Local Search

Local Search:

- Maintain an assignment of a value to each variable.
- At each step, select a "neighbor" of the current assignment (e.g., one that improves some heuristic value).
- Stop when a satisfying assignment is found, or return the best assignment found.

Requires:

- What is a neighbor?
- Which neighbor should be selected?

(Some methods maintain multiple assignments.)

Selecting Neighbors in Local Search

- When the domains are small or unordered, the neighbors of an assignment can correspond to choosing another value for one of the variables.
- When the domains are large and ordered, the neighbors of an assignment are the adjacent values for one of the variables.
- If the domains are continuous, Gradient descent changes each variable proportional to the gradient of the heuristic function in that direction. The value of variable X_i goes from v_i to $v_i - \eta \frac{\partial h}{\partial X_i}$.
  Gradient ascent: go uphill; v_i becomes $v_i + \eta \frac{\partial h}{\partial X_i}$.

Local Search for CSPs

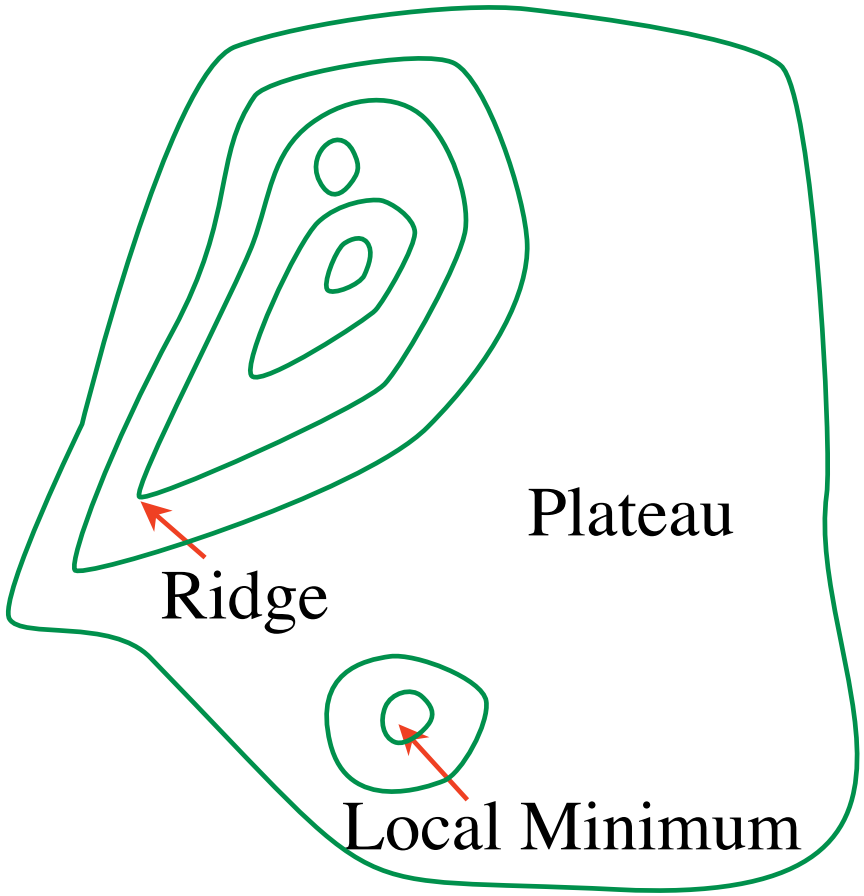
- Aim is to find an assignment with zero unsatisfied constraints.
- Given an assignment of a value to each variable, a conflict is an unsatisfied constraint.
- The goal is an assignment with zero conflicts.
- Heuristic function to be minimized: the number of conflicts.

Greedy Descent Variants

- Find the variable-value pair that minimizes the number of conflicts at every step.
- Select a variable that participates in the most number of conflicts. Select a value that minimizes the number of conflicts.
- Select a variable that appears in any conflict. Select a value that minimizes the number of conflicts.
- Select a variable at random. Select a value that minimizes the number of conflicts.
- Select a variable and value at random; accept this change if it doesn't increase the number of conflicts.

Problems with Greedy Descent

- a local minimum that is not a global minimum
- a plateau where the heuristic values are uninformative
- a ridge is a local minimum where n-step look-ahead might help



Randomized Algorithms

- Consider two methods to find a minimum value:
  - Greedy descent, starting from some position, keep moving down & report minimum value found
  - Pick values at random & report minimum value found
- Which do you expect to work better to find a global minimum?
- Can a mix work better?

