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1. Maxsort, Binary Search, and Inductive Proofs

1. Algorithm Design and Analysis: Maxsort

Develop and analyse an algorithm *Maxsort*, that takes an unsorted array of integer numbers as input and returns the array sorted in descending order, by doing the following:
First, it searches in the whole array for the biggest element. It then puts this element to the beginning of the array (by exchanging it with the element that is there). Then, it searches the whole array excluding the first element for the biggest value, and puts it to the second position, and so on.

1. Formalize the idea of the algorithm by writing it in pseudocode.
2. Find the best and the worst case for the running time of this algorithm, and analyse the number of comparisons and assignments that will happen in each of those cases. *Optional*: What would be the average case, and can you give any estimations for the running time there?
3. What needs to be changed to achieve an ordering in ascending order ("Minsort")?

2. Recursion: Binary Search

Binary search is a popular recursive algorithm to search sorted data.
Suppose you are given an array $A[1..n]$ of integers, that is sorted in ascending order, and suppose that values may occur multiple times in that array (e.g., $[1, 2, 2, 3]$ is a valid array).

1. Develop in pseudocode a recursive algorithm that searches for the position of some occurrence of a value n in A. Hint: Consider how you would search for a name in an alphabetically ordered telephone directory

2. How many recursive calls are made in the best and in the worst case?
3. Modify your algorithm so that instead of the position of just some occurrence of n, it returns the leftmost occurrence.
4. Modify your algorithm so that instead of some occurrence of n, it returns the position of the middlemost occurrence. Can you do that without introducing a second function?

3. Inductive Proofs: Algorithmic Correctness

Consider the following recursive function definitions:

$$f(m,n) = \begin{cases} m & \text{if } n = 1 \\ f(m, n-1) + m & \text{if } n > 1 \end{cases}$$

$$g(m,n) = \begin{cases} m & \text{if } n = 1 \\ g(m, n-1) * m & \text{if } n > 1 \end{cases}$$

$$h(m,n) = \begin{cases} m & \text{if } n = 1 \\ h(m, n-1) + m^2 & \text{if } n > 1 \end{cases}$$

What do these functions calculate? Give inductive proofs for your claims.