

Data Structures and Algorithms

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Part 5

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Data Structures and Algorithms

Part 5

- Dynamic Data Structures
 - Records, Pointers
 - Lists
- Abstract Data Types
 - Stack, Queue
 - Ordered Lists
 - Priority Queue

Data Structures and Algorithms

Part 5

- **Dynamic Data Structures**
 - Records, Pointers
 - Lists
- **Abstract Data Types**
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Records

- Records are used to group a number of (different) fields.
- A *person* record may group *name, age, city, nationality, ssn*.
- The grouping of fields is a basic and often used technique.
- It is available in all programming languages.

Records in Java

- In java a *class* is used to group fields:

```
class rec { int a; int b; };  
public class dummy {  
    static rec r;  
    public static void main(String args[]) {  
        r = new rec();  
        r.a = 15; r.b = 8;  
        System.out.print("Adding a and b yields ");  
        System.out.println(r.a + r.b);  
    }  
}
```

Records in C

- In C a *struct* is used to group fields:

```
struct rec {
    int a;
    int b;
};

struct rec r;

int main() {
    r.a = 5; r.b = 8;
    printf("The sum of a and b is %d\n", r.a + r.b);
}

// gcc -o dummy dummy.c ; ./dummy
```


Recursive Data Structures

- The counterpart of recursive functions are recursively defined data structures.
- Example: list of integers

$$\text{list} = \left\{ \begin{array}{l} \text{integer} \\ \text{integer, list} \end{array} \right\}$$

- In C:

```
struct list {  
    int value;  
    Struct list * tail; };
```

Recursive Data Structures/2

- The **storage space** of recursive data structures is not known in advance.
 - It is determined by the number of elements that will be stored in the list.
 - This is only known during **runtime** (program execution).
 - The list can **grow** and shrink **during** program execution.

Recursive Data Structures/3

- There must be a mechanism to **constrain** the initial **storage space** of recursive data structures (it is potentially infinite).
- There must be a mechanism to **grow and shrink** the storage space of a recursive data structures during program execution.

Pointers

- A common technique is to **allocate** the storage space (memory) **dynamically**.
- That means the storage space is allocated when the **program executes**.
- The compiler only reserves space for an **address** to these dynamic parts.
- These addresses are called **pointers**.

Pointers/2

1

- integer **i**
- pointer **p** to an integer (**55**)
- record **r** with integer components **a** (**17**) and **b** (**24**)
- pointer **s** that points to **r**

Address	Variable	Memory
1af782	i	23
1af783	p	1af789
1af784	r	17
1af785		24
1af786	s	1af784
1af787		
1af788		
1af789		55
1af78a		

Pointers in C

1. To follow (chase, **dereference**) a pointer variable we write `*p`
 - `*p = 12`
2. To get the **address** of a variable `i` we write `&i`
 - `p = &i`
3. To **allocate memory** we use `malloc(sizeof(Type))`
 - `p = malloc(sizeof(int))`
4. To **free storage space** pointed to by a pointer `p` we use `free`
 - `free(p)`

Pointers in C/2

- To declare a pointer to type T we write T*
 - `int* p`
- Note that * is used for two purposes:
 - **Declaring** a pointer variable
`int* p`
 - **Following** a pointer
`*p = 15`
- In other languages these are syntactically different.

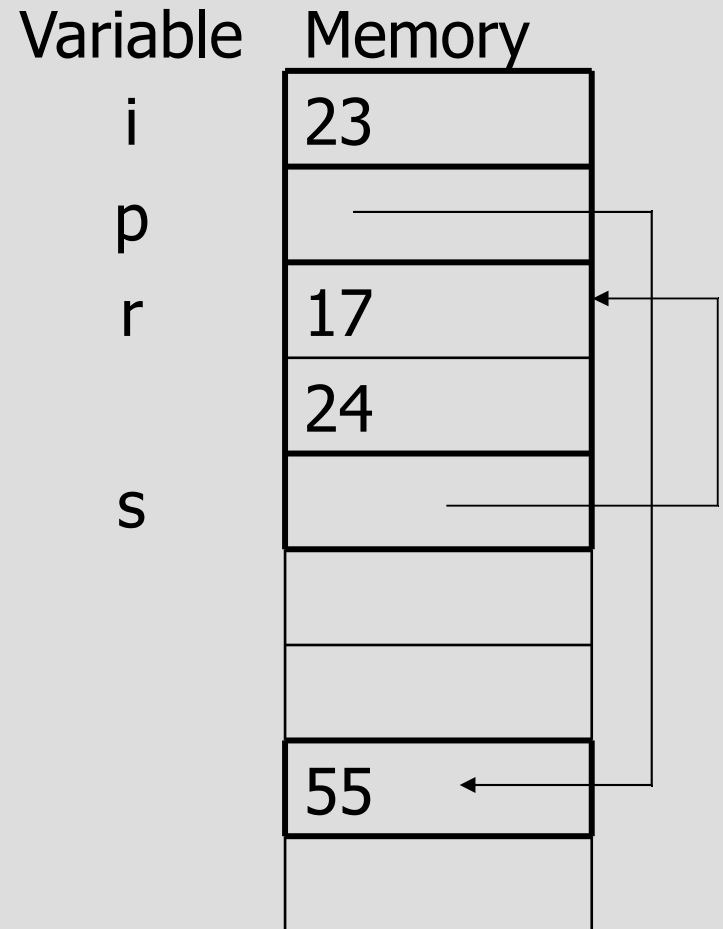
Pointers in C/3

	Address	Variable	Memory
• <code>int i</code> <code>i = 23</code>	1af782	<code>i</code>	23
• <code>int* p</code> <code>p = malloc(sizeof(int))</code>	1af783	<code>p</code>	1af789
<code>*p = 55</code>	1af784	<code>r</code>	17
	1af785		24
• <code>struct rec r</code> <code>rec.a = 17</code> <code>rec.b = 24</code>	1af786	<code>s</code>	1af784
	1af787		
	1af788		
• <code>struct rec* s;</code> <code>s = &r</code>	1af789		55
	1af78a		

