Data Structures and Algorithms

Part 1

Werner Nutt

Acknowledgments

- The course follows the book “Introduction to Algorithms‘”, by **Cormen, Leiserson, Rivest and Stein**, MIT Press [CLRST]. Many examples displayed in these slides are taken from their book.
- These slides are based on those developed by Michael Böhlen for this course.

(See http://www.inf.unibz.it/dis/teaching/DSA/)

- The slides also include a number of additions made by Roberto Sebastiani and Kurt Ranalter when they taught later editions of this course

(See http://disi.unitn.it/~rseba/DIDATTICA/dsa2011_BZ//)

DSA, Part 1: Overview

- Introduction, syllabus, organisation
- Algorithms
- Recursion (principle, trace, factorial, Fibonacci)
- Sorting (bubble, insertion, selection)

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Learning Outcomes

The main things we will learn in this course:

- To *think algorithmically* and
  get the spirit of how algorithms are designed
- To get to know a *toolbox* of *classical* algorithms
- To learn a number of algorithm design *techniques*
  (such as divide-and-conquer)
- To analyze (in a precise and formal way)
  the *efficiency* and the *correctness* of algorithms.

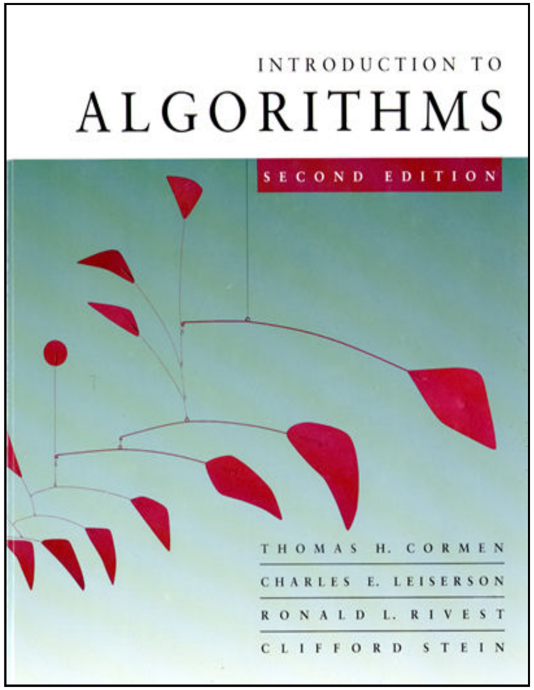
Syllabus (provisional)

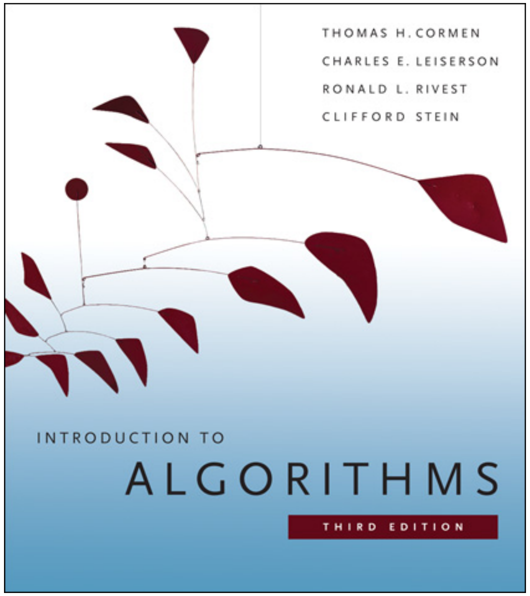
1. Introduction, recursion (chap 1 in CLRS)
2. Correctness and complexity of algorithms (2, 3)
3. Divide and conquer, recurrences (4)
4. Sorting (1, 6, 7)
5. Pointers, lists, sets, abstract data types (10)
6. Trees, red-black trees (12, 13)
7. Hash tables (11)
8. Dynamic programming (15)
9. Graph algorithms (22, 23, 24)
10. NP-Completeness (34)

Literature

Cormen, Leiserson, Rivest
and Stein (CLRS),
Introduction to Algorithms,
Second Edition, MIT Press and
McGraw-Hill, 2001
and
Third Edition, MIT Press, 2009

(See http://mitpress.mit.edu/algorithms/)





Course Organization

- Lectures: Tue 8:30-10:30, Fri 8:30-10:30
- Labs
  - English: Mouna Kacimi, Mouna.Kacimi@unibz.it
    Fri 10:30-12:30
  - German: Simon Razniewski, razniewski@inf.unibz.it
    Mon 8:30-10:30
  - Italian: Valeria Fionda, Valeria.Fionda@unibz.it
    Mon 14:00-16:00
- Home page:
  http://www.inf.unibz.it/~nutt/DSA1112/

Assignments

The assignments are a crucial part of the course

- Each week an assignment has to be solved
- Assignments will be published on Monday and have to be handed in on the Monday of the following week (in the exercise group or in the lecture)
- Late hand-ins are not accepted.
- A number of assignments include programming tasks. It is strongly recommended that you implement and run all programming exercises.
- Assignments will be marked. The assignment mark will count towards the course mark.

Assignment, Exam and Course Mark

- There will be one written exam at the end of the course.
- Students who do not submit exercises
  will be assessed on the exam alone.
- For students who submit all assignments, the final mark will be a weighted average

  50% exam mark + 50% assignment mark
- If students submit fewer assignments,
  the percentage will be lower.
- Assignments for which the mark is lower than the mark of the written exam will not be considered.
- The assignment marks apply to three exam sessions.

General

- Algorithms are first designed on paper
  … and later keyed in on the computer.

- The most important thing is to be simple and precise .

- During lectures:
  - Interaction is welcome; ask questions
    (I will ask you anyway ☺)
  - Additional explanations and examples if desired
  - Speed up/slow down the progress

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What are Algorithms About?

Solving problems in everyday life

- Travel from Bolzano to Munich
- Cook Spaghetti alla Bolognese *(I know, not in Italy,…)*
- Register for a Bachelor thesis at FUB

For all these problems, there are

- instructions
- recipes
- procedures,

which describe a complex operation in terms of

- elementary operations *("beat well …")*
- control structures and conditions *("… until fluffy")*

Algorithms

Problems involving numbers, strings, mathematical objects:

- for two numbers, determine their sum, product, …
- for two numbers, find their greatest common divisor
- for a sequence of strings,
  find an alphabetically sorted permutation of the sequence
- for two arithmetic expressions, find out if they are equivalent
- for a program in Java,
  find an equivalent program in byte code
- on a map, find for a given house the closest bus stop

We call instructions, recipes, for such problems *algorithms*

What have algorithms in common with recipes?
How are they different?

History

- *First algorithm:* Euclidean Algorithm, greatest common divisor, 400-300 B.C.
- *Name:* Persian mathematician Mohammed al-Khowarizmi, in Latin became “Algorismus”

  كتاب الجمع و التفريق بحساب الهند

  = *Kitāb al-Dschamʿ wa-l-tafrīq bi-ḥisāb al-Hind*

  = *Book on connecting and taking apart in the calculation of India*
- *19th century*
  - Charles Babbage: Difference and Analytical Engine
  - Ada Lovelace: Program for Bernoulli numbers
- *20th century*
  - Alan Turing, Alonzo Church: formal models computation
  - John von Neumann: architecture of modern computers

Data Structures and Algorithms

- Data structure
  - Organization of data to solve the problem at hand
- Algorithm
  - Outline, the essence of a computational procedure, step-by-step instructions
- Program
  - implementation of an algorithm in some programming language

Overall Picture

Using a computer to help solve problems.