Randomized Greedy Descent

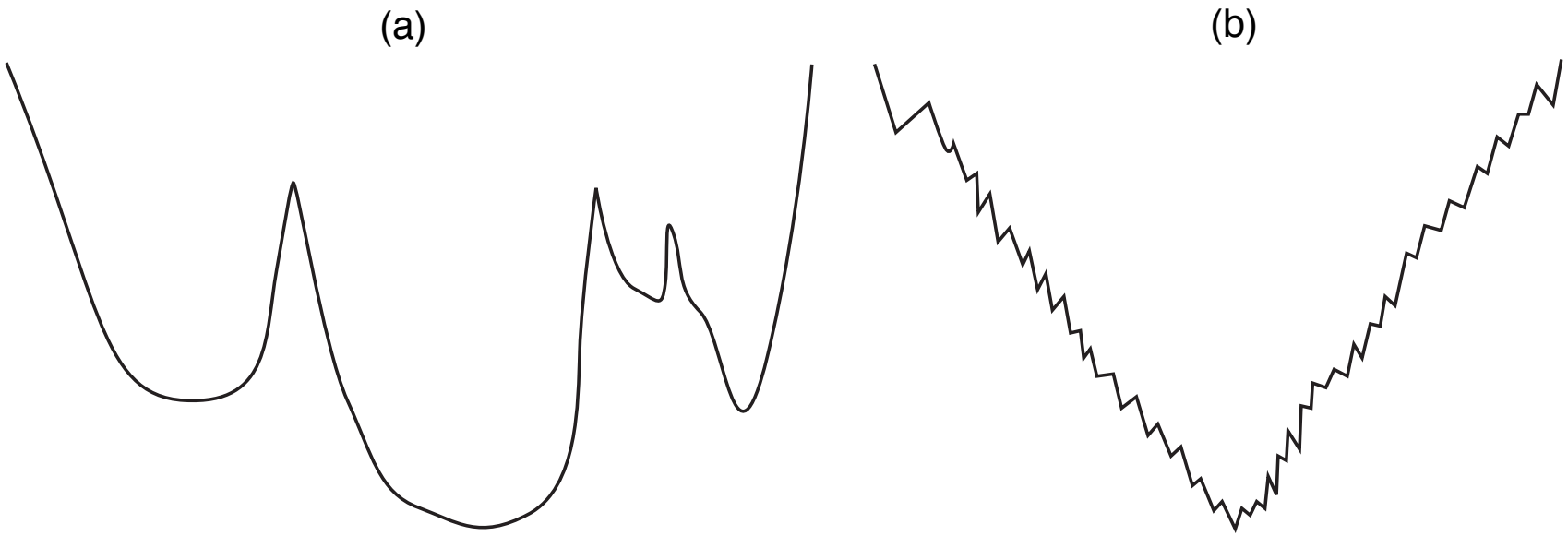
As well as downward steps we can allow for:

- Random steps: move to a random neighbor.
- Random restart: reassign random values to all variables.

Which is more expensive computationally?

1-Dimensional Ordered Examples

Two 1-dimensional search spaces; step right or left:



- Which method would most easily find the global minimum?
- What happens in hundreds or thousands of dimensions?
- What if different parts of the search space have different structure?

Stochastic Local Search

Stochastic local search is a mix of:

- Greedy descent: move to a lowest neighbor
- Random walk: taking some random steps
- Random restart: reassigning values to all variables

Random Walk

Variants of random walk:

- When choosing the best variable-value pair, randomly sometimes choose a random variable-value pair.
- When selecting a variable then a value:
  - Sometimes choose any variable that participates in the most conflicts.
  - Sometimes choose any variable that participates in any conflict (a red node).
  - Sometimes choose any variable.
- Sometimes choose the best value and sometimes choose a random value.

Comparing Stochastic Algorithms

- How can you compare three algorithms when
  - one solves the problem 30% of the time very quickly but doesn't halt for the other 70% of the cases
  - one solves 60% of the cases reasonably quickly but doesn't solve the rest
  - one solves the problem in 100% of the cases, but slowly?