Pointers in C/4

Alternative notation:

Address	Variable	Memory
1af782	i	23
1af783	p	1af789
1af784	r	17
1af785		24
1af786	s	1af784
1af787		
1af788		
1af789		55
1af78a		



Pointers/3

- Pointers are only **one** mechanism to implement **recursive data structures**.
- The programmer does not have to be aware of their existence. The **storage space** can be managed **automatically**.
- In **C** the storage space has to be managed **explicitly**.
- In **Java**
 - an **object** is implemented as a **pointer**.
 - **creation** of objects (new) **automatically** allocates **storage** space.
 - **accessing** an object will **automatically** follow the **pointer**.
 - **deallocation** is done **automatically** (garbage collection).

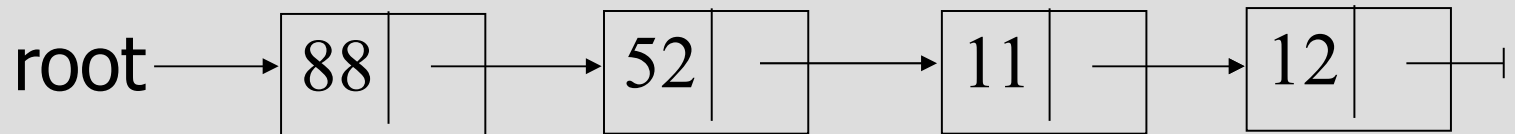
Data Structures and Algorithms

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Lists

- A list of integers:



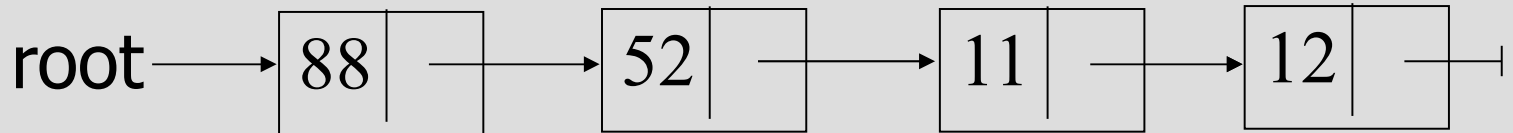
- Corresponding declaration in Java:

```
class node {  
    int val;  
    node next;  
}  
  
node root;
```

- Accessing a field: `p.a`

Lists/2

- A list of integers:



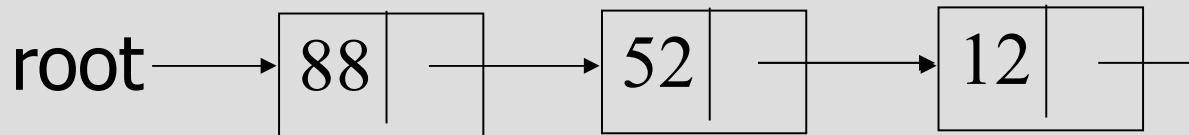
- Corresponding declaration in C:

```
struct node {  
    int val;  
    struct node* next;  
}  
  
struct node* root;
```

- Accessing a field: $(*p) . a = p \rightarrow a$

Lists/3

- Populating the list with integers (java):



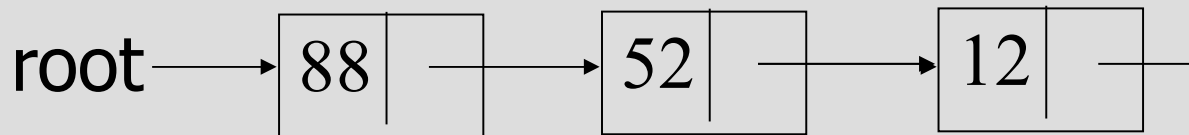
```
root = new node();  
root.val = 88;  
root.next = new node();
```

```
p = root.next;  
p.val = 52;  
p.next = new node();
```

```
p = p.next;  
p.val = 12;  
p.next = null;
```

Lists/4

- Populating the list with integers (C):

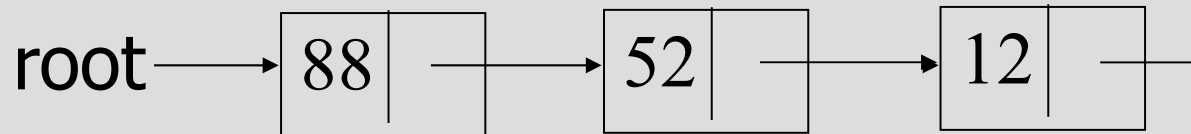


```
root = malloc(sizeof(struct node));  
root->val = 88;  
root->next = malloc(sizeof(struct node));  
  
p = root->next;  
p->val = 52;  
p->next = malloc(sizeof(struct node));  
  
p = p->next;  
p->val = 12;  
p->next = NULL;
```

List Traversal

6
5

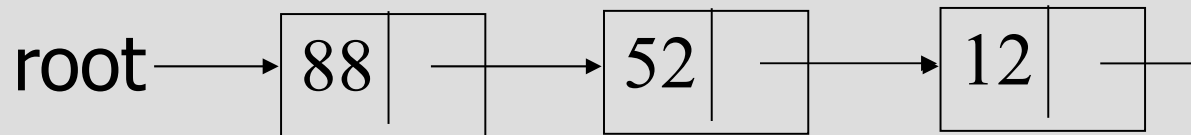
- Print all elements of a list (java):



```
p = root;
while (p != null) {
    System.out.printf("%d,", p.val);
    p = p.next
}
System.out.printf("\n");
```