- Precisely specify the problem
- Designing programs
  - architecture
  - algorithms
- Writing programs
- Verifying (testing) programs

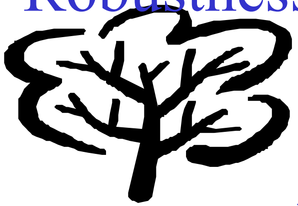
Data Structure and Algorithm Design Goals

Correctness

Efficiency

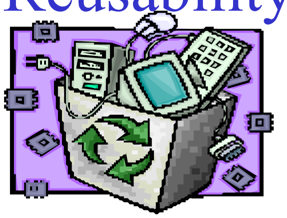
Implementation Goals

Robustness



Adaptability

Reusability

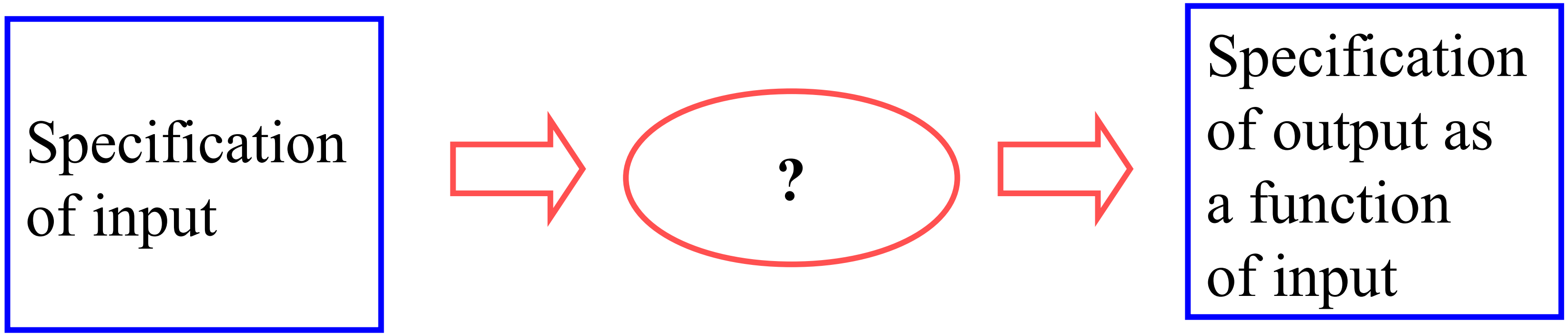


This course is not about:

- Programming languages
- Computer architecture
- Software architecture
- SW design and implementation principles

We will only touch upon
the theory of complexity
and computability.

Algorithmic Problem

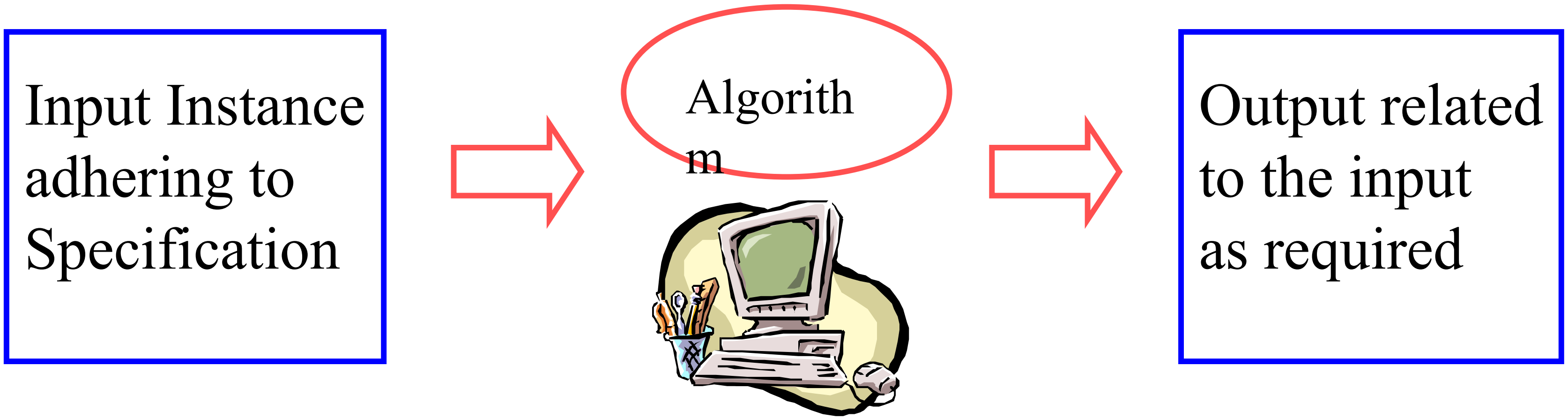


There is an infinite number of possible input *instances* satisfying the specification.

For example: A sorted, non-decreasing sequence of natural numbers, on nonzero, finite length:

1, 20, 908, 909, 100000, 1000000000.

Algorithmic Solution



- Algorithm describes actions on the input instance
- There may be many correct algorithms for the same algorithmic problem.

Definition

An algorithm is a sequence of *unambiguous* instructions for solving a problem, i.e.,

- for obtaining a *required output*
- for any *legitimate input*

in a finite amount of time.

➔ This presumes a mechanism to execute the algorithm

Properties of algorithms:

- Correctness, Termination, (Non-)Determinism,
  Run Time, …

How to Develop an Algorithm

- Precisely define the problem.
  Precisely specify the input and output.
  Consider all cases.

- Come up with a simple plan
  to solve the problem at hand.
  - The plan is independent of a (programming) language
  - The precise problem specification influences the plan.

- Turn the plan into an implementation
  - The problem representation (data structure) influences the implementation

Preconditions, Postconditions

Specify preconditions and postconditions of algorithms:

Precondition:

- what does the algorithm get as input?

Postcondition:

- what does the algorithm produce as output?
- … how does this relate to the input?

Make sure you have considered the special cases:

- empty set, number 0, pointer nil, …

Example 1: Searching

INPUT

- A - (un)sorted sequence of n ($n > 0$) numbers
- q - a single number

$a_1, a_2, a_3, \ldots, a_n;\ q$

2 5 6 10 11; 5

2 5 6 10 11; 9

OUTPUT

- index of number q in sequence A, or *NIL*

j

2

NIL

Searching/2, search1

search1
INPUT: A[1..n] (un)sorted array of integers, q an integer.
OUTPUT: index j such that A[j]=q. *NIL* if for all j ($1 \leq j \leq n$): A[j] $\neq q$

```
j := 1
while j ≤ n and A[j] ≠ q do j++
if j ≤ n then return j
else return NIL
```

- The code is written in pseudo-code and INPUT and OUTPUT of the algorithm are specified.
- The algorithm uses a *brute-force* technique, i.e., scans the input sequentially.

