Logics of agency
Applications of STIT

EASSS 2018
Overview of this chapter

- STIT and obligations: “oughts to do”
- Powers in concurrent games and in STIT
- STIT and knowledge “knowing how to play”
Outline

1. STIT and Deontic Logic

2. Power in STIT theories

3. Knowing how to play
\textbf{BT} structures (reminder)

\textit{BT} structure \( \langle \text{Mom}, < \rangle \):

\begin{itemize}
  \item \textbf{History} = maximally \(<\)-ordered set of moments
  \item \textbf{Hist} = set of all histories
  \item \textbf{H}_m = set of histories passing through the moment \( m \)
  \item Explode \textbf{moments} into \textbf{indexes} (moment/history pairs)
    \begin{itemize}
      \item \( m_0/h_3 \not\models \mathbf{F}p \)
      \item \( m_0/h_1 \models \mathbf{F}p \)
    \end{itemize}
\end{itemize}
A $BT + AC$ model is a tuple $\mathcal{M} = \langle Mom, <, Choice, v \rangle$, where:

- $\langle Mom, < \rangle$ is a $BT$ structure;
- $Choice : \text{Agt} \times Mom \rightarrow \mathcal{P}(\mathcal{P}(\text{Hist}))$
  
  - $Choice(a, m) = \text{repertoire of choices for agent } a \text{ at moment } m$
  - $Choice$ is a function mapping each agent and each moment $m$ into a partition of $H_m$
- $Choice(a, m) : Hist \rightarrow \mathcal{P}(Hist)$
  - For $h \in H_m$: $Choice(a, m)(h) = \text{the particular choice of } a \text{ at index } m/h$.

- Independence of agents/choices: Let $h, m$.
  
  For all collections of $X_a \in Choice(a, m)(h), \bigcap_{a \in \text{Agt}} X_a \neq \emptyset$.

- No choice between undivided histories: if $\exists m' > m \text{ s.t. } h, h' \in H_{m'}$ then $h' \in Choice(a, m)(h)$.

- $v$ is a valuation function $v : \text{Prop} \rightarrow \mathcal{P}(Mom \times Hist)$. 
“Chellas” stit:
\[ M, m/h \models [G \text{ cstit}: \varphi] \iff M, m/h' \models \varphi \text{ for all } h' \in \text{Choice}(G, m)(h) \]

historical necessity:
\[ M, m/h \models \Box \varphi \iff M, m/h' \models \varphi \text{ for all } h' \in H_m \]
A utilitarian deontic model is a tuple \( \mathcal{M} = \langle \text{Mom}, \prec, \text{Choice}, \text{Value}, v \rangle \), where:

- \( \langle \text{Mom}, \prec, \text{Choice}, v \rangle \) is a BT + AC model;
- \text{Value} maps each history \( h \in Hist \) to a real value \( \text{Value} : Hist \rightarrow \mathbb{R} \).

\( \text{Value}(h) \leq \text{Value}(h') \) means that \( h' \) is at least as desirable as \( h \).
Truth value of ought statements

Let $\mathcal{M} = \langle \text{Mom}, <, \text{Choice}, \text{Value}, v \rangle$.

$\mathcal{M}, m/h \models \Box \varphi \iff \exists h' \in H_m$

\[
\begin{cases}
(1) & \mathcal{M}, m/h' \models \varphi \\
(2) & \forall h'' \in H_m: \text{ if } \text{Value}(h') \leq \text{Value}(h'') \text{ then } \mathcal{M}, m/h'' \models \varphi
\end{cases}
\]

($\varphi$ is true for some history, and $\varphi$ is true for all histories at least as desirable.)
Example: “Reparational” oughts

Obligations rising from violations of previous obligations. Example ((Thomason 1984), (Horty 2001)):

- It ought to be the case at the moment $m_1$ that $a$ will soon board a plane to visit his aunt.
- $A$: “$a$ the agent will board the plane”;
- $B$: “$a$ will call his aunt to say that he is not coming”.