List Traversal

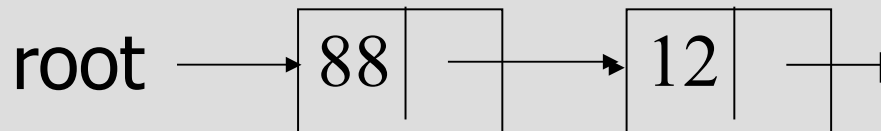
- Print all elements of a list (C):



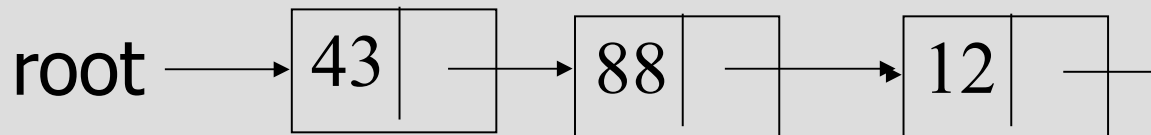
```
p = root;
while (p != null) {
    printf("%d,", p->val);
    p = p->next
}
printf("\n");
```

List Insertion

- Insert 43 at the beginning (Java):

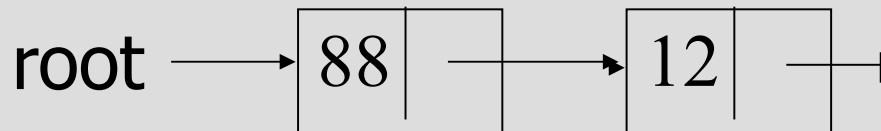


```
p = new node();  
p.val = 43  
p.next = root;  
root = p;
```

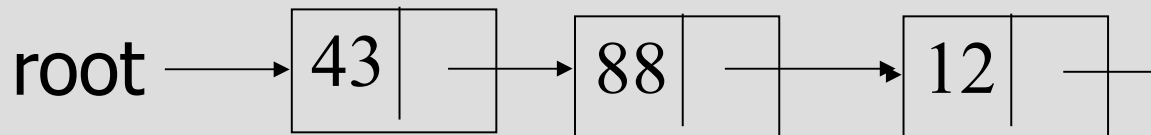


List Insertion

- Insert 43 at the beginning (C):



```
p = malloc(sizeof(struct node));  
p->val = 43  
p->next = root;  
root = p;
```



List Insertion/2

Insert 43 at end (Java): root →

88	→
----	---

 →

12	→
----	---

```
if (root == null) {
    root = new node();
    root.val = 43;
    root.next = null;
} else {
    q = root;
    while (q.next != null) { q = q.next; }
    q.next = new node();
    q.next.val = 43;
    q.next.next = null;
}
```

List Insertion/2

Insert 43 at end (C): root →

88	→
----	---

 →

12	→
----	---

```
if (root == null) {
    root = malloc(sizeof(struct node));
    root->val = 43;
    root->next = null;
} else {
    q = root;
    while (q->next != null) { q = q->next; }
    q->next = malloc(sizeof(struct node));
    q->next->val = 43;
    q->next->next = null;
}
```

List Deletion

Delete element x from a non-empty list (Java):

```
p = root;
if (p.val == x) {
    root = p.next;
} // no need of freeing in java
else {
    while (p.next != null && p.next.val != x) {
        p = p.next;
    }
    tmp = p.next;
    p.next = tmp.next;
}
```

List Deletion

Delete element x from a non-empty list (C):

```
p = root;
if (p->val == x) {
    root = p->next;
    free(p);
} else {
    while (p->next != null && p->next->val != x) {
        p = p->next;
    }
    tmp = p->next;
    p->next = tmp->next;
    free(tmp);
}
```

List

- Cost of operations:
 - Insertion at beginning: $O(1)$
 - Insert at end: $O(n)$
 - Check isEmpty: $O(1)$
 - Delete from the beginning: $O(1)$
 - Search: $O(n)$
 - Delete: $O(n)$
 - Print: $O(n)$

Suggested exercises

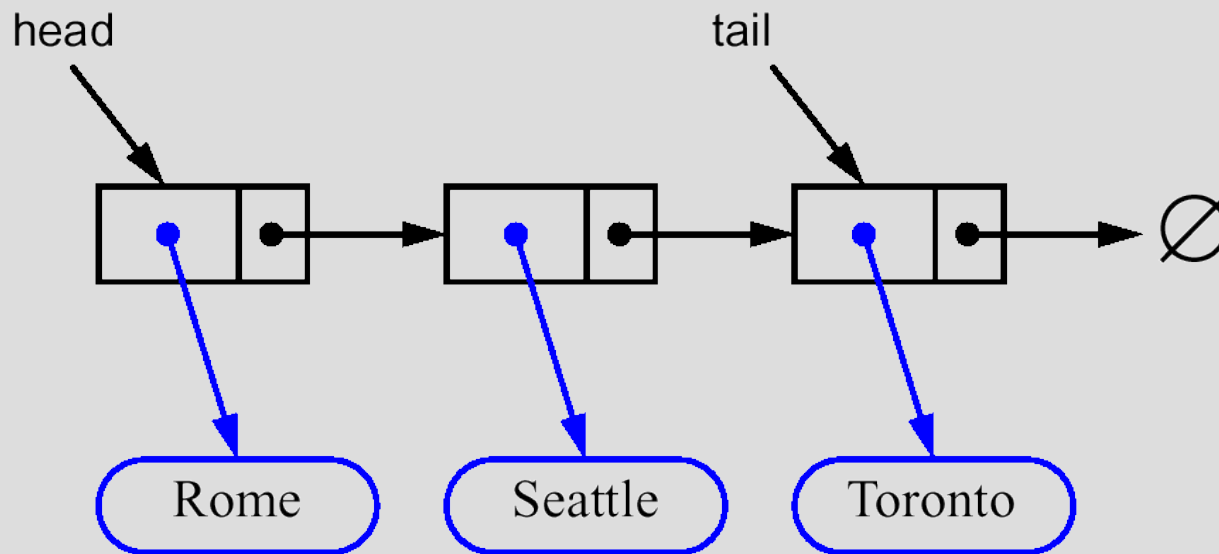
- Implement a linked list with the following functionalities: isEmpty, insertFirst, insertLast, search, deleteFirst, delete, print
- As before, with a recursive version of: insertLast, search, delete, print
 - are recursive versions simpler?
- Implement an efficient version of print which prints the list in reverse order

