

Semantic Web Technologies

Ontologies and Rules with F-Logic

Jos de Bruijn
jos.debruijn@deri.org

KRDB Research Group
Free University of Bolzano, Italy

11 April 2007

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Outline

Logic Programming Basics

Subsets of Logic Programming
Reasoning Problems

Semantics of Logic Programming

Herbrand Semantics
Fixpoint Semantics

Negation in Logic Programs

Querying with Negation
Semantics of Negation

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F-Logic and RDF
Reducing F-Logic Programming to standard LP

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LP Syntax I

- ▶ LP is based on the Horn subset of FOL
- ▶ Vocabulary:
 - ▶ Constants $a, b, c, \text{john}, \text{mary}, 1, \text{"bla"}$
 - ▶ Predicates $p, q, r, \text{married-to}, \text{manager-of}$
 - ▶ Function symbols f, g, h
 - ▶ Variables x, y, z
- ▶ Terms:
 - ▶ Constants
 - ▶ Variables
 - ▶ Constructed terms (i.e. function symbol with arguments)
- ▶ Terms (examples):
 - ▶ $a, x, \text{john}, \text{father-of}(\text{john}), \text{father-of}(x),$
 $\text{father-of}(\text{father-of}(\text{john}))$
- ▶ Atoms:

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LP Syntax II

- ▶ For n -ary predicate symbol p and terms t_1, \dots, t_n , $p(t_1, \dots, t_n)$ is an atom
- ▶ Examples: $p(x)$, $p(a)$, $\text{hasName}(\text{john}, \text{"John"})$,
 $\text{married-to}(\text{john}, \text{mary})$, $\text{hasGrandfather}(x,$
 $\text{father-of}(\text{father-of}(x)))$
- ▶ A **ground** atom is an atom without variables
- ▶ Literals:
 - ▶ A positive literal is an atom: $p(x)$, $q(x)$, ...
 - ▶ A negative literal is a negated atom: $\text{not } p(x)$, $\text{not } q(x)$, ...
 - ▶ A ground literal is a literal without variables
- ▶ Rule:
 - ▶ The **head** of the rule consists of one positive literal H
 - ▶ The **body** of the rule consists of a number of literals B_1, \dots, B_n
 $H \text{ :- } B_1, \dots, B_n$
 - ▶ B_1, \dots, B_n are also called **subgoals**

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LP Syntax III

- ▶ Examples:


```
hasUncle(x,z) :- hasParent(x,y), hasBrother(y,z).
orphan(x) :- not hasParent(x,y).
hasParent(x,f(x)) :- person(x), not orphan(x).
```
- ▶ Note:
 - :- (implication) is sometimes written as \leftarrow
 - , is sometimes written as \wedge :
$$H \leftarrow B_1 \wedge \dots \wedge B_n$$
- ▶ Fact: A ground atom (i.e. rule without a body)
 - ▶ $\text{person}(\text{john})$.
- ▶ Query:
 - ▶ A rule without a head:

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LP Syntax IV

- $\text{?- } B_1, \dots, B_n$.
- ▶ Examples:


```
?- hasUncle(john,z).
?- married-to(john,mary).
?- person(x).
```
- ▶ Consider:


```
u(x,z) :- p(x,y), b(y,z).
p(john,jack).
b(jack,mike).
```
- $\text{?- } u(\text{john}, \text{mike})$.
- $\text{?- } u(\text{john}, x)$.
- ▶ Built-in predicates:
 - ▶ Arithmetic comparators: $>$, $<$, $=<$, $>=$, $=$

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LP Syntax V

- $3 < 4, 5 > x, \dots$
 $<(3,4), >(5,x), \dots$
- ▶ (in)equality: $=, !=$
 $5=x, z=f(x), john!=mary, john=x, jack=father(john)$
- ▶ May only occur in rule bodies
- ▶ Built-in functions:
 - ▶ Arithmetic functions: $+, -, *, /$
 $1+2, 5/6, 5*x, \dots$
 - $+(1,2), /(5,6), *(5,x), \dots$
- ▶ Built-in functions can be reduced to predicates:
 $+(1,x) > *(8,5)$ **reduces to** $y > z, add(1,x,y),$
 $multiply(8,5,z)$

