Logics of agency

The modal view of agency

EASSS 2018

Overview of this chapter

- Generalities and justifications
- Seeing to it that
  - Various modalities and original semantics
- Bringing it about
  - Several semantics
  - A few computational aspects
- Axiomatization of deliberative stit theories, and computational aspects

Outline

The modal view

St. Anselm (11th century): If a does something he does so such that something is true or false. ((Henry 1953), (Chisholm 1964))

- The relevant aspect of agency is what actions bring about.
- No matter how the structure of the action.

The King is responsible for Anselm being in exile

$\Leftrightarrow$

The King sees to it that Anselm is in exile

∇_{King} "Anselm is in exile"

(We use $\nabla_a \varphi$ as a generic notation for "agent a does φ".)

Grounding the modality of agency: Belnap and Perloff's linguistic agenda

Problem definition: distinguish between sentences which involve agency and those which do not.

- Is "Queequeg struck home with his harpoon" agentive for Queequeg?
  - try to uncover general principles for deciding whether a sentence is agentive
- An agentive sentence must emphasize a sort of causality and responsability of an agent for the truth of a state of affairs.

Fundamental theses (Belnap and Perloff 1988)

Definition (Paraphrase thesis)

The sentence φ marks the agentiveness of agent a just in case φ may be usefully paraphrased as "a sees to it that φ".

Definition (Agentiveness thesis)

The sentence "a sees to it that φ" is agentive for a.

Definition (Complement thesis)

The sentence "a sees to it that φ" is grammatical and meaningful for any sentence φ.

Is Queequeg agentive?

sentence	careful English reading
Queequeg struck home with his harpoon.	agentive
Queequeg's harpoon struck home.	non agentive

But these two sentences are equivalently agentive:

- Queequeg sees to it that Queequeg struck home with his harpoon.
- Queequeg sees to it that Queequeg's harpoon struck home.

Some principles of agency

A long history of argument and disagreement (Chellas 69), (Pörn 1970), (Jones & Pörn), (Elgesem 93):

(T)	$\nabla_a \varphi \to \varphi$
(RE)	if $\varphi \leftrightarrow \psi$ then $\nabla_a \varphi \leftrightarrow \nabla_a \psi$

This leaves some place for a variety of modalities, e.g.:

(C)	$(\nabla_a \varphi \wedge \nabla_a \psi) \to \nabla_a (\varphi \wedge \psi)$
(M)	$\nabla_a (\varphi \wedge \psi) \to (\nabla_a \varphi \wedge \nabla_a \psi)$
(N)	$\nabla_a \top$
(No)	$\neg \nabla_a \top$
(QFAFS)[1]	$\nabla_a \nabla_b \varphi \to \nabla_a \varphi$

[1] "quid facit per alium facit per se"

Outline

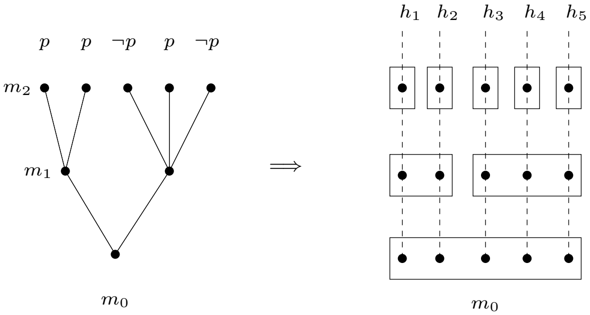
The family of STIT logics

Several logics

- Achievement stit (Belnap and Perloff 1988)
- Deliberative stit (von Kutschera 1986), Chellas stit (Horty and Belnap 1995)
- "Operator of Chellas" (Chellas 1969)
- Strategic achievement stit (Belnap et al. 2001)
- ...
- Strategic Chellas stit of ability (Horty 2001)
- Strategic Chellas stit (Horty 2001)

Ockhamist branching time temporal logic (BT)

BT structure $\langle Mom, < \rangle$:



- History = maximally $<$-ordered set of moments
- $Hist$ = set of all histories
- H_m = set of histories passing through the moment m
- Explode moments into indexes (moment/history pairs)
  - $m_0/h_3 \not\models \mathbf{F}p$
  - $m_0/h_1 \models \mathbf{F}p$

Agents and Choices (AC): selecting sets of histories

Von Wright on action:

> *It would not be right, I think to call acts a kind or species of events. An act is not a change in the world. But many acts may quite appropriately be described as the bringing about or effecting ('at will') of a change. To act is, in a sense, to interfere with 'the course of nature'. ((von Wright 63))*

The notions of a history and history contingency are central to capture the essence of agency.