Variants of linked lists

- Linked lists with explicit head/tail
- Doubly linked lists

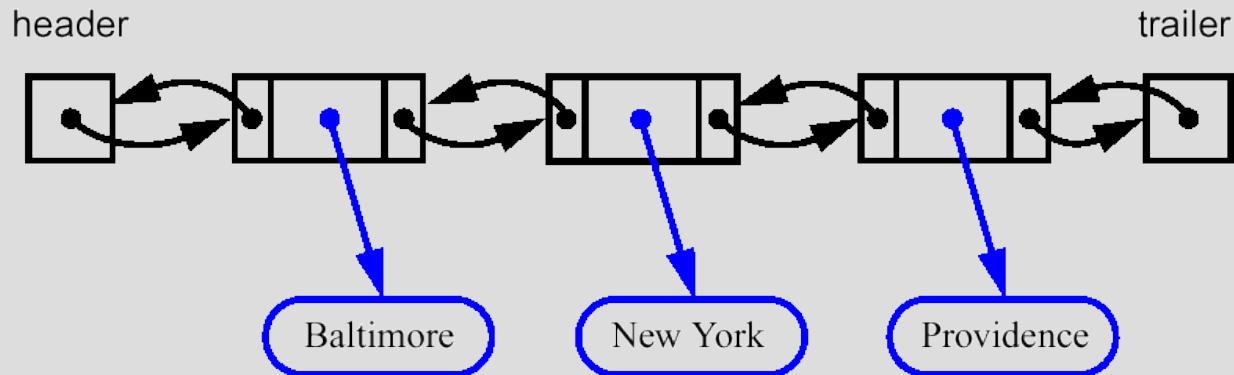
List with Explicit Head/Tail

- Instead of a *root* we can have a *head* and *tail*:

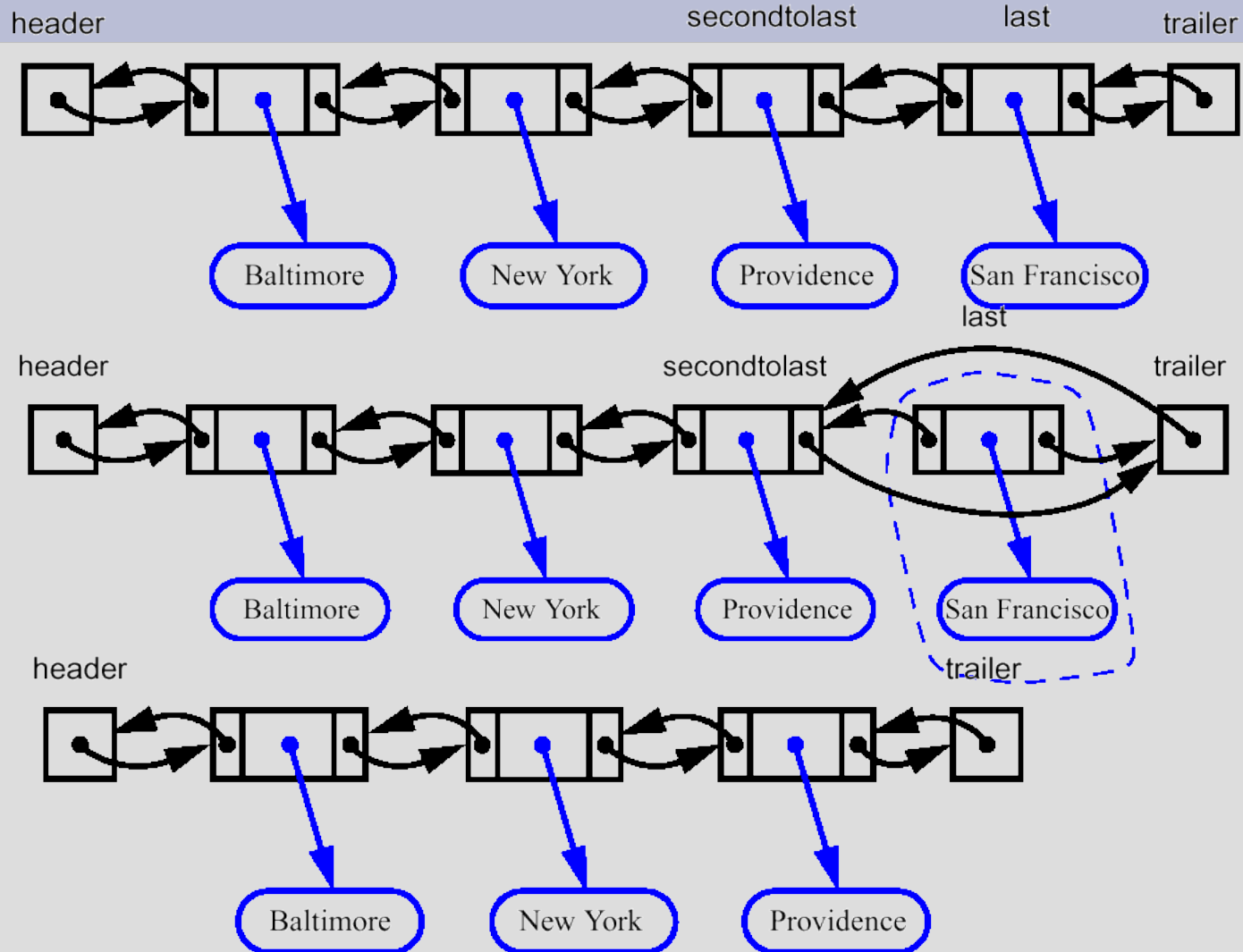


Doubly Linked Lists

- To be able to quickly navigate back and forth in a list we use **doubly linked lists**.



