Walking through the Semantics of Frame-Based Description Languages: a Case Study†

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ABSTRACT

Starting from $\mathcal{FL}^-$, the first formal language belonging to the Frame-Based Description Languages (FDL) family, its expressive power is enhanced by adding new concept- and role-forming operators. The possibility of giving roles a structure will be analyzed, allowing conjunction of roles and introducing thereafter functional roles, which allow a simple form of cardinality as well. Moreover, primitive descriptions are discussed for both concepts and roles. The relevant properties of these languages, with respect to increasing expressive power, such as soundness, completeness and computational complexity of the subsumption procedure are presented formally. Beside these features a meaningful aim of our work is to show a methodology for the analysis of an FDL. Therefore in this paper not only the formal properties (the coherence) of the single languages will be explored, but also the relation of these languages with their effective use in a representational framework (the adequacy). It will be shown how an FDL can lose its expected intuitive properties by augmenting expressiveness: some solutions to this problem are proposed, following constructive criteria. This study concludes with a discussion of the language $\mathcal{KFL}$, the formal basis for the Tbox of a KR system currently in use in a natural language project.

1. INTRODUCTION

All formal FDL's introduced up to the present in the Knowledge Representation arena have taken into account the tradeoff between the expressiveness of the language and the cost of computing subsumption. To manage this tradeoff such languages adopt a restricted expressiveness, making subsumption efficient - e.g. Kandor [Patel-Schneider:84], Krypton [Brachman et al.:85], Classic [Borgida et al.:89], $\mathcal{ALE}$ [Donini et al:90]. However the still unresolved question is: how to get the maximum in expressiveness, without falling into the intractability pit? Unfortunately there is not one all-inclusive answer to this question. There are several distinct ways to enhance a language while preserving computational tractability of subsumption: the widespread range of developed languages in the FDL family is an example. Moreover, the computational cliff *shifts* as the subsumption procedure becomes incomplete, and

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sometimes it moves very far, depending on the degree of incompleteness, as in LOOM [MacGregor:88] or BACK [Peltason et al.:89]. Another way to push the barrier - to enrich expressive power and maintain tractability and completeness of deductive procedure at the same time - is to adopt a weak semantics [Patel-Schneider:87].

The motivating goals must be kept in mind when studying and developing an FDL: *the* ultimate language does not exist. The choice of which way to go depends on the specific needs of a particular research project. We agree with [Doyle, Patil:89] when they say that "representation systems should be rational agents using the user's knowledge and purposes to cooperate with the user to accomplish those purposes".

The goal of this work is a language with a tractable and complete subsumption procedure, whose expressiveness arises from a particular analysis of the different ways to expand an FDL. Not only the formal properties (the *coherence*) of the single languages will be explored, but also the relation of such languages with their effective use in a representational framework (the *adequacy*). We will show how some constructs - though extending expressive power and still maintaining tractability - do not give an intuitive behaviour to the language, from the point of view of both meaning and calculus. This study concludes with a discussion of the $\mathcal{KFL}$ language, the formal basis for the TBOX of a hybrid system - KRAPFEN [Franconi:90] - developed at IRST as the main representation module of the *ALFresco* natural language system.

2. EXTENDING THE EXPRESSIVE POWER OF $\mathcal{FL}^-$: $\mathcal{FL}^\mathcal{A}$

$\mathcal{FL}^-$ is an amply studied FDL [Levesque, Brachman:87]; it was the first attempt to understand the computational cost of expressiveness. In this section we introduce $\mathcal{FL}^\mathcal{A}$, a simple extension of $\mathcal{FL}^-$. The AND-ROLE operator, which allows conjunction of roles, has been added to $\mathcal{FL}^-$. Let us consider the following concepts as an example:

a = (AND person (SOME (AND-ROLE child friend))) represents the class of people with the property of having at least one child who is also a friend.

b = (AND person (ALL (AND-ROLE child friend) rich)) denotes the set of people such that all of their children who are also friends - *if* they have one - are rich.

Note that there is a great difference in meaning between concept **a** and the following:

c = (AND person (SOME child) (SOME friend)); **c** individuates all people having at least one child and at least one friend; these are not necessarily the same individual.

