# Computational Logic

## Query Processing and Optimization

**Free University of Bozen-Bolzano, 2010**

**Werner Nutt**

**(Most slides adapted from Thomas Eiter and Leonid Libkin)**

# Query Processing and Optimization

- *Query optimization*: finding a good way to evaluate a query
- Queries are declarative, and can be translated into procedural languages in more than one way
- Hence one has to choose the best (or at least good) procedural query
- This happens in the context of *query processing*
- A query processor turns queries and updates into sequences of operations on the database

## Query Processing and Optimization Stages

- Queries are translated into an extended relational algebra (operator + execution method): Which algebra expressions will allow for an efficient execution?
- Algebraic operators can be implemented by different methods (Examples?): Which algorithm should be used for each operator?
- How do operators pass data (write into main memory, write on disk, pipeline into other operators)?

Issues:

- Translate the query into extended relational algebra (“query plans”)
- Estimate the execution of the plans: We need to know how data is stored, how it accessed, how large are intermediate results, etc.

Decisions are based on general guidelines and statistical information

# Overview of Query Processing

- Start with a declarative query:

```
SELECT R.A, S.B, T.E
FROM R,S,T
WHERE R.C=S.C AND S.D=T.D AND R.A>5 AND S.B<3 AND T.D=T.E
```

- Translate into an algebra expression:

$$\pi_{R.A,S.B,T.E}(\sigma_{R.A>5 \wedge S.B<3 \wedge T.D=T.E}(R \bowtie S \bowtie T))$$

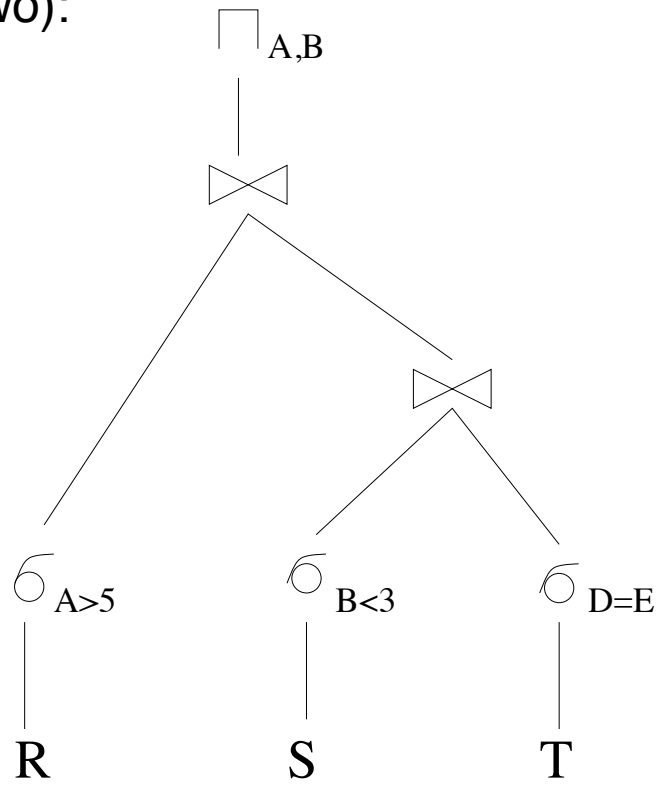
- Optimization step: rewrite to an equivalent but more efficient expression:

$$\pi_{R.A,S.B,T.E}(\sigma_{A>5}(R) \bowtie \sigma_{B<3}(S) \bowtie \sigma_{D=E}(T)))$$

*Why may this be more efficient? Is there a still more efficient plan?*

# Overview of Query Processing (contd)

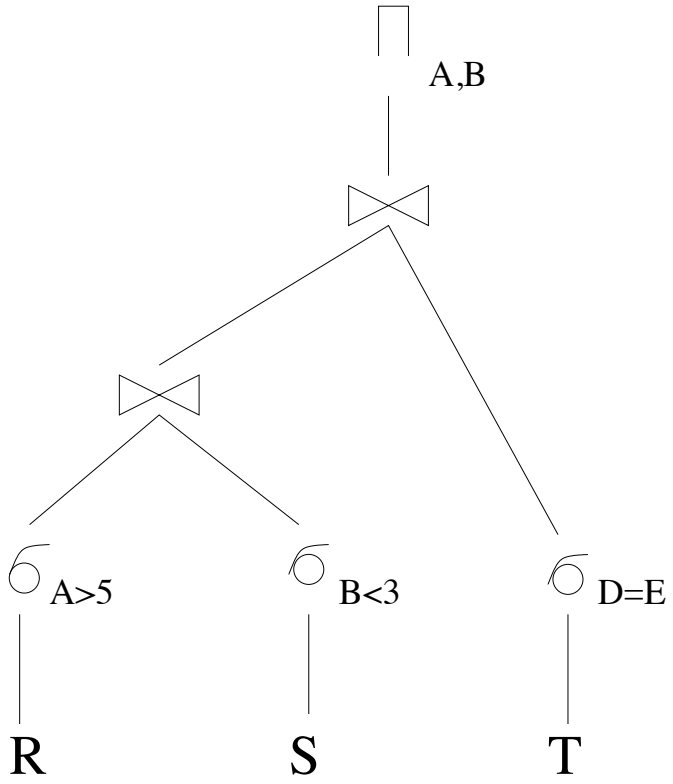
- When evaluating an expression with pushed selections,
  the main choice to make is the order of joins (may influence selections).
- First *query plan* (out of two):




  first join $S$ and $T$, and then join the result with $R$.

