2. Bayesian Classifiers

1. Application of Naive Bayes Classifier

Classify the tuple $X\langle\text{single, light, one}\rangle$ using Naive Bayes given the following examples:

- Healthy:
  - $\langle\text{single, dark, one}\rangle$
  - $\langle\text{single, light, two}\rangle$
  - $\langle\text{double, light, one}\rangle$

- Virulent:
  - $\langle\text{single, dark, two}\rangle$
  - $\langle\text{double, dark, one}\rangle$
  - $\langle\text{double, light, two}\rangle$

2. Problem Modeling

Assume we have two classes:

- Democrats: people should have access to affordable and quality health care
- Republicans: the government should not pay for all health care costs

Classify the following sentences

- Sentence 1: people should have access to hospitals
- Sentence 2: people should not pay for health care
3. From Naive Bayes to Bayesian Networks

1. Looking at the faces shown below, predict whether the last face is smiling or not using Naive Bayes.

2. Draw the Bayesian Network that is equivalent to Naive Bayes.

3. Calculate all the probability tables for each of the Bayesian networks shown in the figure below according to the given examples-faces. Predict the class of the last face according to each of the networks.
4. Conditional Independence

Coughing is a natural reflex that protects your lungs, however if it becomes frequent than it indicates the presence of a disease. There can be many causes for frequent Coughing including: Lung Cancer and Bronchitis.

1. Build a Bayesian Network that models the above problem by considering the following random variables:
   - L: Lung Cancer
   - B: Bronchitis
   - C: Cough

2. Show that \( L \) and \( B \) are independent using the topological property Bayesian networks (i.e., Each node is conditionally independent of its non-descendants given its parents).

3. Assume now that Smoking makes people more subject to Bronchitis. Additionally, Family History is one of the main reasons of Lung Cancer. Update your network assuming that:
   - S: Smoking
   - F: Family History

4. Can you show that \( L \) and \( B \) are independent using the topological property of the Bayesian Network?

5. Compute \( P(L \text{ and } B) \). Are \( L \) and \( B \) still independent?

6. Use the d-separation Algorithm to show that \( L \) and \( B \) are independent. What do you observe?

7. Are \( L \) and \( B \) are conditionally independent given \( C \)? Use the d-separation Algorithm to prove that.

8. Assume now that Smoking can also lead to Lung Cancer which can be diagnosed doing an X ray. If the X ray is positive then, the person has Lung Cancer. Update your network. We assume:
   - X: Positive X ray

9. Given evidence \( E \), which node pairs are conditionally independent?
   - (a) \( E = \{\} \)
   - (b) \( E = \{S\} \)
5. Inference

Given the network above, calculate marginal and conditional probabilities $P(C)$. 

(c) $E = \{L\}$

(d) $E = \{L, B\}$