Analogously concept **b** is very different from **d**:

d = (AND person (ALL child rich) (ALL friend rich)) denoting the set of people such that all of their children and their friends are rich.

2.1. Syntax and Semantics

Let $\mathbf{T} = \mathbf{T}_c \cup \mathbf{T}_r$ be a finite set of atomic concepts and atomic roles, called *terminology*[1]. Let Ω be an arbitrary non empty set, representing the *domain*, and let $\Im_T$ be a total function (the interpretation function) over the terminology such that:

$$\Im_T : \mathbf{T}_c \rightarrow 2^\Omega \qquad\qquad \Im_T : \mathbf{T}_r \rightarrow (\Omega \rightarrow 2^\Omega)$$

Given $\mathbf{T}$ and Ω, the denotation of concept (and role) complex descriptions is obtained by extending the definition of the interpretation function $\Im_T$ recursively over

[1] We follow the FDL formalism introduced by [Levesque, Brachman:87]. Differently from [Nebel:89] we do not introduce names for descriptions here, because the problem of naming goes beyond the scope of this paper. We call the set of non-expandable atomic concepts and roles *terminology*, whereas [Nebel:89] associates this set with the *empty terminology*.

such descriptions:

<concept> ::= <atomic-concept> |
(**AND** <concept>+) |
(**SOME** <role>) |
(**ALL** <role> <concept>)
<role> ::= <atomic-role> |
(**AND-ROLE** <role>+)

$$\Im_T\,[(\mathbf{AND}\ \langle\text{concept}\rangle_1..\langle\text{concept}\rangle_h)] = \bigcap_{i \le h} \Im_T\,[\langle\text{concept}\rangle_i]$$

$$\Im_T\,[(\mathbf{SOME}\ \langle\text{role}\rangle)] = \{\ x \in \Omega : \Im_T\ [\langle\text{role}\rangle]\ (x) \neq \varnothing\ \}$$

$$\Im_T\,[(\mathbf{ALL}\ \langle\text{role}\rangle\ \langle\text{concept}\rangle)] = \{\ x \in \Omega : \Im_T\,[\langle\text{role}\rangle]\ (x) \subseteq \Im_T\,[\langle\text{concept}\rangle]\ \}$$

$$\Im_T\,[(\mathbf{AND\text{-}ROLE}\ \langle\text{role}\rangle_1..\langle\text{role}\rangle_h)] = \boldsymbol{f}\colon \Omega \rightarrow 2^{\Omega} \mid \forall x\ (\boldsymbol{f}(x) = \bigcap_{i \le h} \Im_T\,[\langle\text{role}\rangle_i](x)\ )$$

2.2. The clash of intuitions

Given two concepts $\mathbf{c}_1$ and $\mathbf{c}_2$, $\mathbf{c}_1$ *subsumes* $\mathbf{c}_2$, iff: $\forall\Omega\ \forall\Im_T\ (\Im_T\,[\mathbf{c}_1] \supseteq \Im_T\,[\mathbf{c}_2]\ )$. We now show some theorems that characterize the subsumption relation in $\mathcal{FL}^{\mathcal{A}}$.

THEOREM 1[2]**.** Let c_i be the concept (SOME (AND-ROLE r_1 ..r_i));
if $m \le n$ then c_m subsumes c_n.
THEOREM 2. Let c_i be the concept (ALL (AND-ROLE r_1..r_i) d);
if $m \ge n$ then c_m subsumes c_n.
THEOREM 3. Let c_i be the concept (AND (SOME (AND-ROLE r_1..r_i))
(ALL (AND-ROLE r_1..r_i) d));
$\forall\, n \neq m$ neither c_m subsumes c_n, nor c_n subsumes c_m. ❑