# Overview of query processing (contd)

- Alternative query plan:




  First join $R$ and $S$, then join the result with $T$.

- Both query plans produce the same result *(why?)*.

*Why choose one and not the other?*

## Optimization by Algebraic Equivalences

- Given a relational algebra expression $E$, find another expression $E'$ equivalent to $E$ that is easier (faster) to evaluate.
- Basic question: Given two relational algebra expressions $E_1$, $E_2$, are they equivalent?
- If there were a method to decide equivalence, we could turn it into one to decide whether an expression $E$ is empty, i.e., whether $E(\mathbf{I}) = \emptyset$ for every instance $\mathbf{I}$. *How does one show this?*
- Problem: Testing whether $E \equiv \emptyset$ is undecidable for expressions of full relational algebra i.e., including union and difference. *(Any idea why?)*
- Good news:

  We can still list some useful equalities.

  Equivalence is decidable for important classes of queries (SPJR, SPC)

# Optimization by Algebraic Equivalences (cntd)

Systematic way of query optimization: Apply equivalences

- $\bowtie$ and $\times$ are commutative and associative, hence applicable in any order
- Cascaded projections can be simplified: If the attributes $A_1, \ldots, A_n$ are among $B_1, \ldots, B_m$, then

$$\pi_{A_1,\ldots,A_n}(\pi_{B_1,\ldots,B_m}(E)) \;=\; \pi_{A_1,\ldots,A_n}(E)$$

- Cascaded selections might be merged:

$$\sigma_{C_1}(\sigma_{C_2}(E)) \;=\; \sigma_{C_1 \wedge C_2}(E)$$

- Commuting selection with join. If $c$ only involves attributes from $E_1$, then

$$\sigma_C(E_1 \bowtie E_2) \;=\; \sigma_C(E_1) \bowtie E_2$$

- etc

We will not treat this here.

## Optimization by Algebraic Equivalences (contd)

- Rules combining $\sigma, \pi$ with $\cup$ and $\setminus$
- Commuting selection and union:

$$\sigma_C(E_1 \cup E_2) \;=\; \sigma_C(E_1) \cup \sigma_C(E_2)$$

- Commuting selection and difference:

$$\sigma_C(E_1 \setminus E_2) \;=\; \sigma_C(E_1) \setminus \sigma_C(E_2)$$

- Commuting projection and union:

$$\pi_{A_1,\ldots,A_n}(E_1 \cup E_2) \;=\; \pi_{A_1,\ldots,A_n}(E_1) \cup \pi_{A_1,\ldots,A_n}(E_2)$$

- Question: what about projection and difference?
  Is $\pi_A(E_1 \setminus E_2)$ equal to $\pi_A(E_1) \setminus \pi_A(E_2)$?

## Optimization of Conjunctive Queries

- Reminder:

  Conjunctive queries

  = SPJR queries

  = simple SELECT-FROM-WHERE SQL queries

  (only AND and (in)equality in the WHERE clause)

- Extremely common, and thus special optimization techniques have been developed
- Reminder: for two relational algebra expressions $E_1, E_2$,

  "$E_1 = E_2$" is undecidable.

- But for conjunctive queries, even $E_1 \subseteq E_2$ is decidable.
- Main goal of optimizing conjunctive queries: reduce the number of joins.

# Optimization of Conjunctive Queries: An Example

- Given a relation $R$ with two attributes $A, B$
- Two SQL queries:

Q1

```
SELECT DISTINCT R1.B, R1.A
FROM   R R1, R R2
WHERE  R2.A=R1.B
```

Q2

```
SELECT DISTINCT R3.A, R1.A
FROM   R R1, R R2, R R3
WHERE  R1.B=R2.B AND R2.B=R3.A
```

- Are they equivalent?
- If they are, we can save one join operation.
- In relational algebra:

$$Q_1 = \pi_{2,1}(\sigma_{2=3}(R \times R))$$

$$Q_2 = \pi_{5,1}(\sigma_{2=4 \wedge 4=5}(R \times R \times R))$$

## Optimization of Conjunctive Queries (contd)

- We will show that $Q_1$ and $Q_2$ are equivalent!
- We cannot do this by using our equivalences for relational algebra expression. *(Why?)*
- Alternative idea: CQs are like patterns that have to be matched by a database. If a CQ returns an answer over a db, a part of the db must "look like" the query.
- Use rule based notation to find a representation of part of the db:

$$\begin{aligned} Q_1(x,y) &\text{ :- } R(y,x), R(x,z) \\ Q_2(x,y) &\text{ :- } R(y,x), R(w,x), R(x,u) \end{aligned}$$

## Containment is a Key Property

**Definition. (Query containment)** A query $Q$ is *contained* in a query $Q'$ (written $Q \subseteq Q'$) if

$$Q(\mathbf{I}) \subseteq Q'(\mathbf{I})$$

for every database instance $\mathbf{I}$.