Pseudo-code

A la Java, Pascal, C, or any other imperative language

- Control structures:

  (if then else , while, and for loops)

- Assignment: :=
- Array element access: A[i]
- Access to element of composite type (record or object):

  A.b

CLRS uses b[A]

Searching, Java Solution

```
import  java.io.*;

class search {
  static final int n = 5;
  static int j, q;
  static int a[] = { 11, 1, 4, -3, 22 };

  public static void main(String args[]) {
    j = 0; q = 22;
    while (j < n && a[j] != q) { j++; }
    if (j < n) { System.out.println(j); }
    else { System.out.println("NIL"); }
  }
}
```

Searching, C Solution

```
#include <stdio.h>
#define n 5


int j, q;
int a[n] = { 11, 1, 4, -3, 22 };
int main() {
  j = 0;  q = -2;
  while (j < n && a[j] != q) { j++; }
  if (j < n) { printf("%d\n", j); }
  else { printf("NIL\n"); }
}

// compilation: gcc -o search search.c
// execution:  ./search
```

Searching/3, search2

Another idea:

Run through the array
and set a pointer if the value is found.

search2
INPUT: A[1..n] (un)sorted array of integers, *q* an integer.
OUTPUT: index *j* such that A[*j*]=*q*. *NIL* if for all *j* ($1 \leq j \leq n$): A[*j*] $\neq$ *q*

```
ptr := NIL;
for j := 1 to n do
  if a[j] = q then ptr := j
return ptr;
```

Does it work?

search1 vs search2

Are the solutions equivalent?

- No!

Can one construct an example such that, say,

- search1 returns 3
- search2 returns 7 ?

But both solutions satisfy the specification (or don’t they?)

Searching/4, search3

An third idea:

Run through the array and
return the index of the value in the array.

search3
INPUT: A[1..n] (un)sorted array of integers, *q* an integer.
OUTPUT: index *j* such that A[*j*]=*q*. *NIL* if for all *j* ($1 \leq j \leq n$): A[*j*] $\neq q$

```
for j := 1 to n do
  if a[j] = q then return j
return NIL
```

Comparison of Solutions

Metaphor: shopping behavior when buying a beer:

- **search1**: scan products;
  stop as soon as a beer is found and go to the exit.
- **search2**: scan products until you get to the exit;
  if during the process you find a beer,
  put it into the basket
  (instead of the previous one, if any).
- **search3**: scan products;
  stop as soon as a beer is found
  and exit through next window.

Comparison of Solutions/2

- search1 and search3 return *the same result* (index of the first occurrence of the search value)
- search2 returns the index of the last occurrence of the search value
- search3 does not finish the loop
  (as a general rule, you better avoid this)

Beware: Array Indexes in Java/C/C++

- In pseudo-code, array indexes range from 1 to length
- In Java/C/C++, array indexes range from 0 to length-1
- Examples:
  - Pseudo-code

    ```
    for j := 1 to n do
    ```

    Java:

    ```
    for (j=0; j < a.length; j++) { …
    ```

  - Pseudo-code

    ```
    for j := n to 2 do
    ```

    Java:

    ```
    for (j=a.length-1; j >= 1; j--) { …
    ```

Suggested Exercises

- Implement the three variants of search (with input and output of arrays)
  - Compare the results
  - Add a counter for the number of cycles and return it, compare the result

- Implement them to scan the array in reverse order

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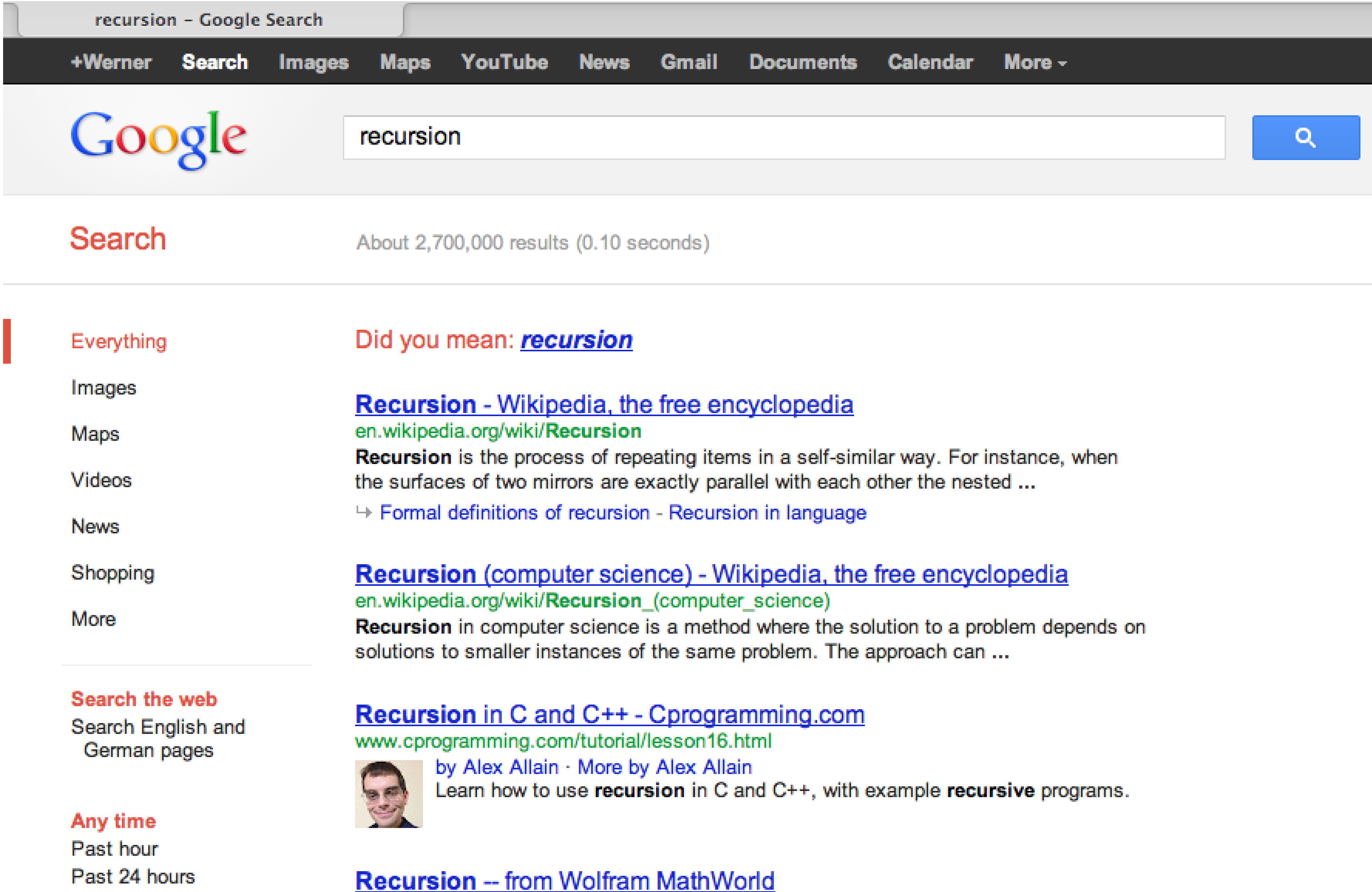
Recursion

An object is recursive if

- a part of the object refers to the entire object, or
- one part refers to another part and vice versa

(mutual recursion)


recursion – Google Search
+Werner
Search
Images
Maps
YouTube
News
Gmail
Documents
Calendar
More
Google
recursion
Search
About 2,700,000 results (0.10 seconds)
Everything
Images
Maps
Videos
News
Shopping
More
Search the web
Search English and German pages
Any time
Past hour
Past 24 hours
Did you mean: recursion
Recursion - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/Recursion
Recursion is the process of repeating items in a self-similar way. For instance, when the surfaces of two mirrors are exactly parallel with each other the nested ...
Formal definitions of recursion - Recursion in language
Recursion (computer science) - Wikipedia, the free encyclopedia
en.wikipedia.org/wiki/Recursion_(computer_science)
Recursion in computer science is a method where the solution to a problem depends on solutions to smaller instances of the same problem. The approach can ...
Recursion in C and C++ - Cprogramming.com
www.cprogramming.com/tutorial/lesson16.html
by Alex Allain · More by Alex Allain
Learn how to use recursion in C and C++, with example recursive programs.
Recursion -- from Wolfram MathWorld

Source: http://bluehawk.monmouth.edu/~rclayton/web-pages/s11-503/recursion.jpg

Recursion/2

- A recursive definition: a concept is defined by referring to itself.