It is possible to infer subsumptions between concepts where SOME and ALL operators occur separately, as for $\mathcal{FL}^{-}$. From the above examples it follows that concept **c** subsumes concept **a** (Theorem 1): looking at the meaning of the concepts, this matches our intuitions.
According to Theorem 2, concept **b** subsumes concept **d**. However, this is just the opposite of the relation expected between **b** and **d**: common sense implies that **b** specializes **d**. Intuition suggests that the extension is reduced by increasing the number of constraints. Instead, the extension of the concept (ALL (AND-ROLE r_1..r_i) c) increases by augmenting the number of roles: this derives from the fact that the set of fillers of the complex role can have an arbitrary cardinality, including zero (there are more individuals that *do not* satisfy the universal condition on fillers).
Theorem 3 shows that nothing can be said about subsumption when we have two concepts, in each of which both SOME and ALL operators occur taking as argument the same role expression. On this matter let us consider the following example:
e = (AND (SOME (AND-ROLE relative friend))
(ALL (AND-ROLE relative friend) female))
f = (AND (SOME relative) (ALL relative female)).
Concept **e** individuates everybody having relative friends (at least one) who are female; **f** represents everybody having relatives (at least one) who are female. Common sense would lead one to say that concept **f** is a generalization of concept **e**. However, this is not true according to the semantics of this language, as shown in Theorem 3.

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2 The proofs of the theorems are omitted: see [Cattoni:89].

For the above example let us consider as domain the set Ω = {John, Jacqueline, Marylin, Bob}. Let us suppose that the interpretation of the concept "female" is the domain subset {Jacqueline, Marylin}, and that of the roles "relative" and "friend" is respectively {<John, Jacqueline>, <John, Bob>} and {<John, Jacqueline>, <John, Marylin>}. From semantics it follows that the interpretation of concept **e** is the set {John}, because Jacqueline is simultaneously relative and friend of John and she is a female; the interpretation of **f** is instead the empty set, because not all relatives of John are females (as for Bob). Therefore it does not hold that **f** subsumes **e**.

3. MORE STRUCTURE TO ROLES: $\mathcal{FL}^{\mathcal{A}+}$

In $\mathcal{FL}^{\mathcal{A}+}$ a new syntactic type is introduced, called functional role (or attribute). The difference between a functional role and the aforesaid relational roles is in the cardinality: through a relational role an element of the domain can have any number of elements associated to it; instead a functional role maps every element to at most one element.

To clarify the difference between relational and functional role let us consider the concepts (AND person (SOME child)), and (AND person (SOME mother)). The first concept represents people having at least one child, while the latter denotes the set of people having at least one mother. Each person can have any number of children; on the other hand we want to represent the fact that one and only one mother corresponds to each person. While "child" is a relational role without particular restrictions, the role "mother" has the feature of *filler singleness*. This difference can be captured within $\mathcal{FL}^{\mathcal{A}+}$: (AND person (SOME (SINGLE-VALUED mother))).

It is known that a language is computationally intractable if it has both the cardinality general operators, AT-LEAST and AT-MOST, and the AND-ROLE operator at the same time [Nebel:88]. In $\mathcal{FL}^{\mathcal{A}+}$ the cardinality of the set of fillers of a role r can range only between the values $(0,\infty)$, $(0,1)$, $(1,1)$, $(1,\infty)$, while preserving tractability of the subsumption procedure.

This new type of role is introduced to avoid the problem seen in the analysis of $\mathcal{FL}^{\mathcal{A}}$ in the previous section: some deductions, which were true by common sense, were not sound with respect to the semantics (Theorem 3). The expressiveness is enhanced by introducing functional roles, and, still maintaining the conjunction of roles, the expected deductions are captured, as will be shown below.

3.1. Syntax and Semantics

With the introduction of functional roles, the interpretation function $\Im_T$ is extended over the terminology $\mathbf{T}_{\hat{r}}$:[3]

$\Im_T : \mathbf{T}_{\hat{r}} \rightarrow (\Omega \rightarrow \Omega)$;[4] where $\mathbf{T} = \mathbf{T}_c \cup \mathbf{T}_r \cup \mathbf{T}_{\hat{r}}$ and $\mathbf{T}_r \cap \mathbf{T}_{\hat{r}} = \emptyset$.

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[3] An <atomic-relational-role> will be referred to as **r**, whereas an expression of the type (SINGLE-VALUED <atomic-functional-role>) will be referred to as $\hat{\mathbf{r}}$.