**Translations:**

- If we can decide containment for a class of queries, then we can also decide equivalence.
- If $\mathcal{Q}$ is a class of queries that is closed under intersection, then the containment problem for $\mathcal{Q}$ can be reduced to the equivalence problem for $\mathcal{Q}$.

## Checking Containment of Conjunctive Queries: Ideas

- We want to check containment of two CQs $Q$, $Q'$
- We consider CQs without equalities and inequalities
  (How serious is this restriction?)
- We also assume that $Q$ and $Q'$ have the same arity and identical vectors of head variables, that is, they are defined as $Q(\vec{x})$ :– $B$, $Q'(\vec{x})$ :– $B'$
- Intuition: a conjunctive query $Q(\vec{x})$ :– $B$ returns an answer over instance $\mathbf{I}$ if $\mathbf{I}$ "matches the pattern $B$"
- Instead of checking containment over all instances, we consider only finitely many test instances (or better, one!)

# Tableau Notation of Conjunctive Queries

- Tableaux notation blurs the distinction between query and database instance
- We first consider queries over a single relation

$$Q_1(x, y) \text{ :- } R(y, x), R(x, z)$$

$$Q_2(x, y) \text{ :- } R(y, x), R(w, x), R(x, u)$$

- Tableaux:

| A | B | |
|---|---|---|
| y | x | |
| x | z | |
| x | y | ← answer line |

| A | B | |
|---|---|---|
| y | x | |
| w | x | |
| x | u | |
| x | y | ← answer line |

- Variables in the answer line are called “distinguished”

# Tableau Homomorphisms

- Tableaux (as well as database instances or first order structures) are compared by homomorphisms
- **Idea:** $T'$ is more general than $T$ if there is a homomorphism from $T'$ to $T$
- **Reminder**: *Terms* are variables or constants
- A homomorphism $\delta$ from Tableau $T_1$ to tableau $T_2$ is a mapping

$$\delta : \{\text{terms of } T_1\} \rightarrow \{\text{terms of } T_2\}$$

such that

  - $\delta(a) = a$ for every constant
  - $\delta(x) = x$ for every distinguished $x$
  - if $(t_1, ..., t_k)$ is a row in $T_1$, then $(\delta(t_1), ..., \delta(t_k))$ is a row in $T_2$

## Tableaux for Queries with Multiple Relations

- So far we assumed that there is only one relation $R$, but what if there are many?
- Construct a tableau for the query:

$$Q(x, y) \text{:-} S(x, y), R(y, z), R(y, w), R(w, y)$$

- We create rows for each relation:

S:

| A | B |
|---|---|
| x | y |

R:

| A | B |
|---|---|
| y | z |
| y | w |
| w | y |

| x | y |
|---|---|

- Formally, a tableau is just a database where variables can appear in tuples, plus a set of distinguished variables.

## Tableaux Homomorphisms: General Case

- Let $T_1$, $T_2$ be tableaux with the same distinguished variables
- A homomorphism $\delta$ from $T_1$ to $T_2$ is a mapping

$$\delta : \{\text{variables of } T_1\} \rightarrow \{\text{terms of } T_2\}$$

  such that
  - $\delta(a) = a$ for every constant
  - $\delta(x) = x$ for every distinguished variable
  - if $\vec{t}$ is a row of $R$ in $T_1$, then $\delta(\vec{t})$ is a row of $R$ in $T_2$, for every relation $R$

## The Homomorphism Theorem for Tableaux

**Homomorphism Theorem**: Let $Q$, $Q'$ be two conjunctive queries without equalities and inequalities that have the same distinguished variables and let $T$, $T'$ be their tableaux. Then

$$Q \subseteq Q' \Leftrightarrow \text{there exists a homomorphism from } T' \text{ to } T$$

## Proof: Necessity

- Assume that $Q \subseteq Q'$
- Note that $T$, ignoring the declaration of distinguished variables, can be seen as a database instance (if we view variables as constants)
- Note also that $\vec{x} \in Q(T)$
- Since $Q \subseteq Q'$, it follows that $\vec{x} \in Q'(T)$
- Since $\vec{x} \in Q'(T)$, there is a valuation $\nu'$ such
  - $\nu'$ satisfies $Q'$ over $T$
  - $\nu'(\vec{x}) = \vec{x}$
- Thus, we obtain a homomorphism $\delta'$ from $T'$ to $T$

  by defining $\delta'(y) = \nu'(y)$ for every variable $y$ in $Q'$

## Proof: Sufficiency

- Assume there is a hom $\delta'$ from $T'$ to $T$. We will show that $Q \subseteq Q'$
- We denote the body of $Q$ as $B$ and the body of $Q'$ as $B'$
- Let $\mathbf{I}$ be a db instance. We show that $Q(\mathbf{I}) \subseteq Q'(\mathbf{I})$
- Let $\vec{a} \in Q(\mathbf{I})$. We show that $\vec{a} \in Q'(\mathbf{I})$
- Since $\vec{a} \in Q(\mathbf{I})$, there is a valuation $\nu$: $\mathbf{var}(Q) \rightarrow \mathbf{dom}(\mathbf{I})$ such that
  - $\nu$ satisfies $B$, the body of $Q$

    (that is, "$\nu(B) \subseteq \mathbf{I}$" in logic programming perspective)
  - $\nu(\vec{x}) = \vec{a}$