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LP Subsets

- ▶ Full logic programming
 - ▶ Function symbols
 - ▶ No negation
- ▶ Datalog as subset
 - ▶ No function symbols
 - ▶ No negation
 - ▶ Safe rules:
 - ▶ All variables are limited
 - ▶ A variable is limited if:
 - ▶ It appears as an argument in an ordinary (non-built-in) predicate of the body, or
 - ▶ The variable X appears in a subgoal $X = a$ or $a = X$, where a is a constant, or
 - ▶ The variable X appears in a subgoal $X = Y$ or $Y = X$, where Y is limited

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LP Subsets (examples)

- ▶ Full Logic Programming:
 $hasParent(x,f(x)) :- person(x).$
 $hasParent(x,y) :- person(x).$
 $bigger(x,y) :- x > y.$
- ▶ Datalog:
 $adult(x) :- person(x), hasAge(x,y), y >= 18.$
 $hasParent(x,y) :- person(x), hasMother(x,y).$
 $hasParent(x,y) :- person(x), hasFather(x,y).$

 $ancestor(x,y) :- hasParent(x,y).$
 $ancestor(x,z) :- ancestor(x,y), ancestor(y,z).$
- ▶ Logic Programming vs. Deductive Databases
 - ▶ Logic Programming: using logic as a programming language (e.g. Prolog)
 - ▶ Deductive Databases: using logic as a database (query) language, e.g. Datalog

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Dependency graph and recursive LP

- ▶ Dependency graphs
 - ▶ Each predicate is a node
 - ▶ There is an arc from predicate p to predicate q if they both occur in a rule where q is in the head and p in a subgoal
- ▶ A program is **recursive** iff its dependency graph contains a cycle
 - ▶ All predicates on such a cycle are called recursive
- ▶ Recursive:
 $ancestor(x,y) :- hasParent(x,y).$
 $ancestor(x,z) :- ancestor(x,y), ancestor(y,z).$
- ▶ Not recursive:
 $hasParent(x,f(x)) :- person(x).$
 $hasParent(x,y) :- person(x).$
 $bigger(x,y) :- x > y.$

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Datalog

- ▶ A logic for querying databases
- ▶ Extensional Database (EDB) consists of facts
 $person(john).$
 $father(john,jack).$
 $brother(jack,mike).$
- ▶ Intentional Database (IDB) consists of non-ground rules
 $adult(x) :- person(x), hasAge(x,y), y >= 18.$
 $parent(x,y) :- person(x), mother(x,y).$
 $parent(x,y) :- person(x), father(x,y).$
 $uncle(x,z) :- parent(x,y), brother(y,z).$
- ▶ Consider the query:
 $?- uncle(john,mike).$

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Reasoning Problems

- ▶ Query answering:
 - ▶ Ground queries, e.g. is Mike an uncle of John?
 $?- uncle(john,mike).$
 - ▶ Non-ground queries, e.g. who are the uncles of John?
 $?- uncle(john,x).$
- ▶ Nonground queries can be reduced to ground queries, e.g.:
 - ▶ Replace $?- uncle(john,x)$ With the three queries:
 $?- uncle(john,john).$
 $?- uncle(john,jack).$
 $?- uncle(john,mike).$
- ▶ Answering ground queries is equivalent to entailment of ground facts:
 - ▶ For program P and ground atom A , does the following entailment hold?

$$P \models A$$

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LP Semantics

- ▶ Model-theoretic semantics
 - ▶ What does a program/database mean in terms of its models?
 - ▶ We define the **minimal Herbrand model**.
- ▶ Computational semantics
 - ▶ How to compute the model of a program/database?
 - ▶ We define the direct consequence operator T_P for program P , which computes all consequences,
- ▶ Correspondence between model-theoretic and computational semantics
 - ▶ The fixpoint of T_P is the minimal Herbrand model!
- ▶ LP semantics is equivalent with First-Order semantics wrt. entailment of **ground facts**
 - ▶ Note: this only hold for logic programming without negation!