- A node of a doubly linked list has a **next** and a **prev** link.



Data Structures and Algorithms

Part 5

- Dynamic Data Structures
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- **Abstract Data Types**
 - Stack, Queue
 - Ordered Lists
 - Priority Queue

Abstract Data Types (ADTs)

- An *ADT* is a mathematically specified entity that defines a set of its *instances*, with:
 - a specific *interface* – a collection of signatures of operations that can be invoked on an instance.
 - a set of *axioms* (*preconditions* and *postconditions*) that define the semantics of the operations (i.e., what the operations do to instances of the ADT, but not how).

ADTs/2

- Why ADTs?
 - ADTs allows to break work into pieces that can be worked on independently – without compromising correctness.
 - They serve as *specifications of requirements* for the building blocks of solutions to algorithmic problems.
 - ADTs encapsulate *data structures* and algorithms that *implement* them.

ADTs/3

- Provides a language to talk on a higher level of abstraction.
- Allows to separate the concerns of *correctness* and the *performance analysis*
 1. Design the algorithm using an ADT
 2. Count how often different ADT operations are used
 3. Choose implementations of ADT operations
- ADT = Instance variables + procedures
(Class = Instance variables + methods)

ADTs/4

- We discuss a number of popular ADTs:
 - Stacks, Queues
 - Ordered Lists
 - Priority Queues
 - Trees (next chapter)
- They illustrate the use of lists and arrays.

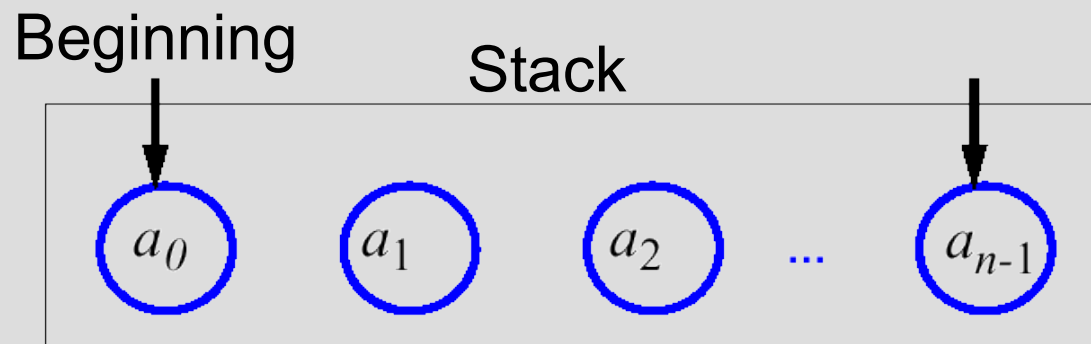
Data Structures and Algorithms

Part 5

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- Abstract Data Types
 - **Stack, Queue**
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Stacks

- In a stack, insertions and deletions follow the **last-in-first-out** (LIFO) principle.
- Thus, the element that has been in the queue for the shortest time are deleted.
 - Example: OS stack, ...
- Solution: Elements are inserted at the **beginning** (push) and removed from the **beginning** (pop).



Stacks/2

- Appropriate data structure:
 - Linked list, one root: good
 - Array: fastest, limited in size
 - Doubly linked list: unnecessary

An Array Implementation

- Create a stack using an array
- A maximum size N is specified.
- The stack consists of an N -element array S and one integer variable *count*:
 - *count*: index of the front element (head)
 - *count* represents the position where to insert next element, and the number of elements in the stack

An Array Implementation/2

```
int size()  
    return count
```

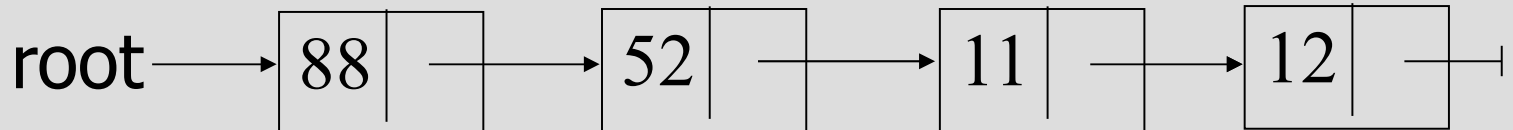
```
int isEmpty()  
    return (count == 0)
```

```
Element pop()  
    if isEmpty() then Error  
    x = S[count-1]  
    count--;  
    return x
```

```
void push(element x)  
    if count==N then Error;  
    S[count] = x;  
    count++;
```

A Linked-List implementation

- A list of integers:



- Insert from the top of the list

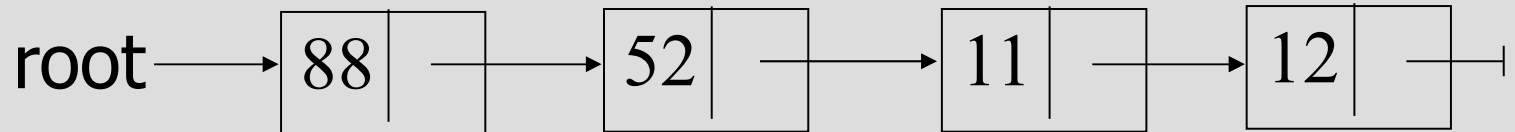
```
push(element x) :
```

```
node p = new node();  
p.info = x;  
p.next = root;  
root = p;
```

- Constant-time operation!

