Open Problems with Part-Whole Relations

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1 Introduction

Among attributes, a fundamental role is often played by various forms of `part-whole` relations, which contribute to describe the structure of composite objects. The representation of such a structural information usually requires a particular semantics together with specialized inference, but rarely do current modeling formalisms and methodologies give it a specific, "first-class" dignity [Simons, 1987; Guarino, 1994].

We have argued in [Artale *et al.*, 1996] that part-whole relations should not be modeled by ordinary attributes in an Object-Oriented formalism like a Description Logic. This rules out simplifying approaches like the one pursued by [Padgham and Lambrix, 1994; Lambrix, 1996] and [Speel and Patel-Schneider, 1994]. They propose a simple extension of description logics, where part-whole relations do not have any particular semantics with respect to standard roles. According to these approaches, the reasoning involving parts is *external* to the standard description logic reasoning services. While terminological subsumption and instance recognition are not affected by the particular meaning of the part-whole roles, ad-hoc reasoning services are needed, like for example the computation of the transitive closure for the *special* part-whole roles in the ABox (see section 3).

In this paper, we will refer to semantically deeper extensions of description logics, and we will point out the expressivity – see [Artale *et al.*, 1996] for a discussion about the conceptual modeling issues involving part-whole relations – and the computational problems of such extensions in the current state of the art.

2 Part-Whole Relation

The first serious attempts to formalize part-whole relations in a description logic consider the part-whole relation as a transitive relation obtained by means of a transitive closure operator applied to a basic primitive *direct* part-whole relation [Baader, 1991; Schild, 1991; Sattler, 1995; Calvanese *et al.*, 1995]. For example, if the has-part role (in symbols, $\succeq$) is modeled as the transitive closure of a primitive direct part relation (in symbols, $\succ_d$), we can describe a car by saying that it has wheels which in turn have tires as their parts:

$$\succeq \doteq (\succ_d)^*$$
$$\texttt{Car} \doteq \exists \succeq . (\texttt{Wheel} \sqcap \exists \succeq . \texttt{Tire})$$

then, the fact that tires are also parts of cars semantically follows: $\texttt{Car} \sqsubseteq \exists \succeq . \texttt{Tire}$. The complexity of satisfiability and subsumption for $\mathcal{ALC}$ with a transitive closure operator has been shown to be EXPTIME-complete [Baader, 1991].

Sometimes, one would like to define parts from wholes or conversely wholes from parts. Thus, the inverse role of the has-part role is needed: $\preceq \doteq \succeq^{-1}$. This can be captured by the $\mathcal{CI}$ (in correspondence with *converse-PDL*, also known as $\mathcal{TSL}$) concept language; satisfiability and subsumption for $\mathcal{CI}$ is again EXPTIME-complete [Schild, 1991].

Another important property of part-whole relations is *well-foundedness*. A relation is well-founded if there is no infinite chain in it: this corresponds to an atomic mereology. Thus, a well-founded transitive relation is a good candidate for a proper part-whole relation: it is finite, antisymmetric, irreflexive and obviously transitive – i.e., a strict partial order with no infinite descending chain. [Calvanese *et al.*, 1995] introduce $\mathcal{CVL}$, an EXPTIME-complete language extending $\mathcal{CI}$ and handling well-foundedness.

Within this approaches, the basic primitive part role denotes a *direct part* relation. Therefore, it should verify the definition of *immediate inferior*, taken from lattice theory. Given a partially ordered set P, we say that a is an *immediate inferior* of b if $a \prec_d b$, but $a \prec_d x \prec_d b$ for no $x \in P$. In order to discard non-intended models – e.g., $\{a \prec_d b,\ a \prec_d c,\ b \prec_d c\}$ – an axiom on roles of the kind $(\prec_d \circ \prec_d \circ \preceq) \sqcap \prec_d \doteq \bot \times \bot$ should be included in the theory, making the logic undecidable. Thus, it is not possible for the primitive direct part relation to satisfy the immediate inferior property.

In order to differentiate among various part relations, [Sattler, 1995] introduces six primitive roles denoting direct parts: `IS-D-COMP`, `IS-D-MEMBER`, `IS-D-SEGMENT`, `IS-D-QUANTITY`, `IS-D-INGREDIENT`, `IS-D-STUFF`, where `D` stands for *direct* – for example, `IS-D-COMPONENT` can be read as "*is a direct component of*". One should take into account the interactions between the various relations so that, for example, the composition of Member/Collection with Component/Object results in the Component/Object relation. In order to express such interactions, complex roles are needed; the language $\mathcal{CI}$ is enough to express these requirements. However, in order to express also disjointness among different part-whole relations, conjunction of roles is needed: such language is at least as expressive as $\mathcal{TSLR}$ [Schild, 1991], which is still an open problem.