## Proof: Sufficiency (cntd.)

- Let $\nu' := \nu \circ \delta$. Then $\nu'$ is a valuation for $Q'$
- Moreover, if $R(\vec{t})$ is an atom of $B'$, then
  - $\delta(R(\vec{t})) = R(\delta(\vec{t}))$ is an atom of $B$, since $\delta$ is a homomorphism, and
  - $\nu'(R(\vec{t})) = \nu(R(\delta(\vec{t})))$ is an atom in $\mathbf{I}$, since $\nu$ satisfies $B$ over $\mathbf{I}$,

  that is, $\nu'(R(\vec{t}))$ is an atom in $\mathbf{I}$
- In summary, since $R(\vec{t})$ was arbitrary, we have that $\nu'(B') \subseteq \mathbf{I}$, that is, $\nu'$ satisfies $B'$ over $\mathbf{I}$
- Also $\nu'(\vec{x}) = \nu(\delta(\vec{x})) = \nu(\vec{x}) = \vec{a}$
- Hence, $\vec{a} \in Q'(\mathbf{I})$

## Applying the Homomorphism Theorem: $Q_1 \equiv Q_2$

## Query Containment: Exercise

Find all containments and equivalences among the following conjunctive queries:

$$
\begin{aligned}
q_1(x,y) &\text{ :- } r(x,y),\ r(y,z),\ r(z,w)\\
q_2(x,y) &\text{ :- } r(x,y),\ r(y,z),\ r(z,u),\ r(u,w)\\
q_3(x,y) &\text{ :- } r(x,y),\ r(z,u),\ r(v,w),\ r(x,z),\ r(y,u),\ r(u,w)\\
q_4(x,y) &\text{ :- } r(x,y),\ r(y,3),\ r(3,z),\ r(z,w)
\end{aligned}
$$

## Query Containment: Complexity

Given two conjunctive queries, how hard is it to test whether $Q \subseteq Q'$?

- It is easy to transform them into tableaux, from either SPJ relational algebra queries, or SQL queries, or rule-based queries. However, . . .

**Theorem.** The following problem is NP-hard.

**Given:** tableaux $T$, $T'$

**Question:** is there a homomorphism from $T'$ to $T$?

**Proof Ideas:** Reductions of graph problems like Hamiltonian Path or Clique.

Also, reduction of 3SAT. (This one is interesting, because it can be modified to prove that containment is harder for generalisations of conjunctive queries, such as CQs with comparisons or CQs over databases with nulls.)

- In practice, query expressions are small, and thus conjunctive query optimization is nonetheless feasible in polynomial time

## The 3-Colorability Problem

**Instance:** A graph $G = (V, E)$

**Question:** Can $G$ be colored with the three colours $\{ r, g, b \}$ in such a way that two adjacent vertices have a distinct colour?

The 3-colorability problem is NP-hard

**Observation:**

A graph $G$ is 3-colourable if and only if there is a graph homomorphism from $G$ to the simplex $S_3$, which consists of of three vertices that are connected to each other.

## Reduction of 3-Colorability to Containment

- Given graph $G = (V, E)$, where
  - $V = \{ v_1, \ldots, v_n \}$ and
  - $E = \{ (v_{i_l}, v_{j_l}) \mid i_l < j_l,\ 1 \leq l \leq m \}$
- We construct queries $Q$, $Q'$ as follows
  - $Q() \text{ :- } e(r, b),\ e(b, r),\ e(r, g),\ e(g, r),\ e(b, g),\ e(g, b)$
  - $Q'() \text{ :- } e(x_{i_1}, x_{j_1}), \ldots, e(x_{i_m}, x_{j_m})$

    where $x_1, \ldots, x_n$ are new variables and

    there is one atom $e(x_{i_l}, x_{j_l})$ for each edge $(v_{i_l}, v_{j_l}) \in E$
- Clearly, $Q \subseteq Q'$ if and only if there is a graph homomorphism from $G$ to $S_3$

## Minimizing Conjunctive Queries

**Goal:** Given a conjunctive query $Q$, find an equivalent conjunctive query $Q'$ with the minimum number of joins.

**Questions:** How many such queries can exist? How different are they?

**Assumption:** We consider only CQs without equalities and inequalities.

We call these queries **simple conjunctive queries** (SCQs) .

If nothing else is said in this chapter, CQs are simple CQs

**Terminology:** If

$$Q(\vec{x}) \text{ :- } R_1(\vec{u}_1), \ldots, R_k(\vec{u}_k)$$

is a CQ, then $Q'$ is a **subquery** of $Q$ if $Q'$ is of the form

$$Q'(\vec{x}) \text{ :- } R_{i_1}(\vec{u}_{i_1}), \ldots, R_{i_l}(\vec{u}_{i_l})$$

where $1 \leq i_1 < i_2 < \ldots < i_l \leq k$.

## Minimization: Background Theory

**Proposition 1.** Let $q$ be a SCQ with $n$ atoms and $q'$ be an equivalent SCQ with $m$ atoms where $m < n$. Then there exists a subquery $q_0$ of $q$ such that $q_0$ has at most $m$ atoms in the body and $q_0$ is equivalent to $q$.

**Proposition 2.** Let $q$ and $q'$ be two equivalent minimal SCQs. Then $q$ and $q'$ are identical up to renaming of variables.