[4] We could also define $\Im_T$ as: $\Im_T : \mathbf{T}_{\hat{r}} \rightarrow (\Omega \rightarrow 2^{\Omega})$, where the codomain 2^{Ω} is constrained into the range of the singletons (that is, the cardinality of the elements of the range is one). We should also notice the different uses of $\perp$ (bottom) and $\emptyset$ in the definitions below. If we define the function associated to a (functional) role as a total function f: $(\Omega \rightarrow 2^{\Omega})$, the elements $x_0 \in \Omega$ where the role is supposed to have no values, are mapped into the empty set $\emptyset$ (that is $f(x_0) = \emptyset$); whereas if we define a functional role as a partial function f: $(\Omega \rightarrow \Omega)$, f is not defined over such elements x_0, and $f(x_0) = \perp$.

Given the new syntax of roles, over such descriptions the semantics become:

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           <role> ::=  <functional-role> |
                       <relational-role>
<functional-role> ::=  (SINGLE-VALUED <atomic-functional-role>) |
                       (AND-ROLE <functional-role>+)
<relational-role> ::=  <atomic-relational-role>
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$\Im_T$ [(**AND-ROLE** $\hat{r}_1..\hat{r}_h$)][5] $= \hat{g}: \Omega \rightarrow \Omega \mid \forall x \left(\hat{g}(x) = \hat{f}_1(x) = ... = \hat{f}_h(x)\right) \vee \left(\hat{g}(x) = \perp\right)$

$\Im_T$ [(**SOME** $\hat{r}$)] $= \{ x \in \Omega \mid \exists y (\hat{f}(x) = y) \}$

$\Im_T$ [(**ALL** $\hat{r}$ <concept>)] $= \{ x \in \Omega \mid \forall y (y = \hat{f}(x) \supset y \in \Im_T$ [<concept>] $)\} =$
$\{ x \in \Omega \mid \hat{f}(x) = \perp \vee \hat{f}(x) \in \Im_T$ [<concept>] $\}$[6]

3.2. The clash of intuitions revisited

THEOREM 4. Let c_i be the concept (SOME (AND-ROLE $\hat{r}_1..\hat{r}_i$));
if $m \leq n$ then c_m subsumes c_n.

THEOREM 5. Let c_i be the concept (ALL (AND-ROLE $\hat{r}_1..\hat{r}_i$) d);
if $m \geq n$ then c_m subsumes c_n.

THEOREM 6. Let c_i be the concept (AND (SOME (AND-ROLE $\hat{r}_1..\hat{r}_i$))
(ALL (AND-ROLE $\hat{r}_1..\hat{r}_i$) d));
if $m \leq n$ then c_m subsumes c_n. ❏

While Theorems 4 and 5 simply generalize Theorems 1 and 2, Theorem 6 says that it is possible to deduce a subsumption relation between two concepts where both the SOME and ALL operators occur, *if* they include only functional roles. This resolves the potential confusion generated by Theorem 3. Let us for example define

c = (AND (SOME (AND-ROLE (SINGLE-VALUED mother)
(SINGLE-VALUED best-friend)))
(ALL (AND-ROLE (SINGLE-VALUED mother)
(SINGLE-VALUED best-friend)) rich));

d = (AND (SOME (SINGLE-VALUED mother)) (ALL (SINGLE-VALUED mother) rich)).

The concept **c** represents everybody having mother as best friend, who is rich; **d** individuates everybody having a rich mother. According to our common sense we would say that concept **d** is a generalization of concept **c**. This fact is captured by the semantics of this language, as shown in Theorem 6, so that **d** subsumes **c**.

4. THE FORMAL LANGUAGE FOR KRAPFEN: $\mathcal{KFL}$

The SOME and ALL operators do not appear in $\mathcal{KFL}$, while the operator EVERY is introduced. Its meaning is clarified considering, for example, the concept (AND person (EVERY child rich)), which represents all persons having at least one child, each of which is rich. As specified in the semantics EVERY is equivalent to the conjunction of SOME and ALL operators of the previous languages; indeed we could express the same concept in $\mathcal{FL}^-$ as (AND person (SOME child) (ALL child rich)).

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[5] Let us adopt the conventions: f_i (x) $= \Im_T$ [r_i] (x) and $\hat{f}_i$ (x) $= \Im_T$ [$\hat{r}_i$] (x).

[6] Remember that $\perp \notin \Omega$.

