

# Semantic Web Technologies

## Ontologies and Rules with F-Logic

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## Outline

### Logic Programming Basics

- Subsets of Logic Programming
- Reasoning Problems

### Semantics of Logic Programming

- Herbrand Semantics
- Fixpoint Semantics

### Negation in Logic Programs

- Querying with Negation
- Semantics of Negation

### F-Logic Extensions

- F-Logic and RDF
- Reducing F-Logic Programming to standard LP

## LP Syntax I

- ▶ LP is based on the Horn subset of FOL
- ▶ Vocabulary:
  - ▶ Constants `a`, `b`, `c`, `john`, `mary`, `1`, `"bla"`
  - ▶ Predicates `p`, `q`, `r`, `married-to`, `manager-of`
  - ▶ Function symbols `f`, `g`, `h`
  - ▶ Variables `x`, `y`, `z`
- ▶ Terms:
  - ▶ Constants
  - ▶ Variables
  - ▶ Constructed terms (i.e. function symbol with arguments)
- ▶ Terms (examples):
  - ▶ `a`, `x`, `john`, `father-of(john)`, `father-of(x)`,  
`father-of(father-of(john))`
- ▶ Atoms:

## LP Syntax II

- ▶ For  $n$ -ary predicate symbol  $p$  and terms  $t_1, \dots, t_n$ ,  $p(t_1, \dots, t_n)$  is an atom
- ▶ Examples:  $p(x)$ ,  $p(a)$ ,  $\text{hasName}(\text{john}, \text{"John"})$ ,  $\text{married-to}(\text{john}, \text{mary})$ ,  $\text{hasGrandfather}(x, \text{father-of}(\text{father-of}(x)))$
- ▶ A **ground** atom is an atom without variables
- ▶ Literals:
  - ▶ A positive literal is an atom:  $p(x)$ ,  $q(x)$ , ...
  - ▶ A negative literal is a negated atom:  $\text{not } p(x)$ ,  $\text{not } q(x)$ , ...
  - ▶ A ground literals is a literal without variables
- ▶ Rule:
  - ▶ The **head** of the rule consists of one positive literal  $H$
  - ▶ The **body** of the rule consists of a number of literals  $B_1, \dots, B_n$   
 $H \text{ :- } B_1, \dots, B_n$
  - ▶  $B_1, \dots, B_n$  are also called **subgoals**

## LP Syntax III

▶ Examples:

```
hasUncle(x,z) :- hasParent(x,y), hasBrother(y,z).  
orphan(x) :- not hasParent(x,y).  
hasParent(x,f(x)) :- person(x), not orphan(x).
```

▶ Note:

$:-$  (implication) is sometimes written as  $\leftarrow$   
, is sometimes written as  $\wedge$ :

$$H \leftarrow B_1 \wedge \dots \wedge B_n$$

▶ Fact: A ground atom (i.e. rule without a body)

▶ `person(john).`

▶ Query:

▶ A rule without a head:

## LP Syntax IV

?- B1, ..., Bn.

▶ Examples:

?- hasUncle(john,z).

?- married-to(john,mary).

?- person(x).

▶ Consider:

$u(x,z) :- p(x,y), b(y,z).$

$p(\text{john}, \text{jack}).$

$b(\text{jack}, \text{mike}).$

?-  $u(\text{john}, \text{mike}).$

?-  $u(\text{john}, x).$

▶ Built-in predicates:

- ▶ Arithmetic comparators:  $>$ ,  $<$ ,  $=<$ ,  $>=$ ,  $=$

## LP Syntax V

$3 < 4, 5 > x, \dots$   
 $<(3,4), >(5,x), \dots$

- ▶ (in)equality:  $=, !=$   
 $5=x, z=f(x), \text{john}!=\text{mary}, \text{john}=x, \text{jack}=\text{father}(\text{john})$ 
  - ▶ May only occur in rule bodies
- ▶ Built-in functions:
  - ▶ Arithmetic functions:  $+, -, *, /$   
 $1+2, 5/6, 5*x, \dots$   
 $+(1,2), /(5,6), *(5,x), \dots$
- ▶ Built-in functions can be reduced to predicates:  
 $+(1,x) > *(8,5)$  **reduces to**  $y > z, \text{add}(1,x,y),$   
 $\text{multiply}(8,5,z)$