**Conclusions:**

- There is essentially one minimal version of each SCQ $Q$.
- We can obtain it by dropping atoms from $Q$'s body.

## An Algorithm for Minimizing SCQs

Given a conjunctive query $Q$, transform it into a tableau $T$.

Minimization algorithm:

$T' := T$;
**repeat until** no change
    choose a row $\vec{t}$ in $T'$;
    **if** there is a homomorphism $\delta : T' \rightarrow T' \setminus \{\vec{t}\}$
    **then** $T' := T' \setminus \{\vec{t}\}$
**end**

Output: The query $Q'$ corresponding to the tableau $T'$

## Questions about the Algorithm

- Does it terminate?
- Is $Q'$ equivalent to $Q$?
- Is $Q'$ of minimal length among the queries equivalent to $Q$?

## Minimizing SPJ/Conjunctive Queries: Example

- $R$ with three attributes $A, B, C$
- SPJ query

$$Q \;=\; \pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\pi_{AB}(R) \bowtie \pi_{AC}(\sigma_{B=4}(R)))$$

- Translate into relational calculus (instead of normalizing):

$$\big(\exists z_1\, R(x, y, z_1) \wedge y = 4\big) \wedge \exists x_1 \Big(\big(\exists z_2\, R(x_1, y, z_2)\big) \wedge \big(\exists y_1\, R(x_1, y_1, z) \wedge y_1 = 4\big)\Big)$$

- Simplify, by substituting the constant, and putting quantifiers forward:

$$\exists x_1, z_1, z_2\, (R(x, 4, z_1) \wedge R(x_1, 4, z_2) \wedge R(x_1, 4, z) \wedge y = 4)$$

- Conjunctive query:

$$Q(x, y, z)\ \text{:–}R(x, 4, z_1), R(x_1, 4, z_2), R(x_1, 4, z), y = 4$$

# Minimizing SPJ/Conjunctive Queries (contd)

- Tableau $T$:

| A | B | C |
|---|---|---|
| $x$ | $4$ | $z_1$ |
| $x_1$ | $4$ | $z_2$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

- Minimization, step 1: Is there a homomorphism from $T$ to

| A | B | C |
|---|---|---|
| $x_1$ | $4$ | $z_2$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

?

- Answer: No. For any homomorphism $\delta$, we have $\delta(x) = x$ (why?), thus the image of the first row is not in the smaller tableau.

## Minimizing SPJ/Conjunctive Queries (contd)

- Step 2: Is $T$ equivalent to

| A | B | C |
|---|---|---|
| $x$ | $4$ | $z_1$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

?

- Answer: Yes. Homomorphism $\delta$: $\delta(z_2) = z$, all other variables stay the same.
- The new tableau is not equivalent to

| A | B | C |
|---|---|---|
| $x$ | $4$ | $z_1$ |
| $x$ | $4$ | $z$ |

or

| A | B | C |
|---|---|---|
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

- Because $\delta(x) = x$, $\delta(z) = z$, and the image of one of the rows is not present.

## Minimizing SPJ/Conjunctive Queries (contd)

- Minimal tableau:

| A | B | C |
|---|---|---|
| $x$ | $4$ | $z_1$ |
| $x_1$ | $4$ | $z$ |
| $x$ | $4$ | $z$ |

- Back to conjunctive query:

$$Q'(x, y, z) \text{ :- } R(x, y, z_1), R(x_1, y, z), y = 4$$

- An SPJ query:

$$\sigma_{B=4}(\pi_{AB}(R) \bowtie \pi_{BC}(R))$$

- Pushing selections:

$$\pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\sigma_{B=4}(R))$$

## Review of the Journey

- We started with

$$\pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\pi_{AB}(R) \bowtie \pi_{AC}(\sigma_{B=4}(R)))$$

- Translated into a conjunctive query
- Built a tableau and minimized it
- Translated back into conjunctive query and SPJ query
- Applied algebraic equivalences and obtained

$$\pi_{AB}(\sigma_{B=4}(R)) \bowtie \pi_{BC}(\sigma_{B=4}(R))$$

- Savings: one join.

## Minimization of Conjunctive Queries: Multiple Relations

- We consider again the query:

$$Q(x, y) :\!- B(x, y), R(y, z), R(y, w), R(w, y)$$

- The tableau was:

B:

| A | B |
|---|---|
| x | y |

R:

| A | B |
|---|---|
| y | z |
| y | w |
| w | y |

| | |
|---|---|
| x | y |

## Minimization with Multiple Relations

- The algorithm is the same as before, but one has to try rows in different relations. Consider the homomorphism where $\delta(z) = w$, and $\delta$ is the identity for all other variables. Applying this to the tableau for $Q$ yields

B:

| A | B |
|---|---|
| x | y |

R:

| A | B |
|---|---|
| y | w |
| w | y |

| x | y |
|---|---|

- This can't be further reduced, as for any homomorphism $\delta$, $\delta(x) = x$, $\delta(y) = y$.
- Thus $Q$ is equivalent to

$$Q'(x, y) \text{ :- } B(x, y), R(y, w), R(w, y)$$

- One join is eliminated.