  E.g., arithmetical expressions (like (3 * 7) - (9 / 3)):

  EXPR := VALUE | (EXPR OPERATOR EXPR)

- A recursive procedure: a procedure which calls itself

  Classical example: factorial, that is n! = 1 * 2 * 3 *...* n

  $n! = n * (n-1)!$

  … or is there something missing?

The Factorial Function

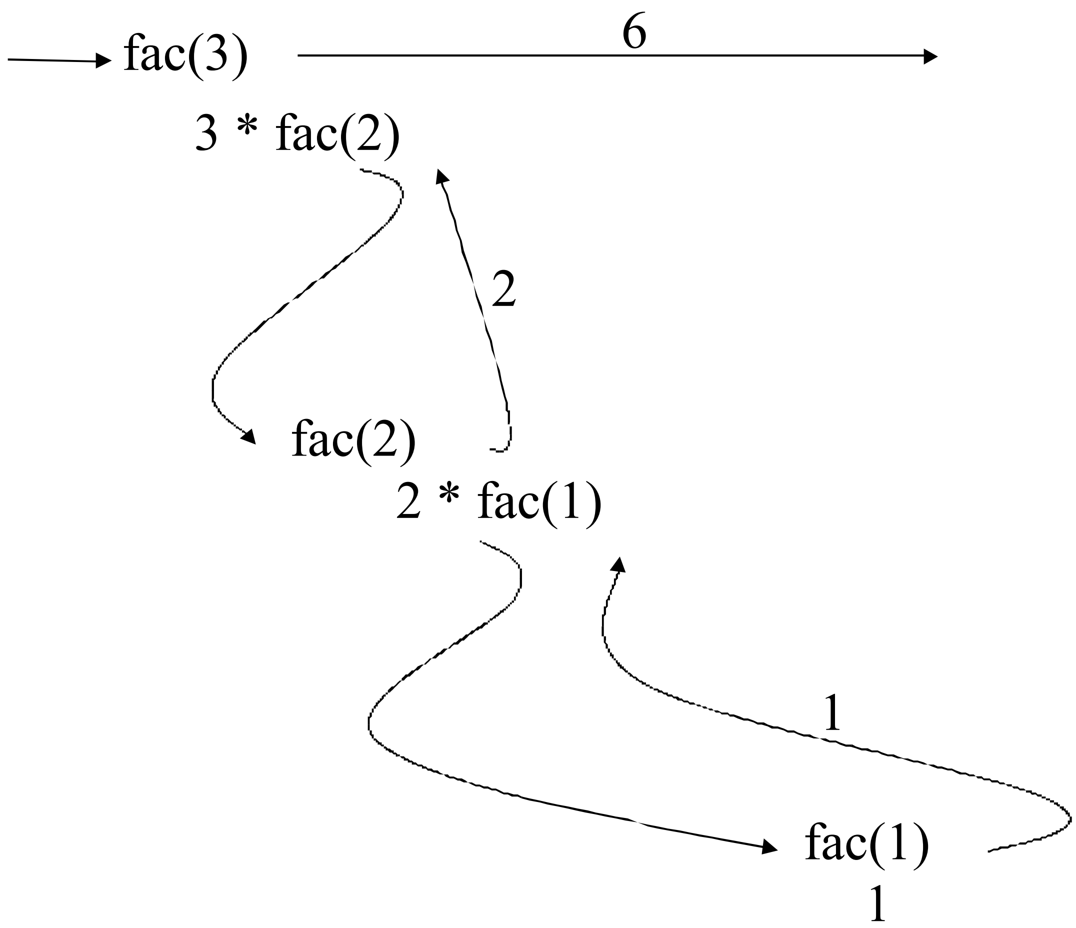
Pseudocode of factorial:

```
fac1
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac1(n)
  if n < 2 then return 1
  else return n * fac1(n-1)
```

This is a recursive procedure. A recursive procedure has

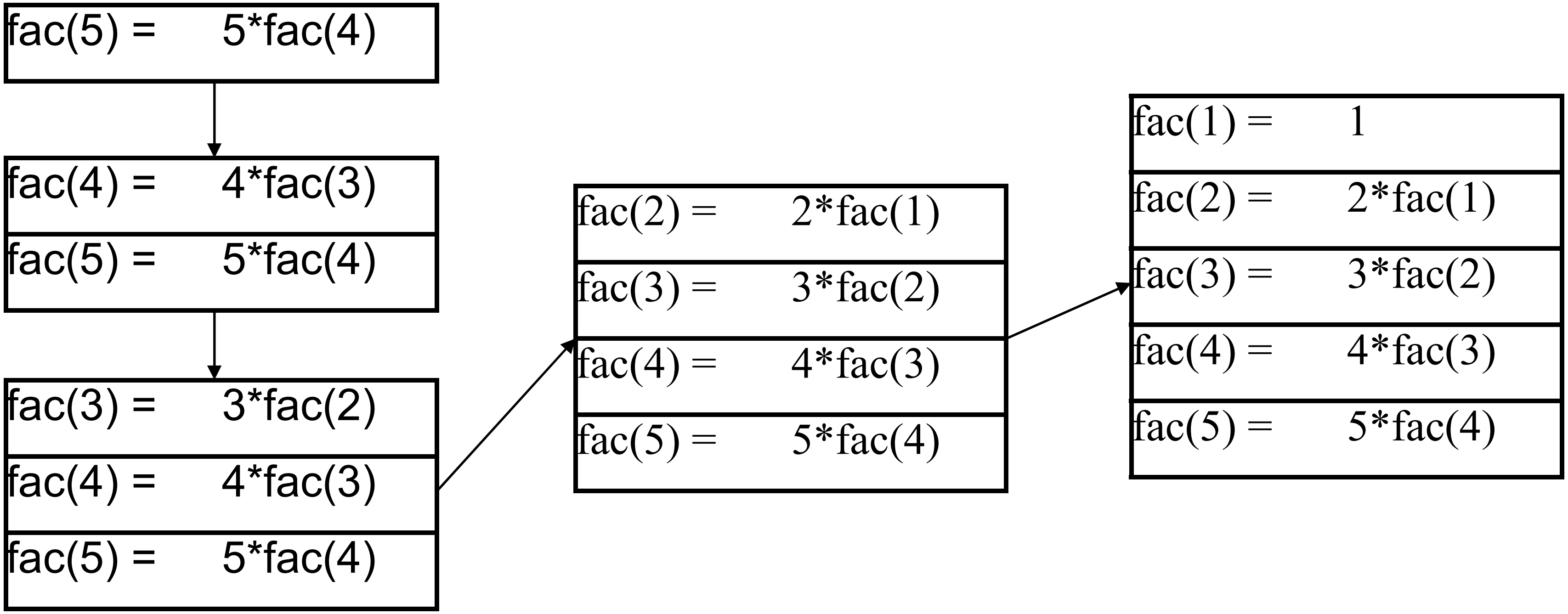
- a termination condition
  (determines when and how to stop the recursion).
- one (or more) recursive calls.