> *When Jones butters the toast (...) the nature of his act, on this view, is to constrain the history to be realized so that it must lie among those in which he butters the toast. Of course, such an act still leaves room for a good deal of variation in the future course of events, and so cannot determine a unique history; but it does rule out all those histories in which he does not butter the toast. ((Belnap et al. 2001))*

$BT + AC$ models

(Already due to (von Kutschera 1986).)

A $BT + AC$ model is a tuple $\mathcal{M} = \langle Mom, <, Choice, v\rangle$, where:

- $\langle Mom, <\rangle$ is a BT structure;
- $Choice$: Agt $\times$ $Mom \to \mathcal{P}(\mathcal{P}(Hist))$;
- v is valuation function v : Prop $\to \mathcal{P}(Mom \times Hist)$.

$Choice$

- $Choice$: Agt $\times Mom \rightarrow \mathcal{P}(\mathcal{P}(Hist))$
  - $Choice(a, m)$ = repertoire of choices for agent a at moment m
  - $Choice$ is a function mapping each agent and each moment m into a partition of H_m
- $Choice(a, m) : Hist \rightarrow \mathcal{P}(Hist)$
  - For $h \in H_m$: $Choice(a, m)(h)$ = the particular choice of a at index m/h.
- Independence of agents/choices: Let h, m.
  For all collections of $X_a \in Choice(a, m)(h)$, $\bigcap_{a \in \text{Agt}} X_a \neq \emptyset$.
- No choice between undivided histories: if $\exists m' > m$ s.t. $h, h' \in H_{m'}$ then $h' \in Choice(a, m)(h)$.

$Choice$ of groups

A coalition (or group) is a set $C \subseteq \mathsf{Agt}$.

We can define:

$$Choice(C, m)(h) = \bigcap_{a \in C} Choice(a, m)(h) \ .$$

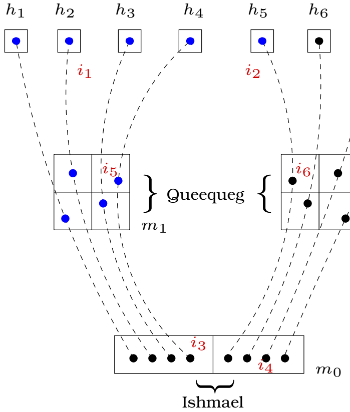
Example: Going Aboard

In Chapter XXI of Melville's Moby Dick, Ishmael and Queequeg go aboard the Pequod deliberately (and Queequeg does not knock Ishmael out). This is the real history.

In some alternate histories:

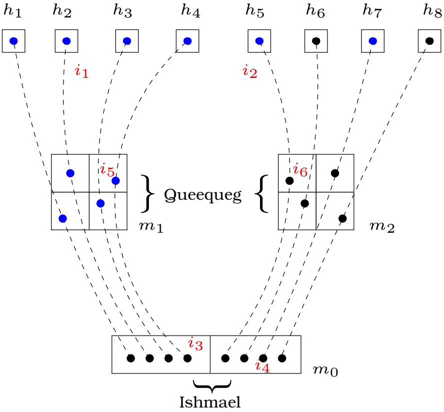
- Ishmael could have stayed on the wharf and walked away
- Queequeg could have knocked him unconscious and dragged him to the cabin aboard

Corresponding BT+AC model



- m_0 Ishmael can go aboard, or stay on the wharf
- m_1 Ishmael can stay on the deck or walk to the cabin
- m_1 Queequeg can do nothing or, knock Ishmael out and drag him to the cabin
- m_2 Ishmael can stay by or, walk away
- m_2 Queequeg can do nothing or, knock Ishmael out and drag him on board to the cabin