## LP Subsets

- ▶ Full logic programming
  - ▶ Function symbols
  - ▶ No negation
- ▶ Datalog as subset
  - ▶ No function symbols
  - ▶ No negation
  - ▶ Safe rules:
    - ▶ All variables are limited
  - ▶ A variable is limited if:
    - ▶ It appears as an argument in an ordinary (non-built-in) predicate of the body, or
    - ▶ The variable  $X$  appears in a subgoal  $X = a$  or  $a = X$ , where  $a$  is a constant, or
    - ▶ The variable  $X$  appears in a subgoal  $X = Y$  or  $Y = X$ , where  $Y$  is limited

## LP Subsets (examples)

▶ Full Logic Programming:

```
hasParent(x,f(x)) :- person(x).  
hasParent(x,y) :- person(x).  
bigger(x,y) :- x > y.
```

▶ Datalog:

```
adult(x) :- person(x), hasAge(x,y), y >= 18.  
hasParent(x,y) :- person(x), hasMother(x,y).  
hasParent(x,y) :- person(x), hasFather(x,y).
```

```
ancestor(x,y) :- hasParent(x,y).  
ancestor(x,z) :- ancestor(x,y), ancestor(y,z).
```

▶ Logic Programming vs. Deductive Databases

- ▶ Logic Programming: using logic as a programming language (e.g. Prolog)
- ▶ Deductive Databases: using logic as a database (query) language, e.g. Datalog

## Dependency graph and recursive LP

- ▶ Dependency graphs
  - ▶ Each predicate is a node
  - ▶ There is an arc from predicate  $p$  to predicate  $q$  if they both occur in a rule where  $q$  is in the head and  $p$  in a subgoal
- ▶ A program is **recursive** iff its dependency graph contains a cycle
  - ▶ All predicates on such a cycle are called recursive

- ▶ Recursive:

```
ancestor(x,y) :- hasParent(x,y).
```

```
ancestor(x,z) :- ancestor(x,y), ancestor(y,z).
```

- ▶ Not recursive:

```
hasParent(x,f(x)) :- person(x).
```

```
hasParent(x,y) :- person(x).
```

```
bigger(x,y) :- x > y.
```

## Datalog

- ▶ A logic for querying databases
- ▶ Extensional Database (EDB) consists of facts

```
person(john).
father(john,jack).
brother(jack,mike).
```
- ▶ Intentional Database(IDB) consists of non-ground rules

```
adult(x) :- person(x), hasAge(x,y), y >= 18.
parent(x,y) :- person(x), mother(x,y).
parent(x,y) :- person(x), father(x,y).
uncle(x,z) :- parent(x,y), brother(y,z).
```
- ▶ Consider the query:

```
?- uncle(john,mike).
```

## Reasoning Problems

- ▶ Query answering:
  - ▶ Ground queries, e.g. is Mike an uncle of John?  
`?- uncle(john,mike).`
  - ▶ Non-ground queries, e.g. who are the uncles of John?  
`?- uncle(john,x).`
- ▶ Nonground queries can be reduced to ground queries, e.g.:
  - ▶ Replace `?- uncle(john,x)` With the three queries:  
`?- uncle(john,john).`  
`?- uncle(john,jack).`  
`?- uncle(john,mike).`
- ▶ Answering ground queries is equivalent to entailment of ground facts:
  - ▶ For program  $P$  and ground atom  $A$ , does the following entailment hold?

$$P \models A$$

## LP Semantics

- ▶ Model-theoretic semantics
  - ▶ What does a program/database mean in terms of its models?
  - ▶ We define the **minimal Herbrand model**.
- ▶ Computational semantics
  - ▶ How to compute the model of a program/database?
  - ▶ We define the direct consequence operator  $T_P$  for program  $P$ , which computes all consequences,
- ▶ Correspondence between model-theoretic and computational semantics
  - ▶ The fixpoint of  $T_P$  is the minimal Herbrand model!
- ▶ LP semantics is equivalent with First-Order semantics wrt. entailment of **ground facts**
  - ▶ Note: this only hold for logic programming without negation!