Tracing the Execution



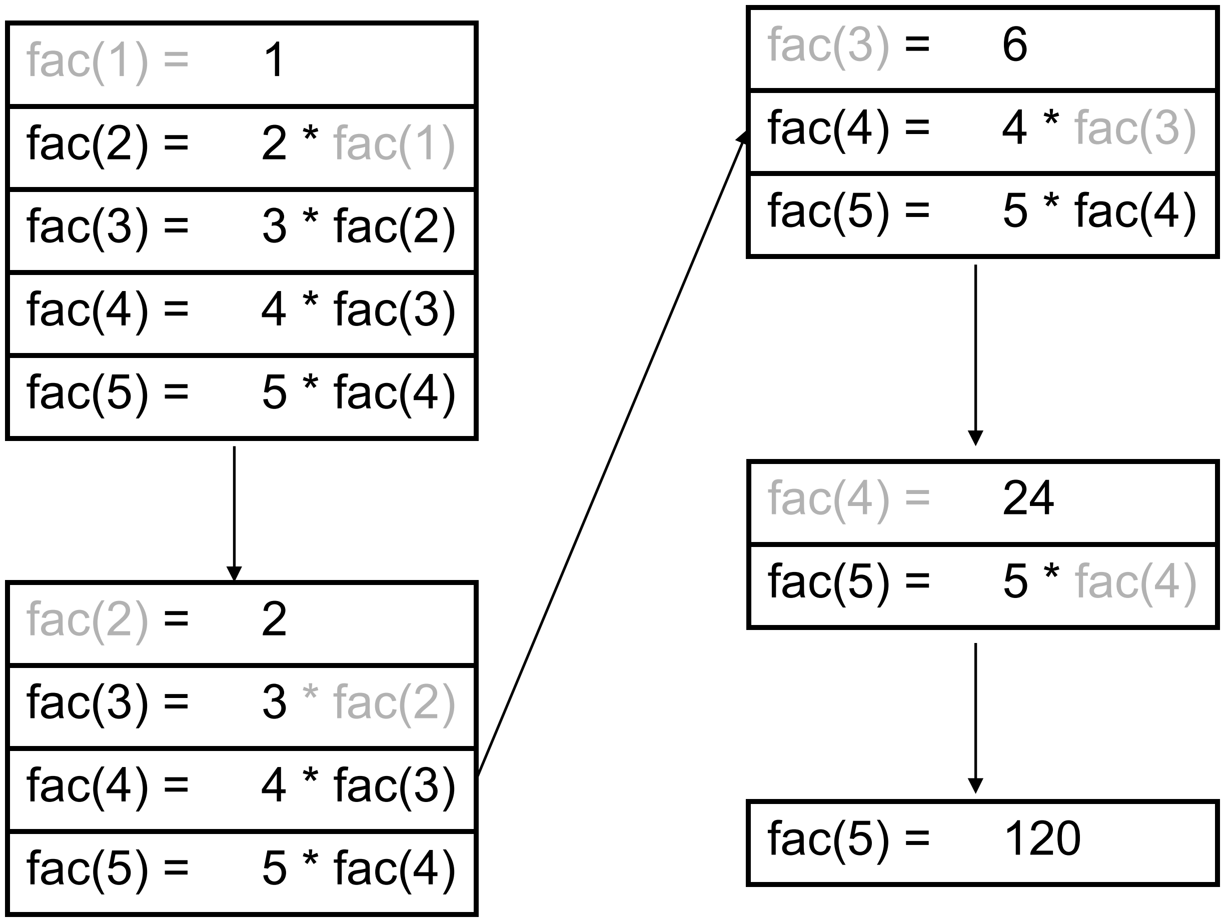
Bookkeeping

The computer maintains an activation stack for active procedure calls (→ compiler construction). Example for fac(5). The stack is built up.



Bookkeeping/2

Then the activation stack is reduced



Variants of Factorial

```
fac2
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac2(n)
  if n = 0 then return 1
  return n * fac2(n-1)
```

```
fac3
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac3(n)
  if n = 0 then return 1
  return n * (n-1) * fac3(n-2)
```

Analysis of the Variants

`fac2` is correct

- The return statement in the if clause terminates the function and, thus, the entire recursion.

`fac3` is incorrect

- Infinite recursion.
  The termination condition is never reached if n is odd:

```
fact(3)
= 3*2*fact(1)
= 3*2*1*0*fact(-1)
= ...
```

Variants of Factorial/2

```
fac4
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac4(n)
  if n <= 1 then return 1
  return n*(n-1)*fac4(n-2)
```

```
fac5
INPUT: n – a natural number.
OUTPUT: n! (factorial of n)

fac5(n)
  return n * fac5(n-1)
  if n < 2 then return 1
```

Analysis of the Variants/2

`fac4` is correct

- The return statement in the if clause terminates the function and, thus, the entire recursion.

`fac5` is incorrect

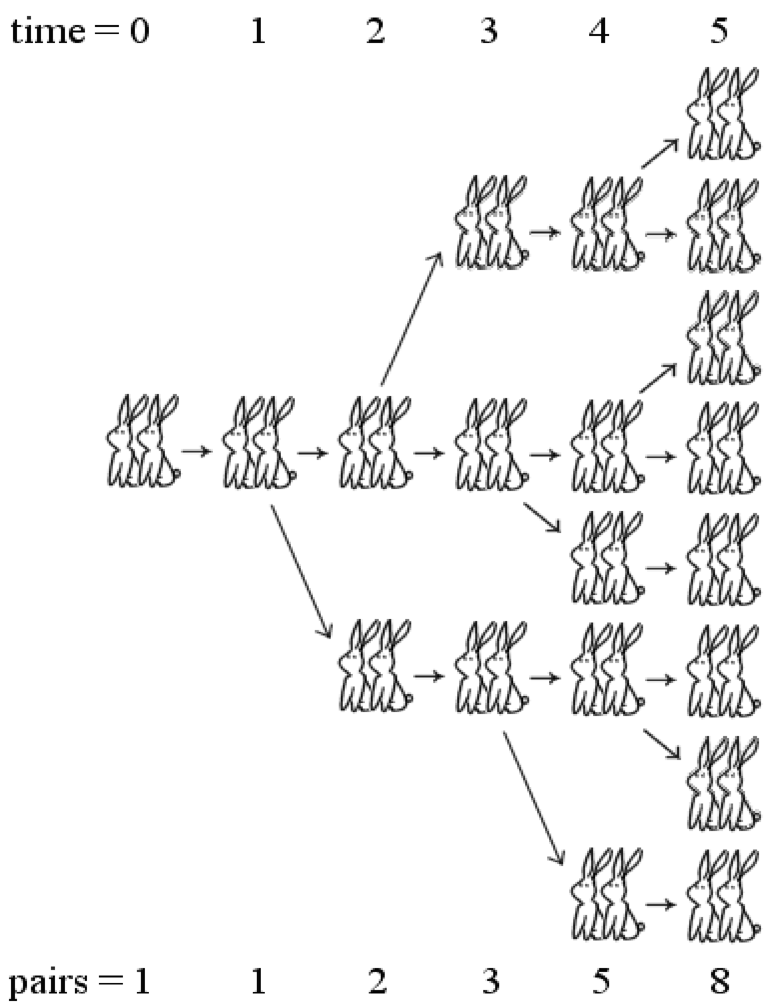
- Infinite recursion.
  The termination condition is never reached.

Counting Rabbits

Someone placed a pair of rabbits
in a certain place,
enclosed on all sides by a wall,
so as to find out
how many pairs of rabbits
will be born there in the course of one year,
it being assumed
that every month
a pair of rabbits produces another pair,
and that rabbits begin to bear
young two months after their own birth.

