n := n+1;
  i := n;
  while i > 1 and A[parent(i)] < key
    A[i] := A[parent(i)]
    i := parent(i)
  A[i] := key
```

Priority Queues/4



Priority Queues/5

- Applications:
  - job scheduling shared computing resources (Unix)
  - Event simulation
  - As a building block for other algorithms
- We used a heap and an array to implement PQ. Other implementations are possible.

Suggested exercises

- Implement a priority queue
- Consider the PQ of previous slides. Draw the status of the PQ after each of the following operations:
    - Insert 17,18,18,19
    - Extract four numbers
    - Insert again 17,18,18,19
- Build a PQ from scratch, adding and inserting elements at will, and draw the status of the PQ after each operation

Summary

- Records, Pointers
- Dynamic Data Structures
  - Lists (root, head/tail, doubly linked)
- Abstract Data Types
  - Type + Functions
  - Stack, Queue
  - Ordered Lists
  - Priority Queues

Next Chapter

- Binary Search Trees
- Red-Black Trees