## Herbrand Semantics: the idea

- ▶ Consider only particular first-order interpretations
  - ▶ Domain = set of ground terms (Herbrand universe)
  - ▶ Interpret ground terms as themselves:  $t^I = t$
- ▶ Intersection of all models is **minimal model**

## Ground Terms, ground atoms and the Herbrand Universe

- ▶ terms not containing any variables are **ground terms**
- ▶ Similarly, a ground atom is an atom not containing variables
- ▶ The **Herbrand Universe**  $U_L$  is the set of all ground terms which can be formed from constants and function symbols in program  $P$ .  
Example:  $a, b, f(a), f(b), f(f(a)), f(f(b)), f(f(f(a))), \dots$
- ▶ The Herbrand Base  $B_P$  is the set of all ground atoms which can be built from predicate symbols in  $P$ , using ground terms from  $U_P$  as arguments.  
Example:  $p(a), p(b), q(a), q(b), p(f(a)), q(f(a)), \dots$

## Herbrand models

- ▶ A Herbrand Interpretation  $I_P$  is a subset of the Herbrand Base  $B_P$ .
- ▶ A Herbrand Model  $M_P$  is a Herbrand Interpretation which makes every formula true, i.e.:
  - ▶ Every fact in  $P$  is in  $M_P$ , and
  - ▶ For every rule  $R$  in  $P$  holds: if every positive literal in the body is in  $M_P$ , then also the head literal is in  $M_P$ .

Note: this only works for positive programs, i.e., programs **without** negation!

- ▶ The semantics of a program  $P$  is characterized in terms of the **least** Herbrand Model, which is the intersection of all possible Herbrand Models.
- ▶ Each positive program has one **unique** least Herbrand Model.

## Fixpoint Semantics

- ▶ Let  $I_P$  be a Herbrand interpretation and  $P$  a positive program:

$$T_P(I) = \{A \in B_P : A : -A_1, \dots, A_n \text{ is a ground instance of a rule in } P \text{ such that } A_1, \dots, A_n \in I_P\}$$

is called the **immediate consequence operator**.

- ▶  $T_P$  is **monotonic**:  $T_P^2(I) = T_P(T_P(I)) \supseteq T_P(I)$
- ▶ Thus,  $T_P$  has a fixpoint  $T_P^\infty$
- ▶ For a positive program, the least fixpoint of  $T_P$  is equivalent to the minimal Herbrand Model  $M_P$ :

$$lfp(T_P) = M_P$$

## Example

$a(x) \text{ :- } b(x).$

$b(x) \text{ :- } e(x).$

$l(karl, x) \text{ :- } a(x).$

$b(franz).$

$e(hansi).$

$$T_P(\emptyset) = \{b(franz), e(hansi)\} \quad (1)$$

$$T_P^2(\emptyset) = T_P(\emptyset) \cup \{a(franz), b(hansi)\} \quad (2)$$

$$T_P^3(\emptyset) = T_P^2(\emptyset) \cup \{l(karl, franz), a(hansi)\} \quad (3)$$

$$T_P^4(\emptyset) = T_P^3(\emptyset) \cup \{l(karl, hansi)\} \quad (4)$$

$$T_P^5(\emptyset) = T_P^4(\emptyset) \quad (5)$$

i.e.  $lfp(T_P) = T_P^4(\emptyset) = M_P$       Note:  $T_P^2(\emptyset) = T_P(T_P(\emptyset))$ ,  $T_P^3 = T_P(T_P^2(\emptyset))$ , etc...

## Example (cont'd)

A simple example where the least Herbrand model is infinite:

$p(a).$

$p(f(x)) \text{ :- } p(x).$

$$\text{lfp}(T_P) = \{p(a), p(f(a)), p(f(f(a))), \dots\}$$

## Negation

- ▶ How do we handle negation in Logic Programs?
- ▶ Negation-as-failure: whenever a fact is not entailed by the knowledge base, its negation is entailed
- ▶ Example:

`p(a).`

`q(x) :- p(x)`

entails:

`p(a).`

`q(a).`

`not p(b).`

`not q(b).`

`not r(a).`

`not r(b).`

`...`

## Examples

- ▶ Using negative literals in rules.
- ▶ Example:

```
p(a).  
q(x) :- not p(x).
```

entails:

```
p(a).  
not q(a).  
not p(b).  
q(b).  
not r(a).  
not r(b).  
...
```

## Examples

- ▶ Example:

`p(a).`

`p(b).`

`r(a).`

`r(c).`

`q(x) :- not p(x), r(x).`

entails:

`p(a).`

`p(b).`

`r(a).`

`r(c).`

`q(c).`

- ▶ We typically leave out the entailed negative facts, since the negation of every fact which is not entailed, is entailed

## Querying

- ▶ Ground queries:
  - ▶ The answer to a ground query  $Q_g$  is "yes" if:
    - ▶ Every positive literal in  $Q_g$  is in the minimal model of the program, and
    - ▶ Every negative literal in  $Q_g$  is not in the minimal model of the program.
  - ▶ Otherwise, the answer is "no".
- ▶ Non-ground queries are instantiated, in the same way as for logic programs without negation