Leonardo di Pisa ("Fibonacci"),
Liber abacci, 1202

Counting Rabbits/2



Source: http://www.jimloy.com/algebra/fibo.htm

Fibonacci Numbers

Definition

- fib(1) = 1
- fib(2) = 1
- fib(n) = fib(n-1) + fib(n-2), n>2

Numbers in the series:

1, 1, 2, 3, 5, 8, 13, 21, 34, …

Fibonacci numbers occur all the time in nature:

- number of sections in apples, oranges, ...
- spirals in flowers and pine cones

Fibonacci Procedure

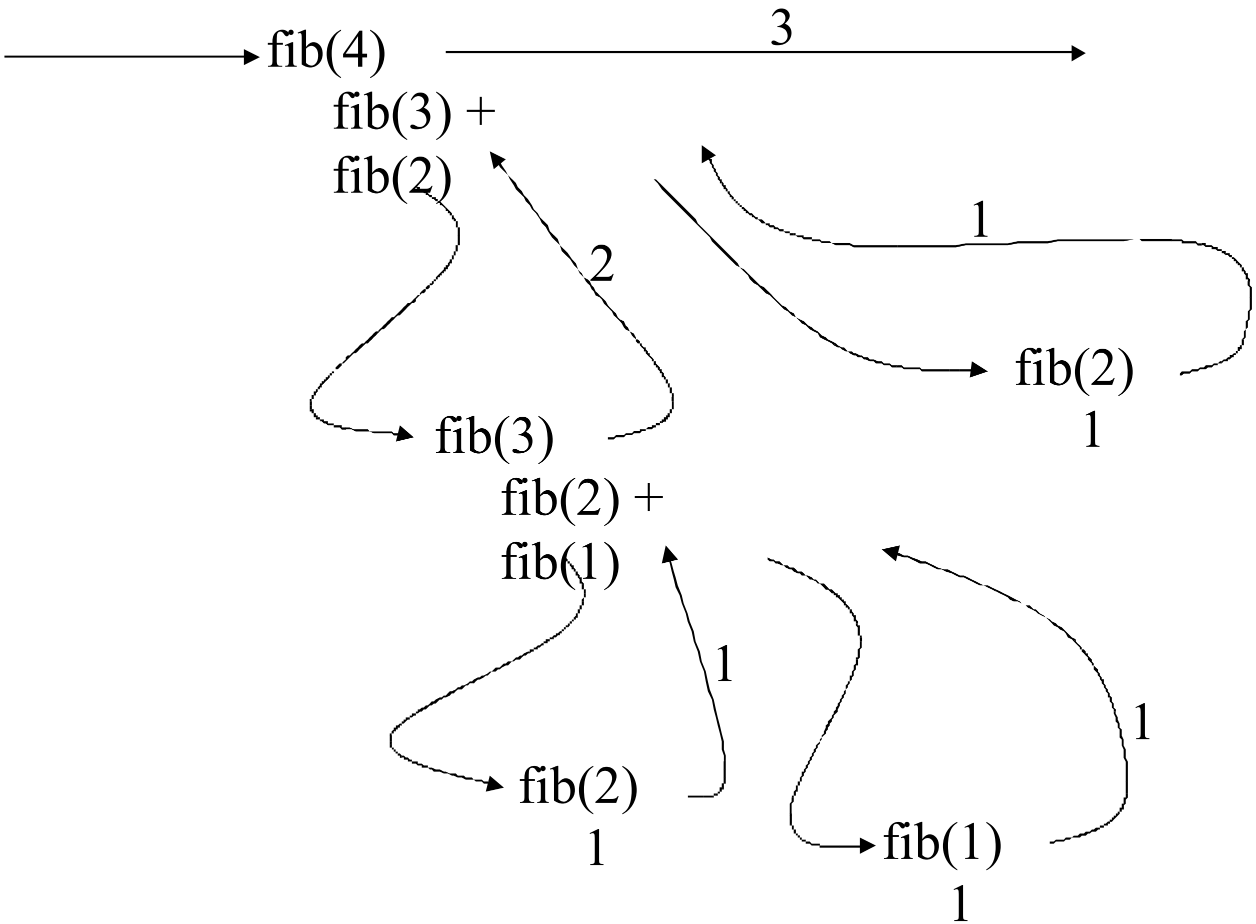
```
fib
INPUT: n – a natural number larger than 0.
OUTPUT: fib(n), the nth Fibonacci number.

fib(n)
  if n ≤ 2 then return 1
  else return fib(n-1) + fib(n-2)
```

A procedure with multiple recursive calls

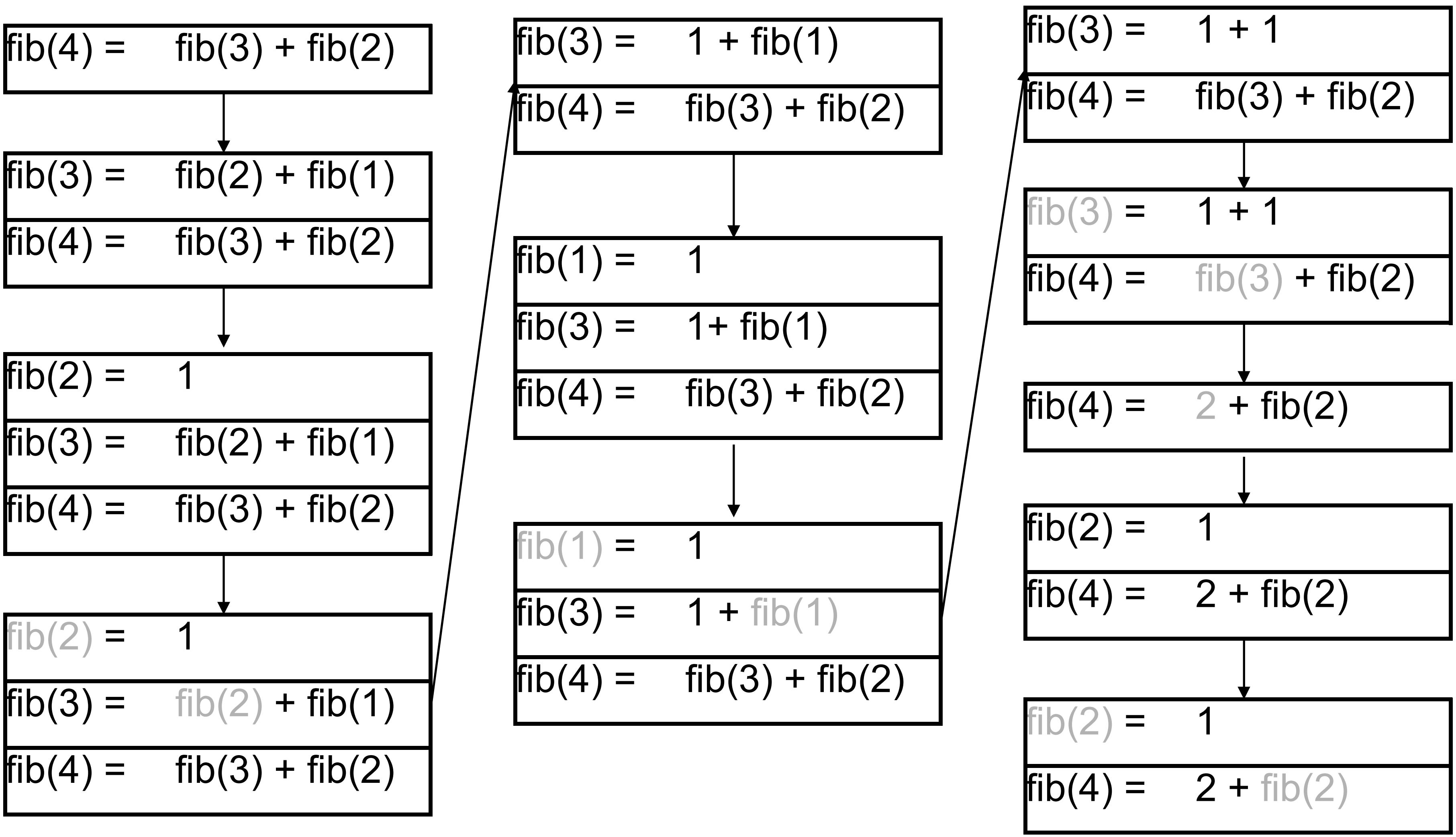
Fibonacci Procedure/2

```
public class fibclassic {

  static int fib(int n) {
    if (n <= 2) {return 1;}
    else {return fib(n - 1) + fib(n - 2);}
  }

  public static void main(String args[]) {
    System.out.println("Fibonacci of 5 is "
                          + fib(5));
  }
}
```

