4. Containment and Minimization of Conjunctive Queries

Instructions: Work in groups of 2 students. You can write up your answers by hand (provided your handwriting is legible) or use a word processing system like Latex or Word. However, experience shows that Word is in general difficult to use for this kind of task. Please, include name and email address in your submission.

1. Simple Conjunctive Queries with Unary Relation Symbols

Recall that simple conjunctive queries have only relational atoms in their body, and no equalities or inequalities. We know that containment of simple conjunctive queries is NP-complete. However, the reduction used queries with binary relation symbols.

What can you say about the difficulty of deciding containment of simple conjunctive queries that have only unary relations in their body (that is, relations of arity 1)? Is this an NP-hard or a polynomial time problem?

To prove NP-hardness, provide a reduction from a known NP-hard problem to the new one. To prove that it is in polynomial time, give an algorithm, show that it solves the problem, and explain why it runs in polynomial time.

(8 Points)

2. Injective and Surjective Mappings

For the proofs in Exercise 3, it will be necessary to make use of some facts about injective and surjective mappings on finite sets. The purpose of this exercise is to review those facts.

Recall that a mapping $f\colon X \to Y$ from a set X to a set Y is *injective* if for all $x, x' \in X$ we have that $f(x) = f(x')$ implies that $x = x'$. In other words, an injective mapping maps any two distinct elements of X to distinct elements of Y. Recall as well that a mapping $f\colon X \to Y$ is called *surjective* if for every $y \in Y$ there exists some $x \in X$ such that $f(x) = y$. In other words, every element y of Y has a preimage in X with respect to f.

Recall as well that for mappings $f\colon X \to Y$ and $g\colon Y \to Z$ the *composition* $g \circ f$ is a mapping from X to Z defined as $(g \circ f)(x) = g(f(x))$.

1. Prove that if $f\colon X \to Y$ and $g\colon Y \to Z$ are injective, then $g \circ f$ is injective.
2. Prove that if $f\colon X \to Y$ and $g\colon Y \to Z$ are surjective, then $g \circ f$ is surjective.
3. If the composition $g \circ f$ is injective, what can you conclude about g and f? Prove your answer.
4. Similarly, if the composition $g \circ f$ is injective, what can you conclude about g and f? Prove your answer.
5. Suppose X is a finite set and $f\colon X \to X$ is injective. What can you conclude about f? Prove your answer.
6. Similarly, suppose X is a finite set and $f\colon X \to X$ is surjective. What can you conclude about f? Prove your answer.

(6 Points)

3. Minimisation of Conjunctive Queries

Recall that *simple conjunctive queries* (SCQs) are conjunctive queries without equalities and inequalities. Recall as well that a conjunctive query q_0 is a *subquery* of another conjunctive query q if q_0 can be obtained from q by dropping some of the atoms in the body of q.
Prove the following two propositions that provide the underpinnings for the algorithm of conjunctive query minimization.

Proposition 1: Let q be a SCQ with n atoms and q' be an equivalent SCQ with m atoms where $m < n$. Then there exists a subquery q_0 of q such that q_0 has at most m atoms in the body and q_0 is equivalent to q.

(8 Points)

Proposition 2: Let q and q' be two equivalent minimal SCQs. Then q and q' are identical up to renaming of variables.

(8 Points)

Submission: 29 April 2009, 10:30 am, at the lecture