At the moment $m_1$, three histories unfold.
- In $h_1$, agent $a$ boards the plane.
- In $h_2$, $a$ does not board the plane and calls his aunt to tell her that he will not be visiting.
- In $h_3$, $a$ does not board the plane and does not call his aunt to tell her that he will not be visiting.
Example: “Reparational” oughts (ctd)

- $Value(h_1) = 10$
- $Value(h_2) = Value(h_2) = 4$
- $Value(h_3) = Value(h_3) = 0$
- $m_1/\_ \models \Box A \land \neg \Box B$
- $m_2/\_ \models \Box B$

(Picture from (Horty 2001))
More theses from (Belnap et al. 2001)

**Definition (Restricted complement thesis)**
A variety of constructions concerned with agents and agency—including deontic statements, imperatives, and statements of intentions, among others—must take agentives as their complements.

**Definition (Stit normal form thesis)**
In investigations of those constructions that take agentives as complements, nothing but confusion is lost if the complements are taken to be all and only stit sentences.
Roderick Chisholm suggests:

“S ought to bring it about that $p$” can be defined as “It ought to be that S brings it about that $p$.” (Chisholm 1964, p. 150)

Agent $a$ ought to see to it that $\varphi$:

$$\Box[a \text{ stit} : \varphi]$$
\( \Diamond A \)

\( \neg \Diamond [acctit : A] \)
Logical principles of utilitarian deontic models

Valid principles:
- $\boxdot$ is a normal modal operator
- $\boxdot \varphi \rightarrow \Diamond \varphi$
- $\boxdot \varphi \rightarrow \Box \Box \varphi$
- $\neg (\Box [a \textit{cstit} : \varphi] \land \Box [b \textit{cstit} : \neg \varphi])$
- $\Box [a \textit{cstit} : \varphi] \rightarrow \Box \varphi$

However,
- $\Box \varphi \rightarrow \Box [a \textit{cstit} : \varphi]$ is not valid
- $\Box \varphi \land \Diamond [a \textit{cstit} : \varphi] \rightarrow \Box [a \textit{cstit} : \varphi]$ is not valid either! (next two slides)
Karen buys a horse

- Karen, wishes to buy a horse, but she has only $10,000 to spend and the horse she wants is selling for $15,000;

- We imagine that Karen offers $10,000 for the horse at the moment $m$ (choice $K_1$);

- It is up to the owner of the horse to decide whether to accept the offer. The history $h_1$ represents a scenario in which the owner accepts Karen’s offer, $h_2$ a scenario in which the offer is rejected;

- $A$ is the statement that Karen will become less wealthy by the amount of $10,000;

- The unique best history is $h_1$, in which the offer is accepted, and, as a consequence, Karen buys the horse and becomes less wealthy by $10,000;

- Since Karen is less wealthy by $10,000 in the unique best history, we must conclude that it ought to be that she is less wealthy by $10,000;

- Of course, Karen also has the ability (throwing away the money) to see to it that she is less wealthy by $10,000 (choice $K_2$);

- But we would not wish to conclude that Karen ought to see to it that she is less wealthy by $10,000.
Karen buys a horse (ctd)

\[
\begin{array}{cccccc}
 & h_1 & h_2 & h_3 & h_4 & h_5 \\
1 & A & 0 & 0 & 0 & 0 \\
\neg A & A & A & \neg A \\
m & K_1 & K_2 & K_3 & \text{Choice}^m_{\alpha}
\end{array}
\]

(Picture from (Horty 2001))

- $\lozenge A$
- $\lozenge [acstit : A]$
- $\neg \lozenge [acstit : A]$
Criticism of the utilitarian deontic model (risk seeking)

- An agent $a$ is faced with two options at the moment $m$: to gamble the sum of five dollars ($K_1$), or to refrain from gambling ($K_2$).
- If $a$ gambles, there is a history in which he wins ten dollars, and another in which he loses his stake;
- If $a$ does not gamble, he preserves his original stake;
- the utility associated with each history at $m$ is entirely determined by the sum of money that $a$ possesses at the end;
- The letter $A$ stands for the proposition that $a$ gambles;
- $\circ [a\; cstit\; A]$ holds at $m$. 