Tracing fib(4)



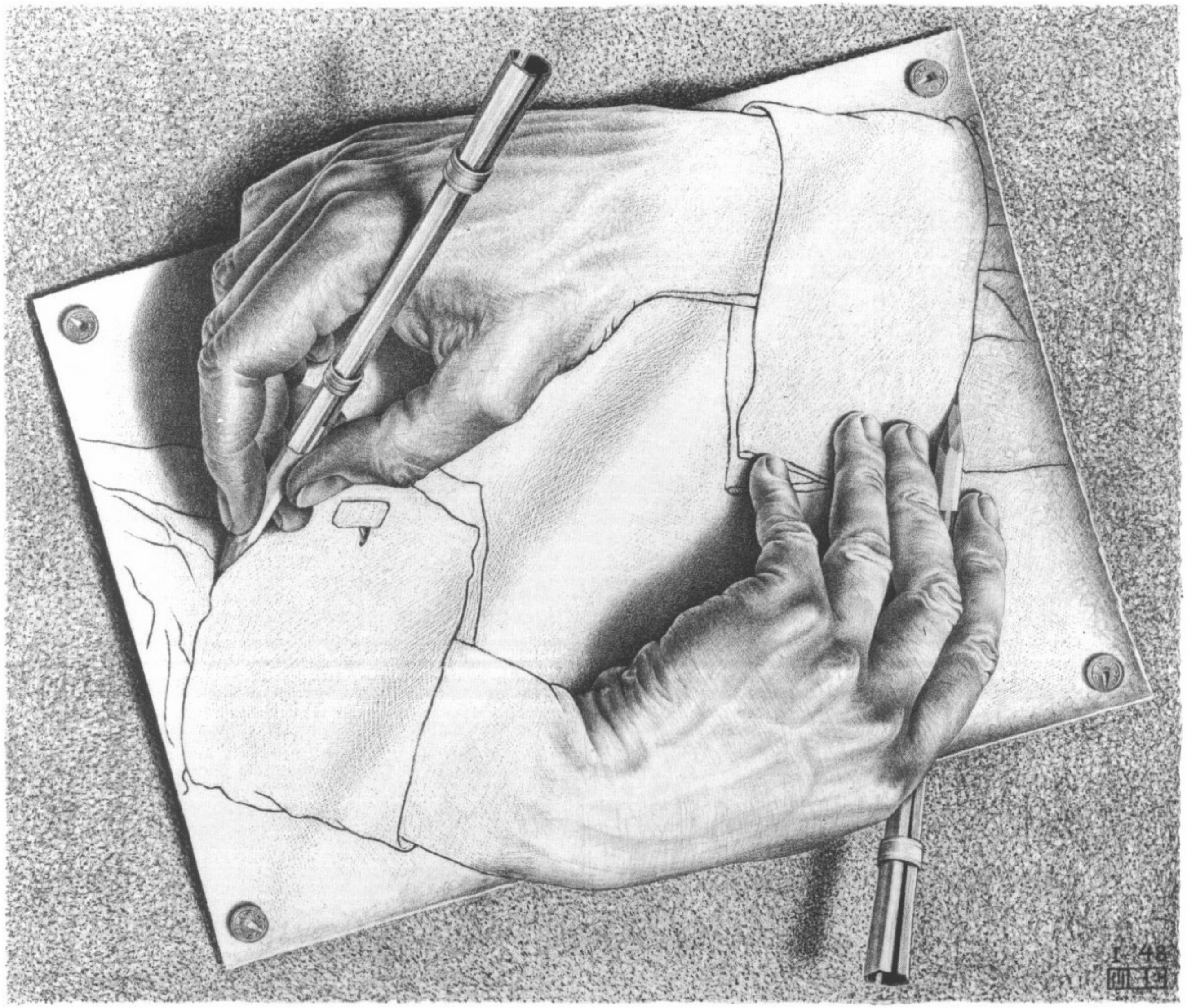
Bookkeeping



Questions

- How many recursive calls are made to compute *fib(n)*?
- What is the maximal height of the recursion stack during the computation?
- How are number of calls and height of the stack related to the size of the input?
- Can there be a recursive procedure for fib with fewer calls?
- How is the size of the result *fib(n)* related to the size of the input *n*?

Mutual Recursion



Source: http://britton.disted.camosun.bc.ca/escher/drawing_hands.jpg

Mutual Recursion Example

- Problem: Determine whether a natural number is even
- Definition of even:
  - 0 is even
  - N is even if N - 1 is odd
  - N is odd if N - 1 is even

Implementation of even

```
even
INPUT: n – a natural number.
OUTPUT: true if n is even; false otherwise

odd(n)
   if n = 0 then return FALSE
   return even(n-1)

even(n)
   if n = 0 then return TRUE
   else return odd(n-1)
```

- How can we determine whether N is odd?

Is Recursion Necessary?

- Theory: You can always resort to iteration and explicitly maintain a recursion stack.
- Practice: Recursion is elegant and in some cases the best solution by far.
- In the previous examples recursion was never appropriate since there exist simple iterative solutions.
- Recursion is more expensive than corresponding iterative solutions since bookkeeping is necessary.
- We shall see: recursion allows for elegant
  divide and conquer algorithms

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Sorting

- Sorting is a classical and important algorithmic problem.
  - For which operations is sorting needed?
  - Which systems implement sorting?
- We look at sorting arrays
  (in contrast to files, which restrict random access)
- A key constraint are the restrictions on the space:
  in-place sorting algorithms (no extra RAM).
- The run-time comparison is based on
  - the number of comparisons (C) and
  - the number of movements (M).

Sorting

- Simple sorting methods use roughly ***n * n*** comparisons
  - Insertion sort
  - Selection sort
  - Bubble sort

- Fast sorting methods use roughly ***n * log n*** comparisons
  - Merge sort
  - Heap sort
  - Quicksort

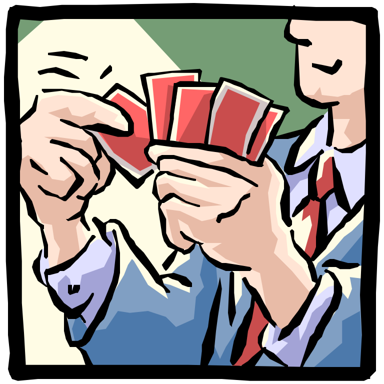
What's the point of studying those simple methods?

