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2. Loop Invariants, Merging of Arrays, Inductive Proofs

1. Loop invariants: SelectKth

The following algorithm (SelectKth) returns the k-smallest element of an unordered input array of integers:

Input: Unsorted array $A[1..n]$ of integers and an integer $k \in \{1..n\}$.

Output: The k-th integer in A.

SELECTKTH(A, k):

```
for i:=1 to k do
  mini:=i
  for j:=i+1 to n do
    if A[j] < A[mini] then
      mini:=j
  key:=A[i]
  A[i]:=A[mini]
  A[mini]:=key
return A[k]
```

We want to prove that at the end of the algorithm the k-smallest element of the input array is the one at position k.

1. State a loop invariant for the inner "for loop".
2. State a loop invariant for the outer "for loop".
3. Prove the claim, using the above defined loop invariants.

2. Algorithm Correctness: Merging of Arrays

Given two ordered arrays in input A and B an interesting tasks is to construct an ordered array by merging the two input arrays.

1. Develop the pseudocode for the merge function.

2. Consider the following **merge** function. Is it correct?

   INPUT: $A[1..n1]$, $B[1..n2]$ sorted arrays of integers
   OUTPUT: permutation C of $A.B$ s.t. $C[1] \leq C[2] \leq ... \leq C[n1 + n2]$

   $1\ i := 1$
   $2\ j := 1$
   3 **for** $k := 1$ **to** $n1 + n2$ **do**
   4 **if** $A[i] <= B[j]$ **then**
   5 $C[k] := A[i]$
   6 $i++$
   7 **else**
   8 $C[k] := B[j]$
   9 $j++$
   10 **return** C

3. Consider the following **merge** function is it correct?

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   OUTPUT: permutation C of $A.B$ s.t. $C[1] \leq C[2] \leq ... \leq C[n1 + n2]$

   $1\ i := 1$
   $2\ j := 1$
   3 **for** $k := 1$ **to** $n1 + n2$ **do**
   4 **if** $j > n2$ **or** $(i <= n1$ **and** $A[i] <= B[j])$ **then**
   5 $C[k] := A[i]$
   6 $i++$
   7 **else**
   8 $C[k] := B[j]$
   9 $j++$
   10 **return** C

3. Inductive Proofs: Fibonacci Numbers

Prove that for every positive integer n, the Fibonacci number $\mathit{fib}(3n)$ is even.