Corresponding BT+AC model



E.g.:

- $H_{m_0} = \{h_1, \ldots, h_8\}$; $H_{m_1} = \{h_1, \ldots, h_4\}$; $H_{m_2} = \{h_5, \ldots, h_8\}$
- Ishmael goes aboard at m_0: $\{h_1, \ldots, h_4\}$
- Ishmael stays on the wharf at m_0: $\{h_5, \ldots, h_8\}$
- Ishmael walks away at m_2: $\{h_7, h_8\}$
- Queequeg knocks Ishmael out and drags him on board to the cabin at m_2: $\{h_5, h_7\}$

Many modalities of agency

How agentive are the characters for “Ishmael is sailing on board the Pequod”?

We will see many formal ways to answer this question.

$BT + AC$ models are enough for:

- Deliberative stit
- Horty’s “Chellas” stit
- Strategic Chellas stit
- Strategic Chellas stit of ability

$BT + AC$ models are not enough for:

- Achievement stit
- Operator of Chellas
- Strategic achievement stit

$BT + AC + I$ models

A $BT + AC + I$ model is a tuple $\mathcal{M} = \langle Mom, <, Choice, v, Instant \rangle$ where:

- $\langle Mom, <, Choice, v, \rangle$ is a $BT + AC$ model
- $Instant$ is a partition of Mom
  - Unique intersection: if $I \in Instant$ and $h \in Hist$ then $I \cap h$ is a singleton $\{m_{I,h}\}$
  - Order preservation: if $m_{I_1,h_1} < m_{I_2,h_1}$ then $m_{I_1,h_2} < m_{I_2,h_2}$

We note $I(m)$ the partition of $Instant$ containing moment m.

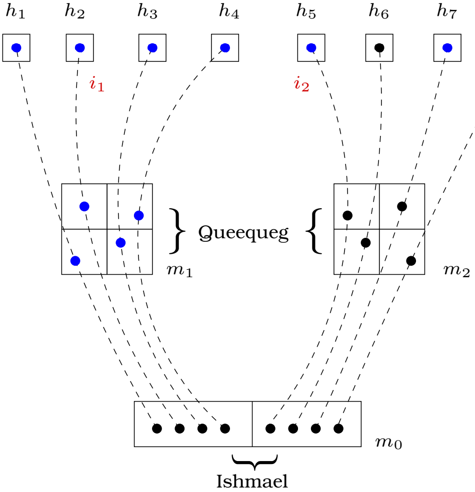
Achievement stit (Belnap and Perloff 1988)

An agent a sees to it that φ if a prior choice of a made sure that φ is true at the current instant, and φ could have been false at this instant had agent a done otherwise.

$M, m/h \models [a\, astit\colon \varphi]$ iff

there is a witness moment $m_0 < m$ such that
(+) $M, m'/h' \models \varphi$ for every m' and h' such that
(i) $Choice(a, m_0)(h) = Choice(a, m_0)(h')$;
(ii) $m' \in h'$ and $I(m) = I(m')$;
(–) $M, m''/h'' \not\models \varphi$ for some m'' and h''
such that $I(m'') = I(m)$ and $m'' \in h''$

astit on our model



$i_1 \models$ [Ishmael $astit : \bullet$]

$i_2 \models \bullet \wedge \neg$[Ishmael $astit : \bullet$]

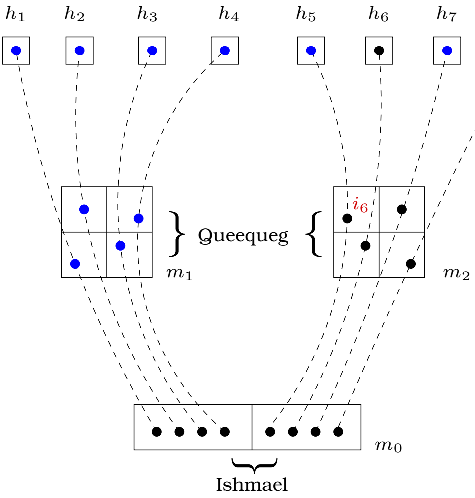
Deliberative stit (von Kutschera 1986)

The witness moment is the current moment: a currently chooses φ but φ was not inevitable.