Example 2: Sorting

INPUT
sequence of *n* numbers

$a_1, a_2, a_3,\ldots,a_n$

2 5 4 10 7



OUTPUT
a permutation of the input sequence of numbers

$b_1, b_2, b_3,\ldots, b_n$

2 4 5 7 10

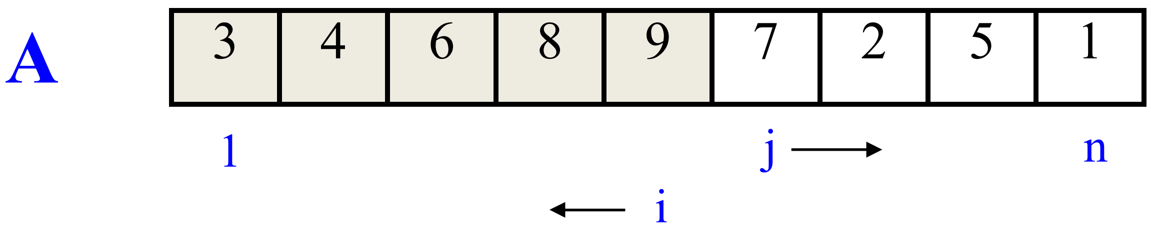
Correctness (requirements for the output)
For any given input the algorithm halts with the output:

- $b_1 \le b_2 \le b_3 \le \ldots \le b_n$
- $b_1, b_2, b_3, \ldots, b_n$ is a permutation of $a_1, a_2, a_3,\ldots,a_n$

Properties of a Sorting Algorithm

- Efficient: has low (worst case) runtime
- In place: needs (almost) no additional space (fixed number of scalar variables)
- Adaptive: performs little work if the array is already (mostly) sorted
- Stable: does not change the order of elements with equal key values
- Online: can sort data as it receives them

Insertion Sort



Strategy

- Start with one sorted card.
- Insert an unsorted card at the correct position in the sorted part.
- Continue until all unsorted cards are inserted/sorted.

44	**55**	**12**	**42**	**94**	**18**	**06**	**67**
44	55	**12**	**42**	**94**	**18**	**06**	**67**
12	44	55	**42**	**94**	**18**	**06**	**67**
12	42	44	55	**94**	**18**	**06**	**67**
12	42	44	55	94	**18**	**06**	**67**
12	18	42	44	55	94	**06**	**67**
06	12	18	42	44	55	94	**67**
06	12	18	42	44	55	67	94

Insertion Sort/2

```
INPUT: A[1..n] – an array of integers
OUTPUT: permutation of A s.t. A[1]≤A[2]≤...≤A[n]

for j := 2 to n do // A[1..j-1] sorted
  key := A[j]; i := j-1;
  while i > 0 and A[i] > key do
    A[i+1] := A[i];  i--;
  A[i+1] := key
```

The number of comparisons during the jth iteration is

– at least 1: Cmin = $\sum_{j=2}^{n} 1$ = *n - 1*

– at most j-1: Cmax = $\sum_{j=2}^{n} j-1$ = *(n*n - n)/2*

Insertion Sort/3

- The number of comparisons during the jth iteration is:
  - j/2 average: Cavg = $\sum_{j=2}^{n} j/2$ = (n*n + n – 2)/4

- The number of movements is Ci+1:
  - Mmin = $\sum_{j=2}^{n} 2$ = 2*(n-1),
  - Mavg = $\sum_{j=2}^{n} j/2+1$ = (n*n + 5n - 6)/4
  - Mmax = $\sum_{j=2}^{n} j$ = (n*n + n - 2)/2

Selection Sort

A

1	2	3	4	5	7	8	9	6

1 j → n
i →

Strategy

- Start empty handed.
- Enlarge the sorted part by switching the first element of the unsorted part with the smallest element of the unsorted part.
- Continue until the unsorted part consists of one element only.

44	55	12	42	94	18	06	67
06	55	12	42	94	18	44	67
06	12	55	42	94	18	44	67
06	12	18	42	94	55	44	67
06	12	18	42	94	55	44	67
06	12	18	42	44	55	94	67
06	12	18	42	44	55	94	67
06	12	18	42	44	55	67	94

Selection Sort/2

```
INPUT: A[1..n] – an array of integers
OUTPUT: a permutation of A such that A[1]≤A[2]≤…≤A[n]

for j := 1 to n-1 do // A[1..j-1] sorted and minimum
  key := A[j]; ptr := j
  for i := j+1 to n do
    if A[i] < key then ptr := i; key := A[i];
  A[ptr] := A[j]; A[j] := key
```

The number of comparisons is independent of the original ordering (this is a less natural behavior than insertion sort):

$$C = \sum_{j=1}^{n-1}(n-j) = \sum_{k=1}^{n-1} k = (n*n - n)/2$$

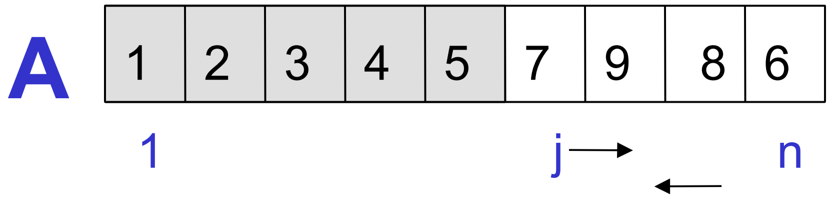
Selection Sort/3

The number of movements is:

Mmin = $\sum_{j=1}^{n-1} 3$ = 3*(n-1)

Mmax = $\sum_{j=1}^{n-1} n\text{-}j+3$ = (n*n – n)/2 + 3*(n-1)

Bubble Sort



Strategy

- Start from the back and compare pairs of adjacent elements.
- Switch the elements if the larger comes before the smaller.
- In each step the smallest element of the unsorted part is moved to the beginning of the unsorted part and the sorted part grows by one.

44	55	12	42	94	18	06	67
06	44	55	12	42	94	18	67
06	12	44	55	18	42	94	67
06	12	18	44	55	42	67	94
06	12	18	42	44	55	67	94
06	12	18	42	44	55	67	94
06	12	18	42	44	55	67	94
06	12	18	42	44	55	67	94

Bubble Sort/2

```
INPUT: A[1..n] – an array of integers
OUTPUT: permutation of A s.t. A[1]≤A[2]≤…≤A[n]

for j := 2 to n do // A[1..j-2] sorted and minimum
  for i := n to j do
    if A[i-1] > A[i] then
      key := A[i-1];
            A[i-1] := A[i];
      A[i]:=key
```

The number of comparisons is indepen-dent of the original ordering:

$$C = \sum_{j=2}^{n} (n - j + 1) = (n*n - n)/2$$

Bubble Sort/3

The number of movements is:

Mmin = 0

Mmax = $\sum_{j=2}^{n} 3(n-j+1)$ = 3*n*(n - 1)/2

Mavg = $\sum_{j=2}^{n} 3(n-j+1)/2$= 3*n*(n - 1)/4

Sorting Algorithms: Properties

Which algorithm has which property?

	Adaptive	Stable	Online
Insertion Sort			
Selection Sort			
Bubble Sort			

Summary

- Precise problem specification is crucial.
- Precisely specify Input and Output.
- Pseudocode, Java, C, … is largely equivalent for our purposes.
- Recursion: procedure/function that calls itself.
- Sorting: important problem with classic solutions.