More generally, it is useful to differentiate among several different part names: while modeling composite individuals it seems natural to give specific names to attributes denoting parts [Artale *et al.*, 1996]. In the domain of artifacts, we can consider for example the individual *car1* of type `Car`. A first modeling choice amount to saying that it has a part which is a wheel:

$$\texttt{Car}(\mathit{car1}) \land \mathit{wheel1} \preceq \mathit{car1} \land \texttt{Wheel}(\mathit{wheel1}).$$

On the other hand, a second modeling choice can make use of the attribute `HAS-WHEEL`:

$$\texttt{Car}(\mathit{car1}) \land \texttt{HAS-WHEEL}(\mathit{car1}, \mathit{wheel1}) \\ \land \texttt{Wheel}(\mathit{wheel1}).$$

In this latter case, we have the hidden assumption that `HAS-WHEEL` is a kind of part attribute:

$$\forall xy.\ \texttt{HAS-WHEEL}(x, y) \longrightarrow y \preceq x.$$

Under such assumption, the latter formulation implies the former. The possibility of expressing hierarchies of roles is needed, and in particular for the transitive role. The sub-part role can be simulated with role conjunction involving the transitive role, which is still – as said before – an open problem.

Another way to represent the part-whole relation, is to have a specific transitive role in the language, without having at the same time the primitive direct part relation. This extension has been analyzed in [Sattler, 1996]; it is in correspondence with the union of multimodal K and $S4$ – i.e. $K4_n$. Satisfiability and susbumption in $\mathcal{ALC}$ with transitive roles have been proved to be PSPACE-complete. This approach seems definitely to be more promising from a complexity point of view. Another point in favor of this approach is the debatable status of the direct part-whole relation in a conceptual modeling framework: we do not see the usefulness of having the direct part-whole relation in an ontology of the physical world.

3 Extensional vs Intensional Reasoning

There are two main approaches in the literature when considering reasoning with part-whole relations. In the first approach – the easier one – complete reasoning over parts and wholes is carried on only for individuals at ABox level. That is, the knowledge on parts and wholes at TBox level is used as a constraint for the ABox statements, and it is ignored for intensional reasoning. This is the approach pursued by [Padgham and Lambrix, 1994; Lambrix, 1996; Speel and Patel-Schneider, 1994]. The main reasoning task is, of course, the computation of the transitive closure for the part-whole roles. What is missed by these approaches are TBox reasoning like

$$\exists \texttt{W}.\exists \texttt{W}.\texttt{C} \sqsubseteq \exists \texttt{W}.\texttt{C},$$

i.e., wholes having sub-parts which have sub-parts themselves of a certain kind are *not* derived as wholes having *directly* sub-parts of that specific kind. The same limit applies also for the structural descriptions (see next section), at least as it is seen in the current systems.

On the other hand, approaches based on the modal interpretations of description logics [Schild, 1991; Sattler, 1995; Calvanese *et al.*, 1995], do suffer of a dual limitation. Several properties which do hold at the TBox level, and which are correctly taken into account in the intensional reasoning process, do not hold any more at ABox level. The most famous example for this is *antisymmetry* of the part-whole relation. Antisymmetry is usually a required property for the part-whole relation, since it captures acyclicity – i.e., an object can not be part of another object having it as part. It is known that antisymmetry does not affect logical implication in modal formulae[1]. That is, a modal theory – at TBox level – can easily take into account antisymmetry, since TBox reasoning does not change. But the problem arises when introducing statements about individuals. Current approaches can not correctly reason at ABox level, by taking into account antisymmetry. In fact, it is not possible to check the inconsistency of an ABox of the following kind – which is a non-intended model for the part-whole relation:

$$a \preceq b,\ b \preceq c,\ c \preceq a.$$

4 Dependency among parts

In order to correctly model the notion of a whole, we cannot limit ourselves to describing its meronymic structure, but we should be able somehow to express *how* the whole is related to the parts, and how the parts are "glued together" to form a whole. Important cases are *essential parts* – the whole is generically dependent on a particular class of parts, *dependent parts* – a part is generically dependent on the whole, *exclusive parts* – there exists at most one whole containing a particular part.