$M, m/h \models [a\, dstit\colon \varphi]$ iff

(+) $M, m/h' \models \varphi$ for all $h' \in Choice(a, m)(h)$
(–) $M, m/h'' \models \neg\varphi$ for some $h'' \in H_m$

dstit on our model



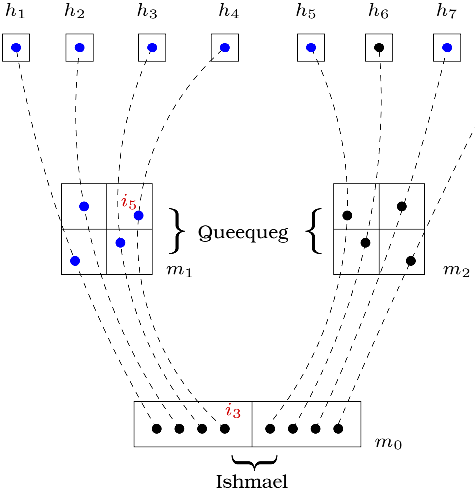
$i_6 \models$ [Queequeg $dstit$: **X**●]

“Chellas” stit (Horty and Belnap 1995)

Like the deliberative stit, but without the negative condition.

$$M, m/h \models [a\, cstit\colon \varphi] \text{ iff } M, m/h' \models \varphi \text{ for all } h' \in Choice(a, m)(h)$$

cstit on our model



$i_3 \models$ [Ishmael $cstit$: **X**●]

$i_5 \models$ [Ishmael $cstit$: ●]

$i_5 \models \neg$[Ishmael $dstit$: ●]

Intermission: "Chelllas" stit to capture choices in matrix games

Consider the moment m_{game}:

	defect$_b$	silent$_b$
defect$_a$	$h_{(-6,-6)}$	$h_{(-10,0)}$
silent$_a$	$h_{(0,-10)}$	$h_{(-2,-2)}$

At $m_{game}/h_{(-6,-6)}$, agent a sees to it that $h_{(-6,-6)} \vee h_{(-10,0)}$:

- $m_{game}/h_{(-6,-6)} \models [a\, cstit\colon h_{(-6,-6)} \vee h_{(-10,0)}]$

At $m_{game}/h_{(-2,-2)}$, the coalition $\{a, b\}$ see to it that $h_{(-2,-2)}$:

- $m_{game}/h_{(-6,-6)} \models [\{a, b\}\, cstit\colon h_{(-2,-2)}]$

The operator of Chellas (Chellas 1969)

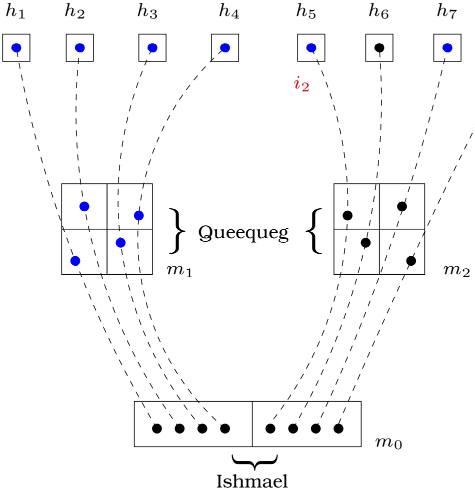
The semantics of the operator of Chellas (Δ_a) requires a discrete time.

$M, m/h \models \Delta_a \varphi$ iff

(let m_{-1} be the moment immediately preceding m)
(+) $M, m'/h' \models \varphi$ for every h' and m' such that
(i) $I(m) = I(m')$ and $m' \in h'$;
(ii) $Choice(a, m_{-1})(h) = Choice(a, m_{-1})(h')$.