## Conjunctive Queries with Equalities and Disequalities

- Equality/Disequality atoms $x = y$, $x = a$, $x \neq z$, etc
- Let $T$, $T'$ be the tableaux of the parts of conjunctive queries $Q$ and $Q'$ with ordinary relations
- **Sufficiency:** $Q \subseteq Q'$ if there exists a homomorphism $\delta : T' \rightarrow T$ such that for each (dis)equality atom $t_1 \theta t_2$ in $Q'$, we have that $\delta(t_1)\theta\delta(t_2)$ is logically implied by the equality and disequality atoms in $Q$
- However, existence of a homomorphims is no more a necessary condition for containment.
- It holds under certain conditions, though.

Note: Deciding whether a set of equality/disequality atoms logically implies an equality/disequality atom is (relatively) easy.

## Queries with Comparisons

For queries with comparison atoms $s \leq t$ we have to refine our semantics

- An *ordered domain* is a nonempty set $\mathcal{D}$ with a linear order (written "$\leq_{\mathcal{D}}$")
- Let $\mathcal{D}$ be fixed. Wlog, $\mathbf{dom} = \mathcal{D}$. That is, from now on, database instances have constants from $\mathcal{D}$.
- Real database languages support many domains, ordered and not ordered, by typing relation symbols and admitting only queries that are well-typed.

## Queries with Comparisons (ctnd)

We consider conjunctive queries

$$Q(\vec{s}) \text{ :- } R, C$$

- with tuples of terms in the head
- whose bodies consist of a set of relational atoms $R$ and a set of comparisons $C$
- whose constants are elements of $\mathcal{D}$
- whose comparisons are interpreted over $\mathcal{D}$

The semantics of such queries is defined in a straightforward manner

## Queries with Comparisons: Examples

$$Q(x) \quad :- \quad P(x, w),\ P(x, x),\ R(x, u),\ w \geq 5,\ x \leq 2$$

$$Q'(x) \quad :- \quad P(x, y),\ R(y, z),\ y \leq 3$$

*How can we determine containment?*

# Query Homomorphism: Definition

Instead of tableau homomorphims, one often defines query homomorphisms.

An **homomorphism** from $Q'(\vec{x}) \text{ :- } R', C'$

to

$$Q(\vec{x}) \text{ :- } R, C$$

is a substitution $\delta$ such that

- $\delta(\vec{x}) = \vec{x}$
- $\delta(R') \subseteq R$
- $C \models \delta(C')$.

Here, $\delta(\vec{x})$, $\delta(R')$ and $\delta(C')$ are the extensions of $\delta$ to complex syntactic entities.

Note that we view $R$, $R'$, $C$, and $C'$ as sets of atoms.

*How should we define "$\models$"?*

## Query Homomorphism: Example

$$Q'(x) \quad :\!- \quad P(x,y),\ R(y,z),\ y \leq 3$$

$$Q(x) \quad :\!- \quad P(x,w),\ P(x,x),\ R(x,u),\ w \geq 5,\ x \leq 2$$

An homomorphism from $Q'$ to $Q$ is

$$\delta(x) = x,\ \ \delta(y) = x,\ \ \delta(z) = u.$$

## Homomorphisms: The General Case

Existence of a homomorphism is not a necessary, but a sufficient condition for containment.

**Theorem:** Let $Q$, $Q'$ be two conjunctive queries, possibly with comparisons and inequalities, such that $Q$ and $Q'$ have the same distinguished variables. Then

$Q \subseteq Q'$ if there exists a homomorphism from $Q'$ to $Q$.

## Containment of Queries with Comparisons

**Classical example:**

$$\begin{aligned} Q'() &\quad :\!- \quad P(u,v),\ u \leq v \\ Q() &\quad :\!- \quad P(y,z),\ P(z,y) \end{aligned}$$

- $Q$ is *contained* in $Q'$,
- but there is no homomorphism from $Q'$ to $Q$.

# Checking Containment of Queries with Comparisons: Idea

$$\begin{aligned} Q'() &\text{ :- } P(u,v),\ u \leq v \\ Q() &\text{ :- } P(y,z),\ P(z,y) \end{aligned}$$

Replace $Q$ with an equivalent union of queries:

$$\begin{aligned} Q_{\{\,y<z\,\}}() &\text{ :- } P(y,z),\ P(z,y),\ y < z \\ Q_{\{\,y=z\,\}}() &\text{ :- } P(y,z),\ P(z,y),\ y = z \\ Q_{\{\,y>z\,\}}() &\text{ :- } P(y,z),\ P(z,y),\ y > z \end{aligned}$$

(Case Analysis)

Check whether $Q_i \subseteq Q'$ for all $Q_i$

## Linearizations

We now make this idea formal.

- Let $\mathcal{D}$ be an ordered domain, $D$ be a set of constants from $\mathcal{D}$, $W$ be a set of variables, and let $T := D \cup W$ denote their union.
- A *linearization* of $T$ over $\mathcal{D}$ is a set of comparisons $L$ over the terms in $T$ such that for any $s, t \in T$ exactly one of the following holds:
  - $L \models_{\mathcal{D}} s < t$
  - $L \models_{\mathcal{D}} s = t$
  - $L \models_{\mathcal{D}} s > t$.
- $L$ partitions the terms into classes such that

  (i) the terms in each class are equal and

  (ii) the classes are arranged in a strict linear order

## Linearizations (cntd)

- **Remark:** A class of the induced partition contains at most one constant
- **Remark:** Whether or not $L$ is a linearization may depend on the domain.