## Query Example

$p(a).$

$p(b).$

$r(a).$

$r(c).$

$q(x) \text{ :- not } p(x), r(x).$

$?- \text{ not } q(a), r(c).$

answer: **YES**

$?- r(c), \text{ not } q(c).$

answer: **NO**

$?- r(X), \text{ not } q(X).$

answer:

$X = a.$

## Outline

- ▶ Characterizing negative information in Herbrand models
- ▶ Characterizing negative information using the Fixpoint operator
- ▶ Problems with the Fixpoint operator under negation
- ▶ A popular solution: stratified negation

## Extending Herbrand Semantics

- ▶ A Herbrand Model  $M_P$  is a Herbrand Interpretation which makes every formula true, i.e.:
  - ▶ Every fact in  $P$  is in  $M_P$ , and
  - ▶ For every rule  $R$  in  $P$  holds: if every positive literal in the body is in  $M_P$  and every negative literal in the body is not in  $M_P$ , then also the head literal is in  $M_P$ .
- ▶ The semantics of a program  $P$  is characterized in terms of the least Herbrand Model.
- ▶ But: **is the least Herbrand model still unique?**

## Extending Herbrand Semantics (II)

Consider the following examples:

$p(a).$

$q(b) \text{ :- not } p(b).$

The minimal Herbrand Model  $M_P$  is:  $\{p(a), q(b)\}$

$p(a).$

$q(x) \text{ :- not } p(x).$

The minimal Herbrand Model  $M_P$  is:  $\{p(a)\}$

## Extending Herbrand Semantics (III)

Consider the following examples:

$r(a).$

$q(x) \text{ :- not } p(x), r(x).$

$p(x) \text{ :- not } q(x), r(x).$

Has two minimal Herbrand Models:  $\{r(a), q(a)\}, \{r(a), p(a)\}$

$a \text{ :- not } b.$

$b \text{ :- not } a.$

Has two minimal Herbrand Models:  $\{a\}, \{b\}$

## Extending $T_P$

- ▶ Let  $I_P$  be a Herbrand interpretation and  $P$  a general program:

$$T_P(I) = \{A \in B_P \mid A : -L_1, \dots, L_i, \text{not } L_{i+1}, \dots, \text{not } L_n \\ \text{is a ground instance of a rule in } P \text{ such that} \\ L_1, \dots, L_i \in I_P \text{ and } L_{i+1}, \dots, L_n \notin I_P\}$$

is the extended immediate consequence operator.

- ▶ Is the least fixpoint of  $T_P$  equivalent to the minimal Herbrand Model  $M_P$ ?
- ▶ **Note: we have already seen cases of several minimal Herbrand models.**

## Extending $T_P$ (cont'd)

Consider the following examples:

$p(a)$ .

$q(b) \text{ :- not } p(b)$ .

Applications of  $T_P$ :

(1)  $\{p(a)\}$

(2)  $\{p(a), q(b)\}$

(2) is the least fixpoint and is equivalent to  $M_P$ .

## Extending $T_P$ (cont'd)

Consider the following examples:

$r(a)$ .

$q(x) :- \text{not } p(x), r(x)$ .

$p(x) :- \text{not } q(x), r(x)$ .

Applications of  $T_P$ :

(1)  $\{r(a)\}$

(2)  $\{r(a), p(a), q(a)\}$

(2) is a fixpoint, but not a Herbrand model! Remember, the Herbrand models are:  $\{r(a), q(a)\}, \{r(a), p(a)\}$

Thus, naive extension of the fixpoint operator is not possible!

## Solution: Stratified Programs

- ▶ Recall: dependency graph
  - ▶ Each predicate is a node
  - ▶ There is an edge from predicate  $p$  to predicate  $q$  if they both occur in a rule where  $q$  is in the head and  $p$  in a subgoal
  - ▶ Edges with negation are marked
- ▶ A predicate is in the same or a higher stratum than all predicates connected to it with a positive edge.
- ▶ If there is a negative edge leading from a predicate  $p$  to a predicate  $q$ , then the stratum of  $q$  is strictly higher than the stratum of  $p$ .
- ▶ If every predicate occurs in at most one stratum, the program is stratifiable.
- ▶ For stratified programs, we can use the extended fixpoint operator  $T_P!$

## Examples of Stratification

$p(a).$   
 $q(b) :- \text{not } p(b).$

Is stratified with:

Stratum 0:  $\{p\}$  Stratum 1:  $\{q\}$

$p(a).$   
 $r(a).$   
 $q(x) :- \text{not } p(x), r(x).$

Is stratified with:

Stratum 0:  $\{p\}$  Stratum 1:  $\{q,r\}$

## Examples of Stratification

$r(a).$

$q(x) :- \text{not } p(x), r(x).$

$p(x) :- \text{not } q(x), r(x).$

Is not stratified. **Why?**

$a :- \text{not } b.$

$b :- \text{not } a.$

Is not stratified. **Why?**

## F-Logic extensions: motivation

- ▶ Predicates not really appropriate to capture object typing and attributes.
- ▶ Consider: John is a Lawyer, has the particular name "John Smith" and has two children named "Mary" and "Jill". Furthermore, Lawyer is a particular kind of Human.
- ▶ Expressed using Logic Programming:

```
lawyer(john).  
hasName(john, "John Smith").  
hasChild(john,mary).  
hasChild(john,jill).  
hasName(jill, "Jill").  
hasName(mary, "Mary").  
person(x) :- lawyer(x).
```
- ▶ This encoding:
  - ▶ Disregards the structure of the natural language sentences
  - ▶ Information about the particular object is no longer grouped

## F-Logic Introduction

- ▶ Consider: John is a Lawyer, has the particular name "John Smith" and has two children named "Mary" and "Jill". Furthermore, Lawyer is a particular kind of Human.
- ▶ Expressed using F-Logic:

```
john:lawyer[name->"John Smith",  
             child->>{mary[name->"Mary"],  
                    jill[name->"Jill"]}].
```

```
lawyer :: person.
```

- ▶ F-Logic also allows specifying signature information:

```
person[name=>string,  
        child=>>person].
```

## F-Logic

- ▶ Frame Logic (F-Logic) allows capturing different information about an object in a frame.
- ▶ Basic building blocks are molecules
- ▶ isa molecules:  $A : B$  (class membership) or  $A :: B$  (subclassing), for objects  $A, B$

For example:

```
john:lawyer, lawyer::person, student::person,  
jack:student, X:Y, john:X
```

## F-Logic (2)

- ▶ attribute molecules:  $A[B \text{ op } C]$  with  $\text{op} \in \{->, ->>, =>, =>>\}$  and objects  $A, B, C$

For example:

Single valued attributes: `john[name->"John Smith"]`,  
`jill[marriedTo->mark]`

Set-values attributes: `john[children->>mary, jill]`

Single valued attribute signature: `X[name=>string]`,  
`employee[salary=>amount]`

Set-values attribute signature:

`person[children=>>person]`, `X[employee=>person]`

## F-Logic (3)

- ▶ Arbitrary terms (functions, variables) can occur in place of object ids, e.g.:

```
father(john) [name->"Jack"], X[name->"John"]
```

- ▶ Molecules starting with the same object ID may be combined and nested:

```
john:lawyer[name->"John Smith";  
             child->>{mary[name->"Mary"];  
             jill[name->"Jill"]}].
```

```
lawyer :: person.
```

- ▶ F-Logic allows the use of predicate symbols, besides the molecules
- ▶ Full F-Logic is a (syntactical) extension to First-Order Logic; we only consider an F-Logic extension to Logic Programming.

## F-Logic Programming

- ▶ An F-Logic vocabulary  $V$  consists of:
  - ▶ Object identifiers:  $a, b, \text{john}, \text{jack}, f, g, \text{father}, \text{etc...}$
  - ▶ Variables:  $X, Y, Z, \dots$
  - ▶ Predicate symbols:  $p, q, \dots$
- ▶ Given an F-Logic vocabulary  $V$ :
  - ▶ Every object identifier is a term
  - ▶ Every variables is a term
  - ▶ If  $f$  is an object identifier and  $t_1, \dots, t_n$  are terms, then  $f(t_1, \dots, t_n)$  is a term, also called a constructed term

Thus, every object identifier can be used as term constructor!

## F-Logic Programming (2)

- ▶ If  $p$  is a predicate symbol and  $t_1, \dots, t_n$  are terms, then  $p(t_1, \dots, t_n)$  is an atom.
- ▶ For  $A, B, C$  are terms, we define molecules as follows:
  - ▶  $A : B$  and  $A : B$  are **isa** molecules
  - ▶  $A [B \text{ op } C]$  with  $\text{op} \in \{->, ->>, =>, =>>\}$  is an attribute molecule
- ▶ A **positive literal** is either an atom  $A$  or a molecule  $M$
- ▶ A **negative literal** is either a negated atom  $\text{not } A$  or a negated molecule  $\text{not } B$

## F-Logic Programming (3)

- ▶ An F-Logic rule is of the form:  $H \text{ :- } B_1, \dots, B_n$ .