Horty's "Chellas" stit $\neq$ the operator of Chellas

Δ on our model



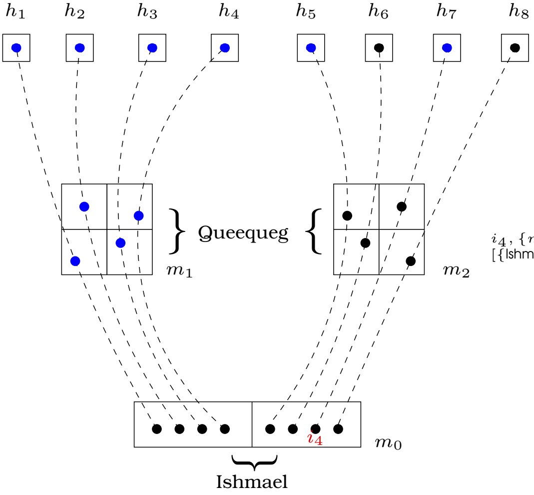
$i_2 \models \Delta_{\text{Queequeg}}$●

$i_2 \models \neg\Delta_{\text{Ishmael}}$●

Strategic stit (informally and with pointers)

- Belnap & Perloff's "strategic achievement stit"
  - (Belnap et al 2001, Ch. 13)
  - "There have been a series of choices by agent a in the past that ensured φ at the current index."
- Horty's "strategic Chellas stit"
  - (Horty 2001, Ch. 7)
  - (Broersen et al. 2006 JELIA) (Tr. & Walther 2012)
  - "The current series of choices (strategy) by agent a ensure φ to be realised."
- Horty's "strategic Chellas stit of ability"
  - (Horty 2001, Ch. 7)
  - Relationship with ATL (Alur et al. 2002) (Broersen et al. 2006 JLC)
  - "There is a series of choices by agent a that (would) ensure φ to be realised."

scstit on our model



$i_4, \{m_0 \mapsto \{h_5, \dots, h_8\}, m_2 \mapsto \{h_7\}\} \models$
[{Ishmael, Queequeg} $scstit$: **XX**●]

Complexity results "seeing to it that" (some pointers)

Reasoning about "seeing to it that" is computationally costly.

- Achievement stit: decidable for one-agent case... without busy choosers
- Individual agency Chellas/deliberative stit: NEXPTIME-complete (Balbiani et al. 08)
- Coalitional agency Chellas/deliberative stit: from NEXPTIME-complete to undecidable (Schwarzentruber et al. 07-11)
- Strategic coalitional agency:
  - satisfiability problem: undecidable (Tr. & Walther 12)
  - model checking problem: non-elementary (Brihaye et al. 07-13)

Taming the complexity:

- Restricting the models: CL-PC (van der Hoek & Wooldridge 05)
- Restricting the coalitions: formulas of "ever growing coalitions" (Schwarzentruber 11), bounded modal depth (Lorini & Schwarzentruber 11)
- Restricting the goal formulas: (Murano et al. 07-)

Outline

Semantics combining normal modalities (Pörn 1970)

- $D_a\varphi$ is true in a world w if φ is true at every hypothetical situation where agent a "does at least as much as he does in w"
- $D'_a\varphi$ is true in w if $\neg\varphi$ is true in every hypothetical situation w' such that "the opposite of everything a does in w is the case in w'"
- Combination of two normal operators in a non-normal modality:
  - $D_a\varphi$: "it is necessary for something a does that φ"
  - $D'_a\varphi$: "but for a's action, it would not be the case that φ"
  - $E_a\varphi \triangleq D_a\varphi \wedge \neg D'_a\neg\varphi$ reads "agent a brings it about that φ".

A controversial semantics

- “one problem with the proposed semantics is that ‘doing at least as much as’ he does in (a world), and the notion of an agent doing ‘the opposite’ of everything he does in (a world), are of dubious intelligibility without substantial further elucidation, and Pörn offers none.” (Horgan 1979)
- “the intuitive significance of this semantics is not altogether clear.” (Segerberg 1992)

Selection functions $M = \langle W, \{f_a\}, V\rangle$

(Elgesem 93): new semantics for agency and ability.

(Elgesem 93), inspired by (Sommerhoff 69)'s control theory:

- W is some set of possible worlds,
- V : Prop $\longrightarrow \mathcal{P}(W)$ is a valuation function
- $f_a : W \times \mathcal{P}(W) \rightarrow \mathcal{P}(W)$ is a selection function for every agent a

The object $f_a(w, X)$ is the set of those worlds where a realizes the ability he has in w to bring about his goal X;

a is able to bring about X at w if $f_a(w, X)$ is nonempty;

a brings about X at w if w belongs to $f_a(w, X)$.