(Picture from (Horty 2001))
Criticism of the utilitarian deontic model (missing obligations)

If we change the utilities:

\[ \neg \Box [a \textit{csit}: A]. \text{Good.} \]

\[ \neg \Box [a \textit{csit}: \neg A]. \]

But we should expect that it is wise not to gamble here.
A first solution with ordered choices

Definition (weak preference over choices)

\[ X \leq Y \text{ iff } \forall h \in X, \forall h' \in Y : \text{Value}(h) \leq \text{Value}(h'). \]

A “ought to stit” operator with ordered choices:

\[ m/h \models [a \ ostit: A] \text{ iff } \exists K \in \text{Choice}(a, m) \text{ such that} \]

1. \( \{m\} \times K \subseteq \|A\| \) and
2. \( \forall K' \in \text{Choice}(a, m) : K' \leq K. \)
Gambling again

\[\neg[a \ ostit : A] \land \neg[a \ ostit : \neg A]\]

\[a \ ostit : \neg A\]
Further problem with multiagency

\( K_2 \) seems preferable for agent \( a \), but it is not the case that \( K_1 \leq K_2 \).
States: choices of others

We define the “strategic contexts” agent $a$ might face.

$$State(a, m) = Choice(\text{Agt} \setminus \{a\}, m) .$$

When there are two players (e.g., previous example):

$$State(a, m) = Choice(b, m) ,$$

and

$$State(b, m) = Choice(a, m) .$$
Choice dominance

Definition (weak choice dominance)

Let $K, K' \in \text{Choice}(a, m)$. $K \preceq_a K'$ iff $K \cap S \preceq K' \cap S$ for every $S \in \text{States}(a, m)$.

On the previous example: $K_1 \prec_a K_2$. 
Optimal choice: a second solution

Define:

$$Optimal(a, m) = \{ K \in Choice(a, m) \mid \forall K' \in Choice(a, m), K \prec_a K' \} .$$

When there is a finite number of choices, this revision of $[a ostit : A]$ of works well:

$$m/h \models [a ostit : A] \text{ iff } \{m\} \times K \subseteq ||A|| \text{ for every } K \in Optimal(a, m) .$$
Further problem with infinite repertories of choices

We’d like to have $[a\ ostit : A]$ and $\neg[a\ ostit : \neg A]$.
But $Optimal(a, m) = \emptyset$...
The “ought to stit” operator

We revise $[a \text{ ostit}: A]$ further into:

$m/h \models [a \text{ ostit}: A]$ iff for every $K \in \text{Choice}(a,m)$, if $\{m\} \times K \not\subseteq \|A\|$, then there is $K' \in \text{Choice}(a,m)$ such that:

1. $K \prec_a K'$, and
2. $\{m\} \times K' \subseteq \|A\|$, and
3. $\{m\} \times K'' \subseteq \|A\|$ for each $K'' \in \text{Choice}(a,m)$ such that $K' \preceq_a K''$.

This is obligation to do. Noted $\bigcirc [a \text{ estit}: A]$ in (Harty 2001, p. 77).
Outline

1. STIT and Deontic Logic
2. Power in STIT theories
3. Knowing how to play
A concurrent game model is a tuple $G = (S, \{\Sigma_a | a \in \text{Agt}\}, o, V)$ where:

- $S$ is a nonempty set of states,
- $\Sigma_a$ is a nonempty set of choices for every agent $a \in \text{Agt}$,
- $o : S \times \prod_{a \in \text{Agt}} \Sigma_a \rightarrow S$ is an outcome function,
- $V : S \rightarrow \mathcal{P}(\text{Prop})$ is a valuation function.

$G, s \models \{C\} \varphi$ iff $\exists \sigma_C \in \Sigma_C, \forall \sigma_{\overline{C}} \in \Sigma_{\text{Agt}\setminus C}$ s.t. $G, o((\sigma_C, \sigma_{\overline{C}})) \models \varphi$

where for every coalition $C \subseteq \text{Agt}$ we note $\Sigma_C = \Pi_{a \in C} \Sigma_a$.