For positive head-literal  $H$  and body-literals  $B_1, \dots, B_n$ .

- ▶ Fact: A ground atom or molecule (i.e. rule without a body):  
 $H$ .
- ▶ Query: A rule without a head:  
 $?- B_1, \dots, B_n$ .

## F-Logic Example

- ▶ Facts (cf. database):

```
bob:empl[name->"Bob";  
         age->40;  
         affiliation->cs1].  
mary:faculty[affiliation->cs1].  
cs1:dept[dname->"CS";  
         mngr->bob]].
```

- ▶ Class information (cf. ontology, database schema):

```
person[name=>string;  
        friends=>>person;  
        children=>>child(person)].  
empl::person[affiliation=>dept;  
              boss=>empl].  
faculty::empl[boss=>(faculty,manager);  
              avgSalary->50000].  
dept[mngr=>empl].
```

## F-Logic Example (cont'd)

► Rule:

```
E[boss->M] :- E:empl, D:dept,  
              E[affiliation->D[mngr->M:empl]].
```

## Translating RDF to F-Logic

- ▶ Translate triples to attribute molecules
- ▶  $\langle A, B, C \rangle \mapsto A [B \rightarrow C]$
- ▶ Axiomatize RDF semantics
  - ▶ Add facts for axiomatic triples
  - ▶ Add rules for entailment rules
- ▶ Treat bNodes as skolem constants
  - ▶ “rigid” bNodes
  - ▶ anonymous resources

## bNodes as Anonymous Resources

- ▶ Unnumbered anonymous resource: `_#`
  - ▶ Globally unique constant identifier
  - ▶ Every occurrence is **fresh** identifier
- ▶ Numbered anonymous resource: `_#1`, `_#2`, ...
  - ▶ Resource has **scope**: formula
  - ▶ Within scope, different occurrences denote the same resource
- ▶ Anonymous resources are **not** existential variables
- ▶ Thus, semantics of bNodes cannot be captured completely

## Reducing F-Logic to LP

- ▶ F-Logic Programming can be reduced to Logic Programming
- ▶ Thus, F-Logic is syntactic sugar; it does not add anything in expressiveness
- ▶ Reducing terms:
  - ▶ Map object IDs to constants
  - ▶ Map constructed terms to functions
  - ▶ Map variables to variables

Thus, terms look exactly the same!

- ▶ Map atoms to atoms

## Reducing F-Logic to LP (2)

- ▶ Each type of molecule is mapped to an atom:

- ▶  $A:B$  maps to `_member(A,B)`
- ▶  $A::B$  maps to `_subclass(A,B)`
- ▶  $A[B \rightarrow C]$  maps to `_svatt(A,B,C)`
- ▶  $A[B \rightarrow \rightarrow C]$  maps to `_mvatt(A,B,C)`
- ▶  $A[B = \rightarrow C]$  maps to `_svattsig(A,B,C)`
- ▶  $A[B = \rightarrow \rightarrow C]$  maps to `_mvattsig(A,B,C)`

Note that the name of the predicates does not really matter.

- ▶ You obtain LP rules by simply replacing terms, atoms and molecules as specified.
- ▶ Axiomatize the semantics of the F-Logic is-a molecules:
 

```

_subclass(X,Z) :- _subclass(X,Y),_subclass(Y,Z).
_member(X,Z) :- _member(X,Y),_subclass(Y,Z).
      
```

## Required reading

- ▶ Michael Kifer: Rules and Ontologies in F-Logic. Reasoning Web 2005: 22-34
- ▶ Jos de Bruijn. Logics for the semantic web. In **Semantic Web Services: Theory, Tools and Applications**. IDEA Publishing, 2007. Sections 4 and 5.

## Further reading

- ▶ Michael Kifer, Georg Lausen, James Wu: Logical Foundations of Object-Oriented and Frame-Based Languages. J. ACM 42(4): 741-843 (1995)
- ▶ Guizhen Yang, Michael Kifer: Reasoning about Anonymous Resources and Meta Statements on the Semantic Web. J. Data Semantics 1: 69-97 (2003)