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Herbrand Semantics: the idea

- ▶ Consider only particular first-order interpretations
 - ▶ Domain = set of ground terms (Herbrand universe)
 - ▶ Interpret ground terms as themselves: $t^I = t$
- ▶ Intersection of all models is **minimal model**

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Ground Terms, ground atoms and the Herbrand Universe

- ▶ terms not containing any variables are **ground terms**
- ▶ Similarly, a ground atom is an atom not containing variables
- ▶ The **Herbrand Universe** U_L is the set of all ground terms which can be formed from constants and function symbols in program P .
 Example: $a, b, f(a), f(b), f(f(a)), f(f(b)), f(f(f(a))), \dots$
- ▶ The Herbrand Base B_P is the set of all ground atoms which can be built from predicate symbols in P , using ground terms from U_P as arguments.
 Example: $p(a), p(b), q(a), q(b), p(f(a)), q(f(a)), \dots$

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Herbrand models

- ▶ A Herbrand Interpretation I_P is a subset of the Herbrand Base B_P .
- ▶ A Herbrand Model M_P is a Herbrand Interpretation which makes every formula true, i.e.:
 - ▶ Every fact in P is in M_P , and
 - ▶ For every rule R in P holds: if every positive literal in the body is in M_P , then also the head literal is in M_P .
 Note: this only works for positive programs, i.e., programs **without** negation!
- ▶ The semantics of a program P is characterized in terms of the **least** Herbrand Model, which is the intersection of all possible Herbrand Models.
- ▶ Each positive program has one **unique** least Herbrand Model.

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Fixpoint Semantics

- ▶ Let I_P be a Herbrand interpretation and P a positive program:

$$T_P(I) = \{A \in B_P : A : -A_1, \dots, A_n \text{ is a ground instance of a rule in } P \text{ such that } A_1, \dots, A_n \in I\}$$

is called the **immediate consequence operator**.

- ▶ T_P is **monotonic**: $T_P^2(I) = T_P(T_P(I)) \supseteq T_P(I)$
- ▶ Thus, T_P has a fixpoint T_P^∞
- ▶ For a positive program, the least fixpoint of T_P is equivalent to the minimal Herbrand Model M_P :

$$lfp(T_P) = M_P$$

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Example

```
a(x) :- b(x).
b(x) :- e(x).
l(karl, x) :- a(x).
b(franz).
e(hansi).
```

$$T_P(\emptyset) = \{b(\text{franz}), e(\text{hansi})\} \quad (1)$$

$$T_P^2(\emptyset) = T_P(\emptyset) \cup \{a(\text{franz}), b(\text{hansi})\} \quad (2)$$

$$T_P^3(\emptyset) = T_P^2(\emptyset) \cup \{l(\text{karl}, \text{franz}), a(\text{hansi})\} \quad (3)$$

$$T_P^4(\emptyset) = T_P^3(\emptyset) \cup \{l(\text{karl}, \text{hansi})\} \quad (4)$$

$$T_P^5(\emptyset) = T_P^4(\emptyset) \quad (5)$$

i.e. $lfp(T_P) = T_P^4(\emptyset) = M_P$ Note: $T_P^2(\emptyset) = T_P(T_P(\emptyset))$, $T_P^3 = T_P(T_P^2(\emptyset))$, etc...

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Example (cont'd)

A simple example where the least Herbrand model is infinite:

```
p(a).  
p(f(x)) :- p(x).
```

$$Ifp(T_P) = \{p(a), p(f(a)), p(f(f(a))), \dots\}$$

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Negation

- ▶ How do we handle negation in Logic Programs?
- ▶ Negation-as-failure: whenever a fact is not entailed by the knowledge base, its negation is entailed
- ▶ Example:

```
p(a).  
q(x) :- p(x)
```

entails:

```
p(a).  
q(a).  
not p(b).  
not q(b).  
not r(a).  
not r(b).  
...
```

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Examples

- ▶ Using negative literals in rules.
- ▶ Example:

```
p(a).  
q(x) :- not p(x).
```

entails:

```
p(a).  
not q(a).  
not p(b).  
q(b).  
not r(a).  
not r(b).  
...
```

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Examples

- ▶ Example:

```
p(a).  
p(b).  
r(a).  
r(c).  
q(x) :- not p(x), r(x).
```

entails:

```
p(a).  
p(b).  
r(a).  
r(c).  
q(c).
```

- ▶ We typically leave out the entailed negative facts, since the negation of every fact which is not entailed, is entailed

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Querying

- ▶ Ground queries:
 - ▶ The answer to a ground query Q_g is "yes" if:
 - ▶ Every positive literal in Q_g is in the minimal model of the program, and
 - ▶ Every negative literal in Q_g is not in the minimal model of the program.
 - ▶ Otherwise, the answer is "no".
- ▶ Non-ground queries are instantiated, in the same way as for logic programs without negation

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Query Example

```
p(a).  
p(b).  
r(a).  
r(c).  
q(x) :- not p(x), r(x).
```

?- not q(a), r(c).

answer: **YES**

?- r(c), not q(c).

answer: **NO**

?- r(X), not q(X).

answer:

X = a.