A Linked-List implementation

- A list of integers:



- Extract from the top of the list

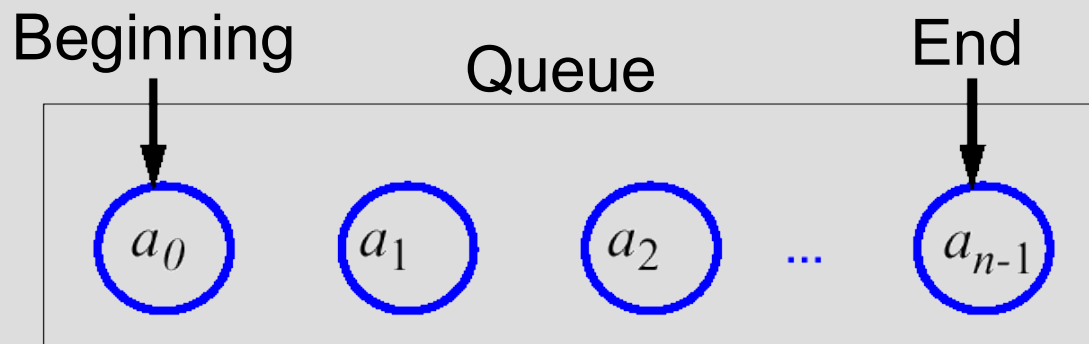
```
Element pop() :
```

```
x = root.info;  
root = root.next;  
Return x;
```

- Constant-time operation!

Queues

- In a queue insertions and deletions follow the **first-in-first-out** (FIFO) principle.
- Thus, the element that has been in the queue for the longest time are deleted.
 - Example: Printer queue, ...
- Solution: Elements are inserted at the **end** (enqueue) and removed from the **beginning** (dequeue).



Queues/2

- Appropriate data structure:
 - Linked list, root: inefficient insertions
 - Linked list, head/tail: good
 - Array: fastest, limited in size
 - Doubly linked list: unnecessary

An Array Implementation

- Create a queue using an array in a circular fashion
- A maximum size N is specified.
- The queue consists of an N -element array Q and two integer variables:
 - f , index of the front element (head, for dequeue)
 - r , index of the element after the last one (tail, for enqueue)



An Array Implementation/2

- “wrapped around” configuration



- what does $f=r$ mean?

An Array Implementation/3

```
int size()  
    return (N-f+r) mod N
```

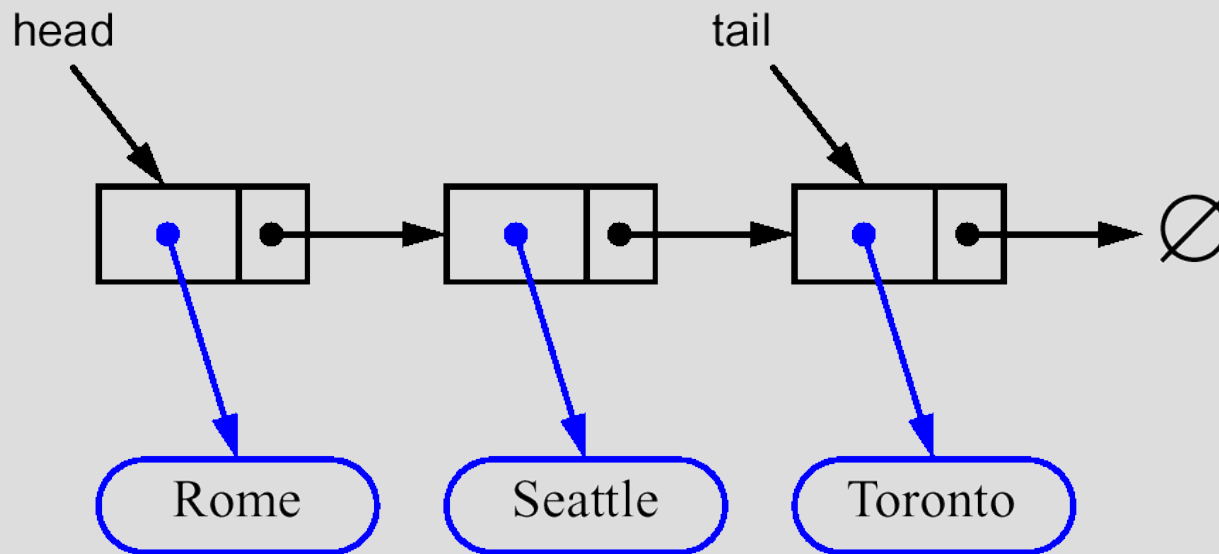
```
int isEmpty()  
    return size() == 0
```

```
Element dequeue()  
    if isEmpty() then Error  
    x = Q[f]  
    f = (f+1) mod N  
    return x
```

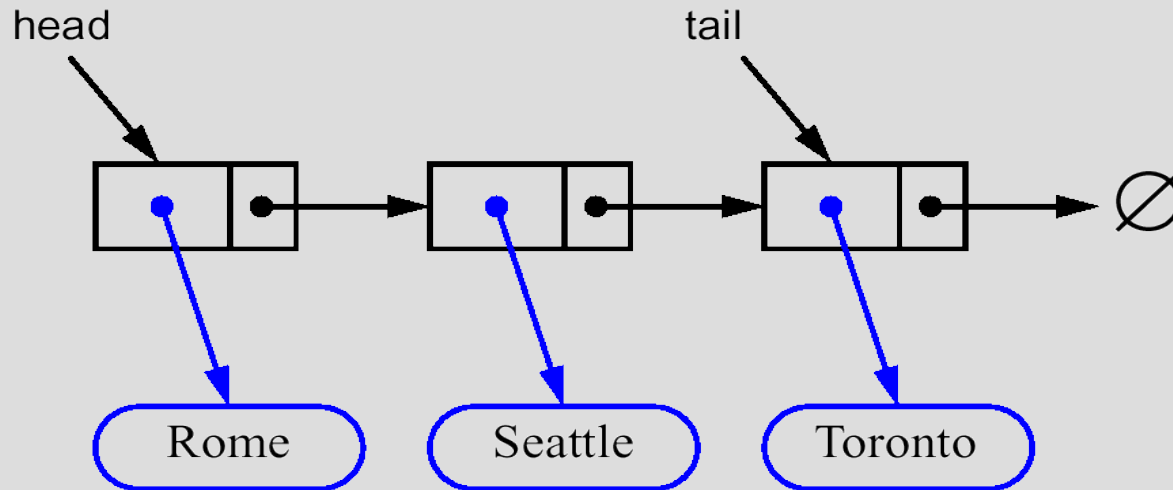
```
void enqueue()  
    if size() == N-1 then Error  
    Q[r] = x  
    r = (r+1) mod N
```