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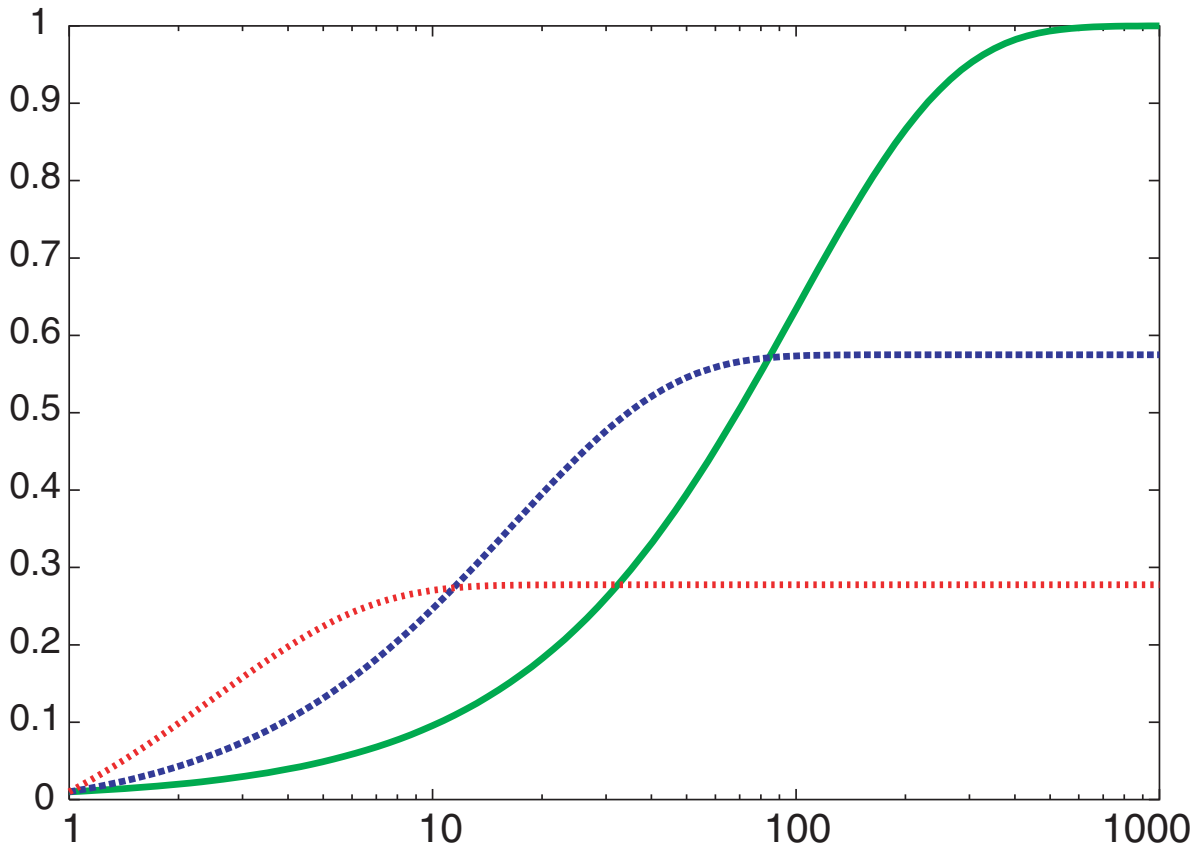
  - one solves 60% of the cases reasonably quickly but doesn't solve the rest
  - one solves the problem in 100% of the cases, but slowly?
- Summary statistics, such as mean run time, median run time, and mode run time don't make much sense.

Runtime Distribution

- Plots runtime (or number of steps) and the proportion (or number) of the runs that are solved within that runtime.



Variant: Simulated Annealing

- Pick a variable at random and a new value at random.
- If it is an improvement, adopt it.
- If it isn't an improvement, adopt it probabilistically depending on a temperature parameter, T.
  - With current assignment n and proposed assignment n' we move to n' with probability $e^{(h(n')-h(n))/T}$
- Temperature can be reduced.

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Probability of accepting a change:

Temperature	1-worse	2-worse	3-worse
10	0.91	0.81	0.74
1	0.37	0.14	0.05
0.25	0.02	0.0003	0.000005
0.1	0.00005	0	0

Tabu lists

- To prevent cycling we can maintain a tabu list of the k last assignments.
- Don't allow an assignment that is already on the tabu list.
- If $k = 1$, we don't allow an assignment of to the same value to the variable chosen.
- We can implement it more efficiently than as a list of complete assignments.
- It can be expensive if k is large.

Parallel Search

A total assignment is called an individual.

- Idea: maintain a population of k individuals instead of one.
- At every stage, update each individual in the population.
- Whenever an individual is a solution, it can be reported.
- Like k restarts, but uses k times the minimum number of steps.

Beam Search

- Like parallel search, with k individuals, but choose the k best out of all of the neighbors.
- When $k = 1$, it is greedy descent.
- When $k = \infty$, it is breadth-first search.
- The value of k lets us limit space and parallelism.

Stochastic Beam Search

- Like beam search, but it probabilistically chooses the k individuals at the next generation.
- The probability that a neighbor is chosen is proportional to its heuristic value.
- This maintains diversity amongst the individuals.
- The heuristic value reflects the fitness of the individual.
- Like asexual reproduction: each individual mutates and the fittest ones survive.

Genetic Algorithms

- Like stochastic beam search, but pairs of individuals are combined to create the offspring:
- For each generation:
  - Randomly choose pairs of individuals where the fittest individuals are more likely to be chosen.
  - For each pair, perform a cross-over: form two offspring each taking different parts of their parents:
  - Mutate some values.
- Stop when a solution is found.

Crossover

- Given two individuals:

$$X_1 = a_1, X_2 = a_2, \ldots, X_m = a_m$$

$$X_1 = b_1, X_2 = b_2, \ldots, X_m = b_m$$

- Select i at random.
- Form two offspring:

$$X_1 = a_1, \ldots, X_i = a_i, X_{i+1} = b_{i+1}, \ldots, X_m = b_m$$

$$X_1 = b_1, \ldots, X_i = b_i, X_{i+1} = a_{i+1}, \ldots, X_m = a_m$$

- The effectiveness depends on the ordering of the variables.
- Many variations are possible.