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Outline

- ▶ Characterizing negative information in Herbrand models
- ▶ Characterizing negative information using the Fixpoint operator
- ▶ Problems with the Fixpoint operator under negation
- ▶ A popular solution: stratified negation

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Extending Herbrand Semantics

- ▶ A Herbrand Model M_P is a Herbrand Interpretation which makes every formula true, i.e.:
 - ▶ Every fact in P is in M_P , and
 - ▶ For every rule R in P holds: if every positive literal in the body is in M_P and every negative literal in the body is not in M_P , then also the head literal is in M_P .
- ▶ The semantics of a program P is characterized in terms of the least Herbrand Model.
- ▶ But: **is the least Herbrand model still unique?**

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Extending Herbrand Semantics (II)

Consider the following examples:

$p(a).$
 $q(b) :- \text{not } p(b).$

The minimal Herbrand Model M_P is: $\{p(a), q(b)\}$

$p(a).$
 $q(x) :- \text{not } p(x).$

The minimal Herbrand Model M_P is: $\{p(a)\}$

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Extending Herbrand Semantics (III)

Consider the following examples:

$r(a).$
 $q(x) :- \text{not } p(x), r(x).$
 $p(x) :- \text{not } q(x), r(x).$

Has two minimal Herbrand Models: $\{r(a), q(a)\}, \{r(a), p(a)\}$

$a :- \text{not } b.$
 $b :- \text{not } a.$

Has two minimal Herbrand Models: $\{a\}, \{b\}$

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Extending T_P

- ▶ Let I_P be a Herbrand interpretation and P a general program:

$T_P(I) = \{A \in B_P \mid A : -L_1, \dots, L_i, \text{not } L_{i+1}, \dots, \text{not } L_n$
is a ground instance of a rule in P such that
 $L_1, \dots, L_i \in I_P$ and $L_{i+1}, \dots, L_n \notin I_P\}$

is the extended immediate consequence operator.

- ▶ Is the least fixpoint of T_P equivalent to the minimal Herbrand Model M_P ?
- ▶ **Note: we have already seen cases of several minimal Herbrand models.**

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Extending T_P (cont'd)

Consider the following examples:

$p(a).$
 $q(b) :- \text{not } p(b).$

Applications of T_P :
(1) $\{p(a)\}$
(2) $\{p(a), q(b)\}$

(2) is the least fixpoint and is equivalent to M_P .

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Extending T_P (cont'd)

Consider the following examples:

```
r(a).  
q(x) :- not p(x), r(x).  
p(x) :- not q(x), r(x).
```

Applications of T_P :

```
(1){r(a)}  
(2){r(a),p(a),q(a)}
```

(2) is a fixpoint, but not a Herbrand model! Remember, the Herbrand models are: $\{r(a), q(a)\}, \{r(a), p(a)\}$

Thus, naive extension of the fixpoint operator is not possible!

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Solution: Stratified Programs

- ▶ Recall: dependency graph
 - ▶ Each predicate is a node
 - ▶ There is an edge from predicate p to predicate q if they both occur in a rule where q is in the head and p in a subgoal
 - ▶ Edges with negation are marked
- ▶ A predicate is in the same or a higher stratum than all predicates connected to it with a positive edge.
- ▶ If there is a negative edge leading from a predicate p to a predicate q , then the stratum of q is strictly higher than the stratum of p .
- ▶ If every predicate occurs in at most one stratum, the program is stratifiable.
- ▶ For stratified programs, we can use the extended fixpoint operator T_P !

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Examples of Stratification

```
p(a).  
q(b) :- not p(b).
```

Is stratified with:

Stratum 0: {p} Stratum 1: {q}

```
p(a).  
r(a).  
q(x) :- not p(x), r(x).
```

Is stratified with:

Stratum 0: {p} Stratum 1: {q,r}

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Examples of Stratification

```
r(a).  
q(x) :- not p(x), r(x).  
p(x) :- not q(x), r(x).
```

Is not stratified. Why?

```
a :- not b.  
b :- not a.
```

Is not stratified. Why?