Selection functions $M = \langle W, \{f_a\}, V\rangle$ (ctd)

The functions f_a have to satisfy the following constraints:

- $f_a(w, X) \subseteq X$, for every $X \subseteq W$ and $w \in W$;
- $f_a(w, X_1) \cap f_a(w, X_2) \subseteq f_a(w, X_1 \cap X_2)$, for every $X_1, X_2 \subseteq W$ and $w \in W$;
- $f_a(w, W) = \emptyset$, for every $w \in W$.

The truth conditions are as follows:

$$\begin{array}{lll} M, w \models p & \text{iff} & w \in V(p); \\ M, w \models E_a\varphi & \text{iff} & w \in f_a(w, ||\varphi||^M); \\ M, w \models C_a\varphi & \text{iff} & f_a(w, ||\varphi||^M) \neq \emptyset. \end{array}$$

where $||\varphi||^M = \{w \in W \mid M, w \models \varphi\}$.

Bringing it about (neighbourhood semantics)

Paraphrased from (McNamara 2000):

- The semantics involves some agents $\mathtt{Agt}$, existing at various possible worlds W.
- In these worlds, an agent often exhibits her agency by bringing certain things about.
  - $EE : W \times \mathtt{Agt} \longrightarrow \mathcal{P}(\mathcal{P}(W))$
- Presumably, she does so by taking certain actions that result in certain propositions being true, the ones she has brought about.

An abstract semantics

Neighbourhoods semantics $M = \langle W, EE, EC, V \rangle$:

- W is some set of possible worlds,
- V : Prop $\longrightarrow \mathcal{P}(W)$ is a valuation function
- $EE : W \times$ Agt $\longrightarrow \mathcal{P}(\mathcal{P}(W))$
- $EC : W \times$ Agt $\longrightarrow \mathcal{P}(\mathcal{P}(W))$

Constraints on the neighborhoud functions:

- $W \notin EE(w, a)$
- $\emptyset \notin EC(w, a)$
- if $X \in EE(w, a)$, then $w \in X$
- if $X \in EE(w, a)$ and $Y \in EE(w, a)$ then $X \cap Y \in EE(w, a)$
- $EE(w, a) \subseteq EC(w, a)$

Truth values:

$$
\begin{array}{lll}
M, w \models p & \text{iff} & w \in V(p); \\
M, w \models E_a\varphi & \text{iff} & ||\varphi||^M \in EE(w, a); \\
M, w \models C_a\varphi & \text{iff} & ||\varphi||^M \in EC(w, a).
\end{array}
$$

Core principles of agency in "bringing-it-about"

(Elgesem 93), (Elgesem 97), (Governatori & Rotolo 2005), ...:

- Propositional logic
- $\vdash \neg E_a \top$
- $\vdash E_a\varphi \wedge E_a\psi \to E_a(\varphi \wedge \psi)$
- $\vdash E_a\varphi \to \varphi$
- if $\vdash \varphi \leftrightarrow \psi$ then $\vdash E_a\varphi \leftrightarrow E_a\psi$

Simple abilities

Individual agency and ability (Elgesem 93):

$$E_a\varphi \to C_a\varphi \ .$$

Coalitional agency and ability (Tr. 2014):

$$E_{G_1}\varphi \wedge E_{G_2}\varphi \to C_{G_1 \cup G_2}(\varphi \wedge \psi) \ .$$

Reasoning about "bringing-it-about"

Decision problem 1: Is φ a valid formula?
Decision problem 2: Is φ a satisfiable formula?