  Consider e.g.,

  $$\{\, 1 < x,\ x < 2 \,\}$$

- A linearization $L$ of $T$ over $\mathcal{D}$ is *compatible* with a set of comparisons $C$ if $L \cup C$ is satisfiable over $\mathcal{D}$

## Linearizations of Conjunctive Queries

- When checking containment of two queries, we have to consider linearizations that contain the constants of *both* queries
- Let

$$Q(\vec{s}) \text{ :- } R, C$$

  be a query with
  - set of comparisons $C$ ranging over $\mathcal{D}$
  - $W$ be the set of variables occurring in $q$
  - $D$ be a set of constants from $\mathcal{D}$ that comprise the constants of $q$
- Then we denote with $\mathcal{L}_D(Q)$ the set of all linearizations of $D \cup W$ that are compatible with the comparisons $C$ of $Q$

## Linearizations of Conjunctive Queries (cntd)

Let $Q$ be as above. Let $L$ be a linearization of $T = D \cup W$ compatible with $C$.

- Note: $L$ defines an equivalence relation on $T$, where each equivalence class contains at most one constant
- A substitution $\phi$ is *canonical* for $L$ if
  - it maps all elements in an equivalence class of $L$ to one term of that class
  - if a class contains a constant, then it maps the class to that constant.
- Then $Q_L$ is obtained from $Q$ by means of a canonical substitution $\phi$ for $L$ as

$$Q_L(\phi(\vec{s})) \text{ :- } \phi R,\ \phi L,$$

  that is,
  - we first replace $C$ with $L$
  - and then "eliminate" all equalities by applying $\phi$

## Linearizations of Conjunctive Queries (cntd)

Consider

$$Q_L(\phi(\vec{s})) \text{ :- } \phi R, \phi L,$$

- We call $Q_L$ a *linearization* of $q$ w.r.t. $L$
- There may be more than one linearization of $q$ w.r.t. $L$, but all linearizations are identical up to renaming of variables
- Note that $\phi$ is a homomorphism from $Q$ to $Q_L$

## Linear Expansions

- A *linear expansion* of $q$ over $D$ is a family of queries $(Q_L)_{L \in \mathcal{L}_D(Q)}$, where each $Q_L$ is a linearization of $q$ w.r.t. $L$.
- If $Q$ and $D$ are clear from the context, or do not matter, we write simply $(Q_L)_L$.

## Containment of Queries with Comparisons

**Theorem (Klug 88):** If

- $Q$, $Q'$ are **conjunctive** queries **with comparisons** over $\mathcal{D}$ with set of constants $D$
- $(Q_L)_L$ is a **linear expansion** of $Q$ over $D$,

then:

$$Q \subseteq Q' \quad \Leftrightarrow \quad \text{for every } Q_L \text{ in } (Q_L)_L, \text{ there is an homomorphism from } Q' \text{ to } Q_L$$

**Theorem (van der Meyden 92):** Containment with comparisons is $\Pi_2^P$-complete.

## Query Optimization and Functional Dependencies

- Additional equivalences hold if db instances satisfy integrity constraints
- We consider here *functional dependencies*
- Example: Let $R$ have attributes $A, B, C$. Assume that $\mathbf{I}(R)$ satisfies $A \rightarrow B$.
- Then it holds that

$$(\pi_{AB}(R) \bowtie \pi_{AC}(R))(\mathbf{I}) \;\; = \;\; R(\mathbf{I})$$

(We have considered the expressions $\pi_{AB}(R) \bowtie \pi_{AC}(R)$ and $R$ as queries.)

- Tableaux can help with these optimizations!
- $\pi_{AB}(R) \bowtie \pi_{AC}(R)$ as a conjunctive query:

$$Q(x, y, z) \text{ :– } R(x, y, z_1), R(x, y_1, z)$$

# Query Optimization and Functional Dependencies (contd)

- Tableau:

| A | B | C |
|---|---|---|
| $x$ | $y$ | $z_1$ |
| $x$ | $y_1$ | $z$ |
| $x$ | $y$ | $z$ |

- Using the FD $A \rightarrow B$ infer $y = y_1$
- Next, minimize the resulting tableau:

| A | B | C |
|---|---|---|
| $x$ | $y$ | $z_1$ |
| $x$ | $y$ | $z$ |
| $x$ | $y$ | $z$ |

$\rightarrow$

| A | B | C |
|---|---|---|
| $x$ | $y$ | $z$ |
| $x$ | $y$ | $z$ |

- And this says that the query is equivalent to $Q'(x, y, z)$:–$R(x, y, z)$, that is, $R$
- This is known as the “chase” technique

## Query Optimization and Functional Dependencies (contd)

- General idea: simplify the tableau using functional dependencies and then minimize.
- Given: a conjunctive query $Q$, and a set of FDs $F$
- Algorithm:

  Step 1. Construct the tableau $T$ for $Q$

  Step 2. Apply algorithm CHASE$(T, F)$

  Step 3. Minimize output of CHASE$(T, F)$

  Step 4. Construct a query from the tableau produced in Step 3

## The CHASE

We assume that all FDs are of the form $X \rightarrow A$, where $A$ is a single attribute. For simplicity, we also assume that the tableau has only a single relation. The generalisation is straightforward.

for each $X \rightarrow A$ in $F$ do
  for each $t_1, t_2$ in $T$ such that $t_1.X = t_2.X$ and $t_1.A \neq t_2.A$ do
    case $t_1.A, t_2.A$ of
    one nondistinguished variable $\Rightarrow$
      replace the nondistinguished variable by the other term
    one distinguished variable, one distinguished variable or constant $\Rightarrow$
      replace the distinguished variable by the other term
    two constants $\Rightarrow$
      output $\perp$ and STOP
  end
end.