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F-Logic extensions: motivation

- ▶ Predicates not really appropriate to capture object typing and attributes.
- ▶ Consider: John is a Lawyer, has the particular name "John Smith" and has two children named "Mary" and "Jill". Furthermore, Lawyer is a particular kind of Human.
- ▶ Expressed using Logic Programming:

```
lawyer(john).  
hasName(john, "John Smith").  
hasChild(john,mary).  
hasChild(john,jill).  
hasName(jill, "Jill").  
hasName(mary, "Mary").  
person(x) :- lawyer(x).
```
- ▶ This encoding:
 - ▶ Disregards the structure of the natural language sentences
 - ▶ Information about the particular object is no longer grouped

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F-Logic Introduction

- ▶ Consider: John is a Lawyer, has the particular name "John Smith" and has two children named "Mary" and "Jill". Furthermore, Lawyer is a particular kind of Human.
- ▶ Expressed using F-Logic:

```
john:lawyer[name->"John Smith",  
             child->>{mary[name->"Mary"],  
                    jill[name->"Jill"]}].  
  
lawyer :: person.  
  
▶ F-Logic also allows specifying signature information:

```
person[name=>string,
 child=>>person].
```


```

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F-Logic

- ▶ Frame Logic (F-Logic) allows capturing different information about an object in a frame.
- ▶ Basic building blocks are molecules
- ▶ isa molecules: $A : B$ (class membership) or $A :: B$ (subclassing), for objects A,B

For example:

```
john:lawyer, lawyer::person, student::person,  
jack:student, X:Y, john:X
```

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F-Logic (2)

- ▶ attribute molecules: $A[B \text{ op } C]$ with $\text{op} \in \{->, ->>, =>, =>>\}$ and objects A,B,C

For example:

```
Single valued attributes: john[name->"John Smith"],  
jill[marriedTo->mark]
```

```
Set-values attributes: john[children->>mary, jill]
```

```
Single valued attribute signature: X[name=>string],  
employee[salary=>amount]
```

```
Set-values attribute signature:  
person[children=>>person], X[employee=>person]
```

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F-Logic (3)

- ▶ Arbitrary terms (functions, variables) can occur in place of object ids, e.g.:

```
father(john)[name->"Jack"], X[name->"John"]
```

- ▶ Molecules starting with the same object ID may be combined and nested:

```
john:lawyer[name->"John Smith";  
  child->>{mary[name->"Mary";  
  jill[name->"Jill"]}] .
```

```
lawyer :: person.
```

- ▶ F-Logic allows the use of predicate symbols, besides the molecules
- ▶ Full F-Logic is a (syntactical) extension to First-Order Logic; we only consider an F-Logic extension to Logic Programming.

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F-Logic Programming

- ▶ An F-Logic vocabulary V consists of:
 - ▶ Object identifiers: a, b, john, jack, f, g, father, etc...
 - ▶ Variables: X, Y, Z, ...
 - ▶ Predicate symbols: p, q, ...
 - ▶ Given an F-Logic vocabulary V :
 - ▶ Every object identifier is a term
 - ▶ Every variables is a term
 - ▶ If f is an object identifier and t_1, \dots, t_n are terms, then $f(t_1, \dots, t_n)$ is a term, also called a constructed term
- Thus, every object identifier can be used as term constructor!

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F-Logic Programming (2)

- ▶ If p is a predicate symbol and t_1, \dots, t_n are terms, then $p(t_1, \dots, t_n)$ is an atom.
- ▶ For A, B, C are terms, we define molecules as follows:
 - ▶ $A : B$ and $A :: B$ are isa molecules
 - ▶ $A[B \text{ op } C]$ with $\text{op} \in \{->, ->>, =>, =>>\}$ is an attribute molecule
- ▶ A **positive literal** is either an atom A or a molecule M
- ▶ A **negative literal** is either a negated atom $\text{not } A$ or a negated molecule $\text{not } B$

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F-Logic Programming (3)

- ▶ An F-Logic rule is of the form: $H :- B_1, \dots, B_n$.
- For positive head-literal H and body-literals B_1, \dots, B_n .
- ▶ Fact: A ground atom or molecule (i.e. rule without a body): H .
- ▶ Query: A rule without a head:
 $?- B_1, \dots, B_n$.