A Linked-List Implementation

- Use linked-list with head and tail
- Insert in tail, extract from head



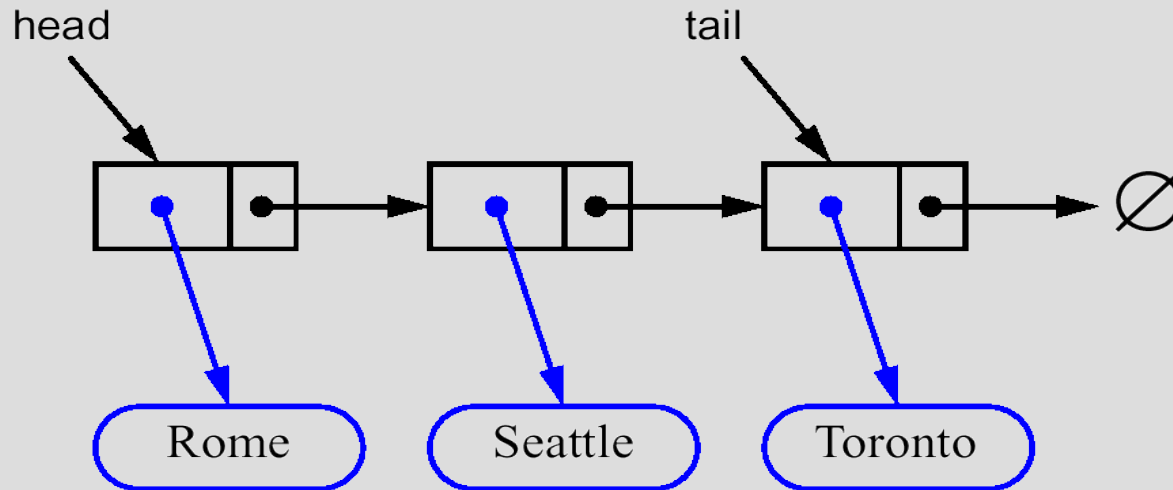
A Linked-List implementation/2



- Insert at the end of the list: $O(1)$

```
enqueue(element x):  
node p = new node();  
p.info = x; p.next = null;  
tail.next=p;  
tail=tail.next;
```


A Linked-List implementation/3



- Extract from the top of the list: $O(1)$

```
Element dequeue():  
x = root.info;  
root = root.next;  
Return x;
```

Suggested exercises

- Implement stack and queue as arrays
- Implement stack and queue as linked lists, with the same interface as the array implementation

Suggested exercises/2

- Suppose a queue of integers is implemented with an array of 8 elements: draw the outputs and status of such array after the following operations:

- enqueue 2,4,3,1,7,6,9
- dequeue 3 times
- Enqueue 2,3,4

Can we enqueue any more element?

- Try the same with a stack
- Try similar examples (also with a stack)

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 - Queue
 - **Ordered Lists**
 - Priority Queue

Ordered List

- In an ordered list elements are ordered according to a key.
- Example functions on ordered list:
 - `init()`
 - `isEmpty()`
 - `Search(element x)`
 - `delete(element x)`
 - `print()`
 - `insert(element x)`

Ordered List/2

- Declaration of an ordered list identical to unordered list
- Some operations (search, and hence insert and delete) are slightly different

Ordered List/3

- Insertion into an ordered list (C):

```
void insert(struct node* l, int x) {
    struct node* p;
    struct node* q;

    if (root == NULL || root->val > x) {
        p = malloc(sizeof(struct node));
        p->val = x;
        p->next = root;
        root = p;
    } else {
        ...
    }
}
```

Ordered List/4

- Insertion into an ordered list (C):

```
void insert(struct node* l, int x) {  
    ...  
} else {  
    p = root;  
    while (p->next != NULL && p->next->val < x)  
        p = p->next;  
    q = malloc(sizeof(struct node));  
    q->val = x;  
    q->next = p->next;  
    p->next = q;  
}  
}
```


Ordered List/5

- Insertion into an ordered list (java):

```
void insert(node l, int x) {
    node p;
    node q;

    if (root == NULL || root.val > x) {
        p = new node();
        p.val = x;
        p.next = root;
        root = p;
    } else {
        ...
    }
}
```

Ordered List/5

- Insertion into an ordered list (java):

```
void insert(node l, int x) {  
    ...  
} else {  
    p = root;  
    while (p.next != NULL && p.next.val < x)  
        p = p.next;  
    q = new node();  
    q.val = x;  
    q.next = p.next;  
    p.next = q;  
}  
}
```

Ordered List

- Cost of operations:
 - Insertion: $O(n)$
 - Check isEmpty: $O(1)$
 - Search: $O(n)$
 - Delete: $O(n)$
 - Print: $O(n)$

Suggested exercises

- Implement an ordered list with the following functionalities: isEmpty, insert, search, delete, print
- Implement also deleteAllOccurrences()
- As before, with a recursive version of: insert, search, delete, print
 - are recursive versions simpler?
- Implement an efficient version of print which prints the list in reverse order

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 - **Priority Queue**

Priority Queues

- A priority queue is an *ADT* for maintaining a set S of elements, each with an associated value called key.
- A PQ supports the following operations
 - **Insert**(S,x) insert element x in set S ($S := S \cup \{x\}$)
 - **ExtractMax**(S) returns and removes the element of S with the largest key
- One way of implementing it: a heap