## Query Optimization and Functional Dependencies: Example 2

- $R$ is over $A, B, C$; $F = \{ B \to A \}$
- $Q = \pi_{BC}(\sigma_{A=4}(R)) \bowtie \pi_{AB}(R)$
- $Q$ as a conjunctive query:

$$Q(x, y, z) \text{ :- } R(4, y, z), R(x, y, z_1)$$

- Tableau:

| A | B | C |
|---|---|---|
| $4$ | $y$ | $z$ |
| $x$ | $y$ | $z_1$ |
| $x$ | $y$ | $z$ |

$\xrightarrow{\text{CHASE}}$

| A | B | C |
|---|---|---|
| $4$ | $y$ | $z$ |
| $4$ | $y$ | $z_1$ |
| $4$ | $y$ | $z$ |

$\xrightarrow{\text{minimize}}$

| A | B | C |
|---|---|---|
| $4$ | $y$ | $z$ |
| $4$ | $y$ | $z$ |

- Final result: $Q(x, y, z)$ :- $R(x, y, z), x = 4$, that is, $\sigma_{A=4}(R)$.

## Query Optimization and Functional Dependencies: Example 3

- Same $R$ and $F$; the query is:

$$Q \;=\; \pi_{BC}(\sigma_{A=4}(R)) \bowtie \pi_{AB}(\sigma_{A=5}(R))$$

- As a conjunctive query:

$$Q(x, y, z) \text{ :- } R(4, y, z), R(x, y, z_1), x = 5$$

- Tableau:

| A | B | C |
|---|---|---|
| $4$ | $y$ | $z$ |
| $5$ | $y$ | $z_1$ |
| $5$ | $y$ | $z$ |

$\overset{\text{CHASE}}{\rightarrow}$ $\bot$

- Final result: $\bot$ (the empty query)
- This equivalence does not hold *without* the FD $B \rightarrow A$

## Query Optimization and Functional Dependencies: Example 4

- Sometimes simplifications are quite dramatic
- Same $R$, FD is $A \rightarrow B$, the query is

$$Q \;=\; \pi_{AB}(R) \bowtie \pi_{A}(\sigma_{B=4}(R)) \bowtie \pi_{AB}(\pi_{AC}(R) \bowtie \pi_{BC}(R))$$

- Convert into conjunctive query:

$$Q(x, y) \;\text{:-}\; R(x, y, z_1), R(x, y_1, z), R(x_1, y, z), R(x, 4, z_2)$$

# Query Optimization and Functional Dependencies: Example 4 (contd)

- Tableau:

| A | B | C |
|---|---|---|
| $x$ | $y$ | $z_1$ |
| $x$ | $y_1$ | $z$ |
| $x_1$ | $y$ | $z$ |
| $x$ | $4$ | $z_2$ |
| $x$ | $y$ | |

$\xrightarrow{\text{CHASE}}$

| A | B | C |
|---|---|---|
| $x$ | 4 | $z_1$ |
| $x$ | 4 | $z$ |
| $x_1$ | 4 | $z$ |
| $x$ | 4 | $z_2$ |
| $x$ | 4 | |

$\xrightarrow{\text{minimize}}$

| A | B | C |
|---|---|---|
| $x$ | 4 | $z$ |
| $x$ | 4 | |

## Query Optimization and Functional Dependencies: Example 4 (contd)

- | A | B | C |
  |---|---|---|
  | $x$ | 4 | $z$ |

  is translated into $Q(x, y) \text{ :- } R(x, y, z), y = 4$

  | | |
  |---|---|
  | $x$ | 4 |

- or, equivalently $\pi_{AB}(\sigma_{B=4}(R))$.
- Thus,

  $$\pi_{AB}(R) \bowtie \pi_A(\sigma_{B=4}(R)) \bowtie \pi_{AB}(\pi_{AC}(R) \bowtie \pi_{BC}(R)) \;=\; \pi_{AB}(\sigma_{B=4}(R))$$

  in the presence of FD $A \rightarrow B$.
- Savings: 3 joins!
- This cannot be derived by algebraic manipulations, nor conjunctive query minimization without using CHASE.

## Questions about the CHASE

- Does the CHASE algorithm terminate? What is the run time?
- What is the relation between a tableau and its CHASE'd version?
- Query containment wrt a set of FD's:
  - How can we define this problem?
  - Can we decide this problem?
- Query minimsation wrt to a set of FDs
- Consider SCQs: we know from previous exercises that all such queries are satisfiable. Is the same true if we assume only database instances that satisfy a given set of FDs F?