Axiomatics of Coalition Logic

- Propositional Logic
- \( \langle C \rangle \top \)
- \( \neg \langle C \rangle \bot \)
- \( \neg \langle \emptyset \rangle \neg \varphi \rightarrow \langle \text{Agt} \rangle \varphi \)
- \( \langle C \rangle (\varphi \land \psi) \rightarrow \langle C \rangle \varphi \)
- \( \langle C_1 \rangle \varphi \land \langle C_2 \rangle \psi \rightarrow \langle C_1 \cup C_2 \rangle (\varphi \land \psi) \), when \( C_1 \cap C_2 = \emptyset \)
- if \( \vdash \varphi \leftrightarrow \psi \) then \( \vdash \langle C \rangle \varphi \leftrightarrow \langle C \rangle \psi \)
◊[a astit : ϕ] does not capture any kind of power.
◊[a cstit : ϕ] does.

How to embed Coalition Logic?
A discrete-deterministic STIT

**Hypothesis (discreteness)**

Given a moment $m_1$, there exists a successor moment $m_2$ such that $m_1 < m_2$ and there is no moment $m_3$ such that $m_1 < m_3 < m_2$.

$m/h \models X\varphi$ iff $\varphi$ is true at the moment immediately after $m$ on $h$

**Hypothesis (determinism)**

$\forall m \in Mom, \exists m' \in Mom (m < m' \text{ and } \forall h \in H_{m'}, \text{Choice}(\text{Agt}, m)(h) = H_{m'})$
Translation of Coalition Logic to discrete-deterministic STIT

\[ tr(p) = \square p, \text{ for } p \in \text{Prop} \]
\[ tr(\neg \varphi) = \neg tr(\varphi) \]
\[ tr(\varphi \lor \psi) = tr(\varphi) \lor tr(\psi) \]
\[ tr([C]\varphi) = [C]X tr(\varphi) \]

In STIT terminology

“the coalition C has the power to \varphi”

can be paraphrased by

“it is historically possible that C sees to it that next \varphi”

**Theorem** ([Broersen, Herzig, Troquard 2006])

\( tr \) is a correct embedding of CL into discrete-deterministic STIT.
Example: Ann and Bill switch the light

- Four states: $m_0, m_1, m_2, m_3$
- $li$ = light is on (at $m_3$)
- $f$ = lamp is functioning (at $m_2$ and $m_3$)
- At moment $m_0$, agent $a$ has a choice between *repairing* a broken lamp ($\rho_a$) or *remaining passive* ($\lambda_a$). Agent $b$ has the vacuous choice of *remaining passive* ($\lambda_b$).
- If $a$ chooses not to repair, the system reaches $m_1$. If $a$ chooses to repair, the system reaches $m_2$.
- In $m_1$, $m_2$ and $m_3$ both $a$ and $b$ can choose to *toggle* a light switch ($\tau_a$ and $\tau_b$) or *not toggle* ($\lambda_a$ and $\lambda_b$).
- If $a$ repairs at $m_0$ then $a$ and $b$ ‘play toggling’ between $m_2$ and $m_3$
Game model
Corresponding STIT model
Beyond Coalition Logic

From Alternating-time Temporal Logic to Strategic Chellas stit of ability

ATL (Alur, Henzinger, Kupferman 2002):
- concurrent game models
- $\langle\langle C\rangle\rangle X \varphi \mid \langle\langle C\rangle\rangle \varphi U \psi$

Corresponds to (Broersen, Herzig, Tr. 2006, JLC)):
- Models: discrete-time and deterministic STIT
- $p \leadsto \Box p$,
- $\langle\langle C\rangle\rangle X \varphi \leadsto \Diamond_s [C \text{ scstit: } X \varphi]$,
- $\langle\langle C\rangle\rangle \varphi U \psi \leadsto \Diamond_s [C \text{ scstit: } \varphi U \psi]$. 
1. STIT and Deontic Logic
2. Power in STIT theories
3. Knowing how to play
CL models vs. \( BT + AC \) models

Coalition Logic

- Concurrent game models
- Neighborhood models (effectivity structures)
- Idea: associate a strategic game (form) to every world

In \( BT + AC \) models, indexes represent both

- the current state of affairs of the world, and
- the current choice/commitment of agents
Ann toggles

- At $m_0$, the light is off: $m_0 \models \neg li$
- Ann can toggle or skip
- $m_0 \models \langle Ann \rangle li$
  at $m_0$, “Ann is able to achieve $li$”
Poor blind Ann – a CL account

- As before, the light is off: $m_0 \models \neg li$
- Ann is blind and cannot distinguish a world where the light is on from a world where the light is off
- $m_0 \models K_{Ann}[Ann]li$
  at $m_0$, “Ann knows she is able to achieve $li$”
Adding knowledge

A logical language of action and knowledge should be able to distinguish the following scenarios:

1. The agent $a$ knows it has a particular action/choice in its repertoire that ensures $\varphi$, possibly without knowing which choice to make to ensure $\varphi$.