Theorem ((Tr. 14))

Reasoning about

$$\begin{cases} \textit{individual agency} \\ \textit{individual agency and ability} \\ \textit{coalitional agency and ability} \end{cases}$$

can be solved in space polynomial in the size of φ *(in PSPACE).*

Outline

Historic necessity

Historic necessity:

$$m/h \models \Box\varphi \text{ iff } \forall h' \in H_m : m/h' \models \varphi$$

- $[a\, dstit\colon \varphi] \leftrightarrow [a\, cstit\colon \varphi] \wedge \neg\Box\varphi$
- $[a\, cstit\colon \varphi] \leftrightarrow [a\, dstit\colon \varphi] \vee \Box\varphi$
- $\Box\varphi \leftrightarrow$ ... (see a few slides ahead)

Preparation: $S5$ modal logic (Lewis, Langford 1932)

- $S5$ is characterized by equivalence frames (reflexive, transitive, and symmetrical).
- Axiomatics: (K, T, 4, B), (K, D, T, 4, 5)...

  K $\blacksquare(\varphi \to \psi) \to (\blacksquare\varphi \to \blacksquare\psi)$
  T $\blacksquare\varphi \to \varphi$
  4 $\blacksquare\varphi \to \blacksquare\blacksquare\varphi$
  5 $\blacklozenge\varphi \to \blacksquare\blacklozenge\varphi$
  B $\varphi \to \blacksquare\blacklozenge\varphi$
  D $\blacksquare\varphi \to \blacklozenge\varphi$

Lemma

$A_1 A_2 \ldots A_k \varphi \leftrightarrow A_k \varphi$, $A_i \in \{\blacksquare, \blacklozenge\}$.

Preparation: Product S5$\otimes$S5 (Marx 1999), (Gabbay et al. 2003)

S5$\otimes$S5 is the bi-modal logic axiomatized by:

- S5(0): all S5 principles for $[0]$
- S5(1): all S5 principles for $[1]$
- the permutation axioms
  - $\langle 1\rangle\langle 0\rangle\varphi \to \langle 0\rangle\langle 1\rangle\varphi$,
  - $\langle 0\rangle\langle 1\rangle\varphi \to \langle 1\rangle\langle 0\rangle\varphi$;
- Church-Rosser axioms
  - $\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\varphi$,
  - $\langle 1\rangle[0]\varphi \to [0]\langle 1\rangle\varphi$.

Xu's Ldm axiomatics of *individual* Chellas stit

Convenient notation:

- $[a]\varphi$ instead of $[\{a\}\, cstit: \varphi]$

S5($\Box$)	axiom schemas of S5 for $\Box$
S5($[i]$)	axiom schemas of S5 for every $[i]$
($\Box \to [i]$)	$\Box\varphi \to [i]\varphi$
(AIA$_k$)	$(\Diamond[0]\varphi_0 \wedge \ldots \wedge \Diamond[k]\varphi_k) \to \Diamond([0]\varphi_0 \wedge \ldots \wedge [k]\varphi_k)$

Theorem ((Xu 1994))

Ldm is sound and complete w.r.t. $BT + AC$ models.

A convenient truth

Clearly via the semantics and the completeness theorem:

$$\vdash [1][0]\varphi \to \Box\varphi$$

- Advanced (?) problem: derive it from Ldm.
  I do not know the solution.

The other way round holds too!

- Simple exercise: derive it from Ldm.

Then

- $\vdash \Box\varphi \leftrightarrow [1][0]\varphi$
- we can get rid off the $\Box$ operator!

Alternative Ldm

- Independence of agents in Ldm: (AIA$_k$)
  $\Diamond[0]\varphi_0 \wedge \ldots \Diamond[k]\varphi_k \to \Diamond([0]\varphi_0 \ldots [k]\varphi_k)$
- Alternative axiomatization of Ldm (Balbiani, Herzig, Tr. 2008):

S5(i)	enough S5-theorems, for every $[i]$
Def($\Box$)	$\Box\varphi \leftrightarrow [1][0]\varphi$
(GPerm$_k$)	$\langle l\rangle\langle m\rangle\varphi \to \langle n\rangle \bigwedge_{i \in \mathsf{Agt}\setminus\{n\}} \langle i\rangle\varphi$

- (GPerm$_k$) captures independence of agents

Alternative semantics

All axiom schemes are in the Sahlqvist class, and therefore have a standard possible worlds semantics.