The expressive power of $\mathcal{KFL}$ from this point of view is less than that of $\mathcal{FL}^{\mathcal{A}+}$; in fact, the concepts defined by EVERY operator can be expressed by means of the conjunction of SOME and ALL, but not vice versa. The reason for this choice follows. Let us consider for example the concept (AND person (ALL child rich)), which represents everybody whose children, if he has children, are all rich. Still this definition includes people who have NO children at all, for example a catholic priest. In our opinion, to fully capture their expected meaning, the properties defining a concept should be used as the *slots* of a frame (i.e. the concept *must* have the property defined by the slot). Moreover the EVERY operator, as explained in the next section, cancels, in a way, the clash of intuitions generated by Theorems 2 and 5. That is why in $\mathcal{KFL}$ we cannot express a relation having minimum cardinality equal to zero: the cardinality of the set of fillers of a role can range only between the values $(1,\infty)$ or $(1,1)$.
On the other hand there is no loss of expressiveness with regard to the SOME operator; it is easy to verify the equivalence between the $\mathcal{FL}^{\mathcal{A}+}$ expression (SOME r) and the $\mathcal{KFL}$ concept (EVERY r ANYTHING). The concept ANYTHING individuates the entire set of elements in the domain. Among other things it represents the top of the taxonomy induced by subsumption relation: every concept is a (direct or indirect) specification of ANYTHING.

4.1. Syntax and Semantics

<concept> ::= <atomic-concept> |
ANYTHING |
(**AND** <concept>$^+$) |
(**EVERY** <role> <concept>) |
(**PRIMITIVE** <concept> <index>)
<role> ::= <functional-role> |
<relational-role> |
(**PRIMITIVE** <role> <index>)
<functional-role> ::= (**SINGLE-VALUED** <atomic-functional-role>) |
(**AND-ROLE** <functional-role>$^+$)
<relational-role> ::= <atomic-relational-role>

$\Im_T$ [**ANYTHING**] $= \Omega$
$\Im_T$ [(**PRIMITIVE** $\langle$concept$\rangle$ $\langle$index$\rangle$)] $\subseteq \Im_T [\langle\text{concept}\rangle]$
$\Im_T$ [(**PRIMITIVE** r $\langle$index$\rangle$)] $= \boldsymbol{f}: \Omega \to 2^{\Omega} \mid \forall x\, (\boldsymbol{f}(x) \subseteq \Im_T [r])$
$\Im_T$ [(**PRIMITIVE** $\hat{r}$ $\langle$index$\rangle$)] $= \hat{f}: \Omega \to \Omega \mid \forall x\, (\hat{f}(x) = \Im_T [\hat{r}]) \vee (\hat{f}(x) = \perp)$
$\Im_T$ [(**EVERY** r $\langle$concept$\rangle$)] $= \{ x \in \Omega : \varnothing \neq \boldsymbol{f}(x) \subseteq \Im_T [\langle\text{concept}\rangle] \}$
$\Im_T$ [(**EVERY** $\hat{r}$ $\langle$concept$\rangle$)] $= \{ x \in \Omega : \hat{f}(x) \in \Im_T [\langle\text{concept}\rangle]\} =$
$\{ x \in \Omega : \exists y\, (y = \hat{f}(x)) \wedge \forall y\, (y = \hat{f}(x) \supset y \in \Im_T [\langle\text{concept}\rangle])\}$.

4.2. Towards a rational representation language

THEOREM 7. Let c_i be the concept (EVERY (AND-ROLE $\hat{r}_1..\hat{r}_i$) d);
if $m \leq n$ then c_m subsumes c_n.
THEOREM 8. If r is a <role>, the following concepts are equivalent:
$c_1 =$ (AND (EVERY r a) (EVERY r b))
$c_2 =$ (EVERY r (AND a b)).

THEOREM 9. If $(\mathbf{S}_1 \cap \mathbf{S}_2) \neq \varnothing$ then the following concepts are equivalent[7]:
$c_1 =$ (AND (EVERY (AND-ROLE $\mathbf{S}_1$) a) (EVERY (AND-ROLE $\mathbf{S}_2$) b)),
$c_2 =$ (EVERY (AND-ROLE ($\mathbf{S}_1 \cup \mathbf{S}_2$)) (AND a b)).