2. The agent $a$ ‘knows how to’ / ‘can’ / ‘has the power to’ ensure $\varphi$. 
Two readings of “having a strategy”

- \( tr(K_C\{C\} \varphi) = K_C \Diamond [C] X \varphi \)  
  Group \( C \) knows (\( K \)) there is (\( \exists \)) a choice s.t. for all (\( \forall \)) possible outcomes \( \varphi \)

  - Alternating-time Epistemic Temporal Logic ATEL
    (Wooldridge, van der Hoek 2002)

- We might want: \( \Diamond K_C [C] X \varphi \)  
  There is a choice (\( \exists \)), s.t. group \( C \) knows (\( K \)) that for all (\( \forall \)) possible outcomes \( \varphi \)

  - ATEL does not deal with de re strategies (Jamroga 2003), (Schobbens 2004)
  - Several corrections (Schobbens 2004), (Jamroga, van der Hoek 2004), (Jamroga, Ågotnes 2006, 2007)
  - First semantics with STIT (Herzig, Troquard 2006)
Epistemic STIT

Language.

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \lor \varphi \mid X\varphi \mid [C] \varphi \mid K_a \varphi \]

\(BT + AC + K\)-models are tuples \(\mathcal{M} = (\text{Mom}, <, \text{Choice}, \sim, V)\) where:

- \((\text{Mom}, <, \text{Choice}, V)\) is an \(BT + AC\)-model.

- \(\sim \subseteq (\text{Mom} \times \text{Hist}) \times (\text{Mom} \times \text{Hist})\) is a collection of equivalence relations \(\sim_i\) (one for every agent \(i \in \text{Agt}\)) over indexes.

Extra operators:

- \(\mathcal{M}, m/h \models K_i \varphi\) iff for all \(m'/h' \sim_i m/h\), \(\mathcal{M}, m'/h' \models \varphi\)

Every \(K_i\) is a standard epistemic modality. (Hintikka 1962)
Poor blind Ann again

Epistemic relations are over indexes instead of moments.

- $m_i/h_j \models K_{Ann} \lozenge [Ann] X \phi$
  Ann knows she has an action that leads to a lighten moment.

- $m_i/h_j \not\models \lozenge K_{Ann} [Ann] X \phi$
  Ann does not know how to achieve it.
Rajeev Alur, Thomas A. Henzinger, and Orna Kupferman.
Alternating-time temporal logic.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
Embedding Alternating-time Temporal Logic in Strategic STIT Logic of Agency.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
From Coalition Logic to STIT.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
A normal simulation of coalition logic and an epistemic extension.

Jan Broersen, Andreas Herzig, and Nicolas Troquard.
What groups do, can do, and know they can do: an analysis in normal modal logics.

Valentin Goranko and Wojciech Jamroga.
Comparing semantics of logics for multi-agent systems.

Valentin Goranko, Wojciech Jamroga, and Paolo Turrini.
Strategic games and truly playable effectivity functions.

J. F. Horty.
*Agency and Deontic Logic.*
References II

Andreas Herzig and Nicolas Troquard.
Knowing how to play: uniform choices in logics of agency.

Wojciech Jamroga and Thomas Ågotnes.
Constructive knowledge: what agents can achieve under imperfect information.

Wojciech Jamroga and Wiebe van der Hoek.
Agents that know how to play.

Marc Pauly.
A modal logic for coalitional power in games.

Pierre-Yves Schobbens.
Alternating-time logic with imperfect recall.

Wiebe van der Hoek and Michael Wooldridge.
Tractable multiagent planning for epistemic goals.