Kripke models are of the form $M = \langle W, R, V \rangle$, where

- W is a nonempty set of possible worlds;
- R is a mapping associating to every $i \in \mathsf{Agt}$ an equivalence relation R_i on W;
- V is a mapping from Prop to the set of subsets of W.

We have the usual truth condition:

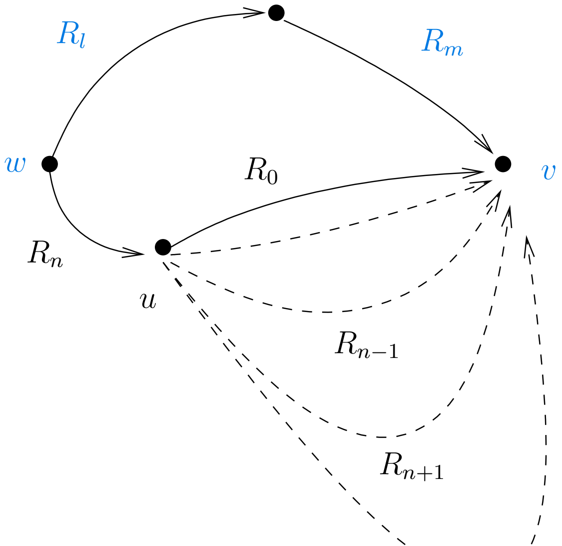
$$M, w \models [i]\varphi \text{ iff } M, u \models \varphi \text{ for every } u \text{ such that } \langle w, u \rangle \in R_i$$

We impose that R satisfies the general permutation property.

Alternative semantics (ctd)

Definition (general permutation property)

For all $w, v \in W$ and for all $l, m, n \in \mathsf{Agt}$, if $\langle w, v\rangle \in R_l \circ R_m$ then there is $u \in W$ such that: $\langle w, u\rangle \in R_n$ and $\langle u, v\rangle \in R_i$ for every $i \in \mathsf{Agt} \setminus \{n\}$.



Link with product logic and complexity

If $\mathsf{Agt} = \{0, 1\}$ then the validities are axiomatized by:

- Def($\Box$): $\Box\varphi \leftrightarrow [1][0]\varphi$
- S5(0)
- S5(1)
- (GPerm$_1$), two instances:
  - $\langle 1\rangle\langle 0\rangle\varphi \rightarrow \langle 0\rangle\langle 1\rangle\varphi$
  - $\langle 0\rangle\langle 1\rangle\varphi \rightarrow \langle 1\rangle\langle 0\rangle\varphi$

Moreover,

- Permutation $\langle 1\rangle\langle 0\rangle\varphi \leftrightarrow \langle 0\rangle\langle 1\rangle\varphi$
- Church-Rosser $\langle 0\rangle[1]\varphi \rightarrow [1]\langle 0\rangle\varphi$, $\langle 1\rangle[0]\varphi \rightarrow [0]\langle 1\rangle\varphi$

can be proved.

Proof of Church-Rosser

1. $\langle 0\rangle\langle 1\rangle[1]\varphi \to \langle 1\rangle\langle 0\rangle[1]\varphi$ (GPerm$_1$)
2. $\langle 0\rangle[1]\varphi \to \langle 1\rangle\langle 0\rangle[1]\varphi$ (S5(1))

3. $\langle 1\rangle\langle 0\rangle[1]\varphi \to \langle 0\rangle\langle 1\rangle[1]\varphi$ (GPerm$_1$)
4. $[1]\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\langle 1\rangle[1]\varphi$ (K(1))
5. $\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\langle 1\rangle[1]\varphi$ (S5(1))
6. $\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle[1]\varphi$ (S5(1))
7. $\langle 1\rangle\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\varphi$ (S5(1))

8. $\langle 0\rangle[1]\varphi \to [1]\langle 0\rangle\varphi$ (From 2 and 7)

Link with product logic and complexity (ctd)

- Hence the logic of the two-agent Ldm is nothing but the product $S5^2 = S5 \otimes S5$ (Marx 1999), (Gabbay et al. 2003)
- NEXPTIME-complete.

Fortunately, adding more agents does not lead to a more complex logic:

Theorem ((Balbiani, Herzig, Tr. 2008))

(Full) Ldm is NEXPTIME-complete.

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