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F-Logic Example

- ▶ Facts (cf. database):

```
bob:empl[name->"Bob";  
         age->40;  
         affiliation->cs1].  
mary:faculty[affiliation->cs1].  
cs1:dept[dname->"CS";  
         mngr->bob]].
```
- ▶ Class information (cf. ontology, database schema):

```
person[name=>string;  
        friends=>>person;  
        children=>>child(person)].  
empl::person[affiliation=>dept;  
             boss=>empl].  
faculty::empl[boss=>(faculty,manager);  
             avgSalary->50000].  
dept[mngr=>empl].
```

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F-Logic Example (cont'd)

- ▶ Rule:

```
E[boss->M] :- E:empl, D:dept,  
             E[affiliation->D[mngr->M:empl]].
```

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Translating RDF to F-Logic

- ▶ Translate triples to attribute molecules
- ▶ $\langle A,B,C \rangle \mapsto A[B \rightarrow C]$
- ▶ Axiomatize RDF semantics
 - ▶ Add facts for axiomatic triples
 - ▶ Add rules for entailment rules
- ▶ Treat bNodes as skolem constants
 - ▶ "rigid" bNodes
 - ▶ anonymous resources

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bNodes as Anonymous Resources

- ▶ Unnumbered anonymous resource: `_#`
 - ▶ Globally unique constant identifier
 - ▶ Every occurrence is **fresh** identifier
- ▶ Numbered anonymous resource: `_#1`, `_#2`, ...
 - ▶ Resource has **scope**: formula
 - ▶ Within scope, different occurrences denote the same resource
- ▶ Anonymous resources are **not** existential variables
- ▶ Thus, semantics of bNodes cannot be captured completely

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Reducing F-Logic to LP

- ▶ F-Logic Programming can be reduced to Logic Programming
- ▶ Thus, F-Logic is syntactic sugar; it does not add anything in expressiveness
- ▶ Reducing terms:
 - ▶ Map object IDs to constants
 - ▶ Map constructed terms to functions
 - ▶ Map variables to variables
- Thus, terms look exactly the same!
- ▶ Map atoms to atoms

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Reducing F-Logic to LP (2)

- ▶ Each type of molecule is mapped to an atom:
 - ▶ `A:B` maps to `_member(A,B)`
 - ▶ `A::B` maps to `_subclass(A,B)`
 - ▶ `A[B->C]` maps to `_svatt(A,B,C)`
 - ▶ `A[B->>C]` maps to `_mvatt(A,B,C)`
 - ▶ `A[B=>C]` maps to `_svattsig(A,B,C)`
 - ▶ `A[B=>>C]` maps to `_mvattsig(A,B,C)`
- Note that the name of the predicates does not really matter.
- ▶ You obtain LP rules by simply replacing terms, atoms and molecules as specified.
- ▶ Axiomatize the semantics of the F-Logic is-a molecules:

```
_subclass(X,Z) :- _subclass(X,Y), _subclass(Y,Z).  
_member(X,Z) :- _member(X,Y), _subclass(Y,Z).
```

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Summary

Logic Programming Basics

- Subsets of Logic Programming
- Reasoning Problems

Semantics of Logic Programming

- Herbrand Semantics
- Fixpoint Semantics

Negation in Logic Programs

- Querying with Negation
- Semantics of Negation

F-Logic Extensions

- F-Logic and RDF
- Reducing F-Logic Programming to standard LP

Required reading

- ▶ Michael Kifer: Rules and Ontologies in F-Logic. Reasoning Web 2005: 22-34
- ▶ Jos de Bruijn. Logics for the semantic web. In **Semantic Web Services: Theory, Tools and Applications**. IDEA Publishing, 2007. Sections 4 and 5.

Further reading

- ▶ Michael Kifer, Georg Lausen, James Wu: Logical Foundations of Object-Oriented and Frame-Based Languages. J. ACM 42(4): 741-843 (1995)
- ▶ Guizhen Yang, Michael Kifer: Reasoning about Anonymous Resources and Meta Statements on the Semantic Web. J. Data Semantics 1: 69-97 (2003)