THEOREM 10. Let r be a <role>, a ≠ ANYTHING, and c_1 and c_2 the concepts:
$c_1 =$ (EVERY r a),
$c_2 =$ (EVERY (PRIMITIVE r) a);
if r is a functional role then c_1 subsumes c_2,
otherwise neither c_1 subsumes c_2 nor c_2 subsumes c_1. ❑

Theorem 7 claims that the extension of (EVERY (AND-ROLE $\hat{r}_1..\hat{r}_i$) c) is reduced by increasing the number of roles, as intuition suggests, whereas that of (ALL (AND-ROLE $\hat{r}_1..\hat{r}_i$) c) grows. It is possible to infer subsumption between two concepts, including the EVERY operator, when roles are not disjoint, as long as these are functional; the most interesting case concerns concepts including EVERY operators applied to subsuming role sets. Let us consider for instance:
a = (AND person (EVERY (SINGLE-VALUED father) rich)),
b = (AND person (EVERY (AND-ROLE (SINGLE-VALUED best-friend)
(SINGLE-VALUED father)) rich));
Concept **a** represents everybody whose father is rich, whereas concept **b** refers to everybody whose father is his best friend and is also rich. According to common sense one would assume that everybody whose own father and best friend is rich must necessarily have a rich father. Thus **a** subsumes **b**; this fact is captured by Theorem 7.
The PRIMITIVE operator for concepts (and roles) is introduced to account, semantically, for the incompleteness of a definition [Israel:83]. A primitive concept is defined by means of necessary (but not sufficient) conditions. For example[8]
(AND (EVERY living-in Sicily)
(PRIMITIVE (AND bird (EVERY has-color (AND yellow light-color)))))
denotes a specific (even if not explicitly given) *subset* of the light yellow birds (e.g. a canary or a goldfinch) living in Sicily. Without the specification of being primitive, that concept would represent the class of all light yellow birds living in Sicily. It is easy to verify that the concept representing anything having a light colour (EVERY has-color light-color) subsumes the former concept.
The extension of an EVERY expression is reduced by increasing the number of constraints (like primitiveness) on its role, only if it is a functional role; the same explanation given above and in section **2.2** applies here. From Theorem 10 it follows, for example, that (EVERY friend rich) does not subsume the concept (EVERY (PRIMITIVE friend) rich); whereas the concept (EVERY (SINGLE-VALUED best-friend) rich) subsumes (EVERY (PRIMITIVE (SINGLE-VALUED best-friend)) rich).
It is provable that there are sound, complete and tractable - $\mathcal{O}(n^3)$ - algorithms for computing subsumption between concepts and between roles in $\mathcal{KL}$ [Cattoni:89]. Theorems 8 and 9 guarantee the soundness of the flat type conversion step in the subsumption algorithm. It is worth noting that the languages described so far cannot represent incoherent concepts (or roles): there exist always at least a non-empty interpretation of a concept. This fact leads to a simplification of the computational effort

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7 If $\mathbf{S} = \{\hat{r}_1..\hat{r}_i\}$ we write (AND-ROLE $\mathbf{S}$) instead of (AND-ROLE $\hat{r}_1..\hat{r}_i$).

8 For simplicity, every occurrence of the expression (PRIMITIVE <x>) will be considered equivalent to the expression (PRIMITIVE <x> *index*), where *index* is always a freshly generated index. This means that the possibility to refer to an already generated primitive concept (or role), without using an explicit name for such a description does not exist [Franconi et al.:90].

for the subsumption procedure.

5. CONCLUSIONS

This paper gives a methodology for the analysis and the development of a particular kind of *representational service* - a frame-based description language. We were interested in both theoretical and functional aspects: we believe this is only a starting point for arriving at a *rational* representation system, in the sense of [Doyle, Patil:89]. Thus the relationships between expressiveness, functional adequacy and formal properties of the deductive procedure have been analyzed. We have pointed out strong interrelations between the expressive power of a language and its effective use in a representational framework: expressiveness should not be compared only with completeness and tractability of subsumption. In conclusion, we propose a language in which there is a proper compromise between these three aspects.

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