Priority Queues/2

- Removal of max takes constant time on top of Heapify $\Theta(\log n)$

```
ExtractMax (A)
```

```
// removes & returns largest elem of A
```

```
max := A[1]
```

```
A[1] := A[n]
```

```
n := n-1
```

```
Heapify(A, 1, n)
```

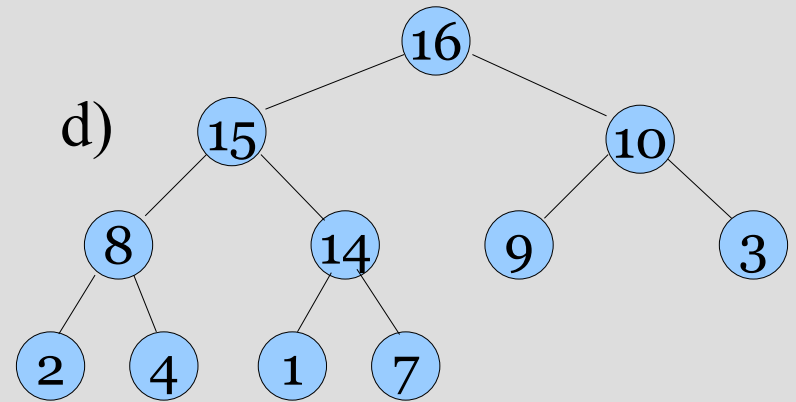
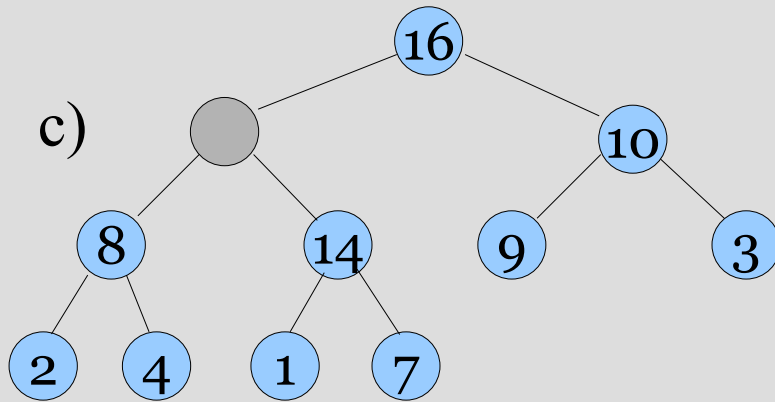
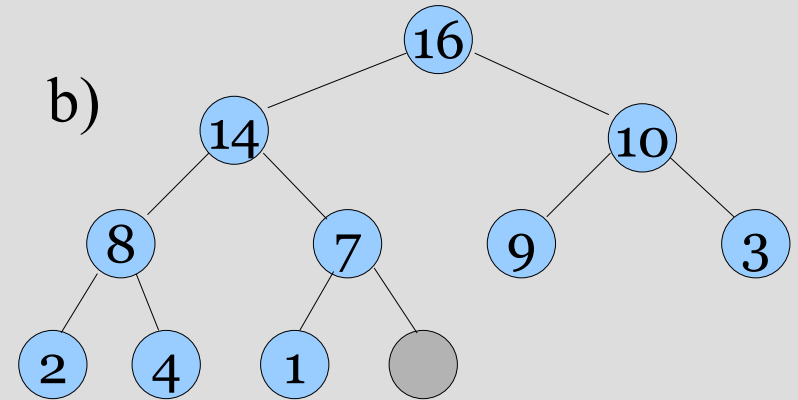
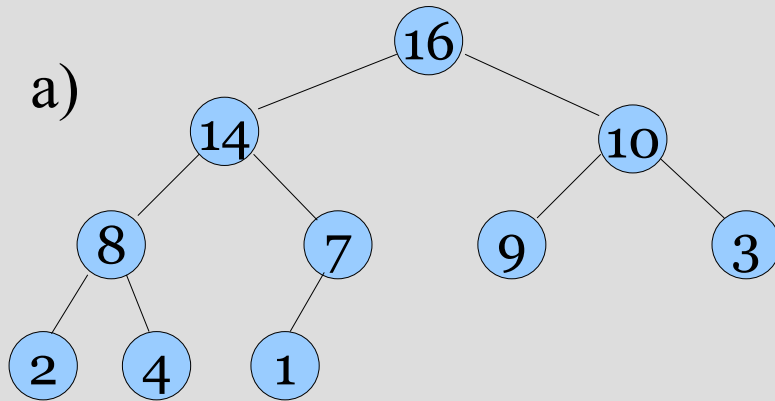
```
return max
```

Priority Queues/3

- Insertion of a new element
 - enlarge the PQ and propagate the new element from last place "up" the PQ
 - tree is of height $\log n$, running time: $\Theta(\log n)$

```
Insert (A, key)
  n := n+1;
  i := n;
  while i > 1 and A[parent(i)] < key
    A[i] := A[parent(i)]
    i := parent(i)
  A[i] := key
```


Priority Queues/4



Priority Queues/5

- Applications:
 - job scheduling shared computing resources (Unix)
 - Event simulation
 - As a building block for other algorithms
- We used a heap and an array to implement PQ. Other implementations are possible.

Suggested exercises

- Implement a priority queue
- Consider the PQ of previous slides. Draw the status of the PQ after each of the following operations:
 - Insert 17,18,18,19
 - Extract four numbers
 - Insert again 17,18,18,19
- Build a PQ from scratch, adding and inserting elements at will, and draw the status of the PQ after each operation

Summary

- Records, Pointers
- Dynamic Data Structures
 - Lists (root, head/tail, doubly linked)
- Abstract Data Types
 - Type + Functions
 - Stack, Queue
 - Ordered Lists
 - Priority Queues

Next Week

- Binary Search Trees
- Red-Black Trees