Such features, characterizing the interdependence between the parts and wholes, can be expressed using qualified number restrictions on complex roles [Sattler, 1995; Baader and Sattler, 1996]. For example, one could say that a given tank is used *exclusively* by the reactors of a given system in the following way:

$$\begin{array}{l} \texttt{Tank} \sqsubseteq \\ \quad (=\ 1\ (\texttt{IS-D-COMP} \circ \texttt{IS-D-COMP})\ \texttt{System}) \sqcap \\ \quad \forall \texttt{IS-D-COMP}.(\neg\texttt{Reactor} \sqcup \exists \texttt{IS-D-COMP}.\texttt{System}) \end{array}$$

[1] Modal formulae can not distinguish between antisymmetric and not antisymmetric structures.

The limit of this representation is the requirement that there exists a chain of direct part relations with a *predefined* length connecting the part with the whole. Thus, this works only if the Tank has been actually defined as being a *direct* part of a *direct* part – the Reactor – of a System.

A more general notion of exclusive part is needed. It is an open problem verifying whether it is possible to express in description logics the notion of exclusive parts, according to the following definition: x is an exclusive part of y iff x is part of y and for all w such that x is part of w then w is part of y. A possible limited way to express it – using qualification – is shown by the following example, where the Tank is an exclusive part of a given System:

$$\texttt{Tank} \sqsubseteq (=\ 1 \preceq \texttt{System}) \sqcap \forall\preceq.(=\ 1 \preceq \texttt{System})$$

Having qualified number restrictions is an hard problem anyway. [De Giacomo and Lenzerini, 1995] show the complexity of satisfiability for $\mathcal{CIQ}$ – a language admitting qualified number restrictions only on boolean combination of primitive roles and inverses of them – being in the EXPTIME class. It is not known what it happens if a complex role – like the composition of two direct part roles or the transitive "$\preceq$" role – appears in the number restriction: these are languages including $\mathcal{ALC}^{+}\mathcal{N}(\circ)$ – recently argued to be undecidable – and $\mathcal{ALC}^{+}\mathcal{N}(+)$ as defined in [Baader and Sattler, 1996].

Another kind of dependency between parts – known in the KR community as *structural description* – characterizes the integrity of the whole. It will suffice here to report the classical example of an arch which can be considered as a whole made out of inter-related parts. For an arch, its parts should satisfy the following constraints: the lintel is SUPPORTED-BY the uprights, and each upright is ON-THE-SIDE-OF and NOT-CONNECTED-WITH the other. Both [Padgham and Lambrix, 1994] and [Speel and Patel-Schneider, 1994] propose an extension of description logics, where *constraints* among role fillers – and in particular among parts – can be asserted in the definition of concepts:

$$\begin{aligned}&\texttt{Arch} \sqsubseteq\\ &\quad (= 1\ \texttt{LINTEL}) \sqcap\\ &\quad (= 2\ \texttt{UPRIGHT}) \sqcap\\ &\quad (\text{constraint}\ \texttt{LINTEL SUPPORTED-BY UPRIGHT})\end{aligned}$$

Such an extension recalls a simplified version of role value map, and it is proved to be decidable in [Hanschke, 1992], where a sound and complete algorithm is provided.

5 Distributing over Parts

In this section, we will consider dependence relationships between properties of parts and wholes and how they can be captured using the *plural quantifiers* [Franconi, 1993].

We speak of *downward distributivity* when there are properties which the parts inherit from the whole. For instance certain locative properties (e.g., "being in the car park") of the whole hold also for its parts:

$$\texttt{Car} \doteq \exists(\lhd\texttt{LOCATION}).\texttt{Car-Park} \sqcap \exists\succeq.\texttt{Engine} \sqcap \ldots$$

i.e., a car has, among other things, an engine and it is located in a car park. Please note the use of the "$\lhd$" *distributive plural quantifier*, which expresses the left distributive reading:

$$(\lhd R)(a,b) \quad \text{iff} \quad \forall x.\ \succeq(a,x) \rightarrow R(x,b)$$

Given the definition of car, it follows that the engine is also in the car park:

$$\begin{aligned}&\texttt{Car} \sqsubseteq \exists\texttt{LOCATION}.\texttt{Car-Park}\\ &\texttt{Car} \sqsubseteq \exists\succeq.(\texttt{Engine} \sqcap \exists\texttt{LOCATION}.\texttt{Car-Park})\end{aligned}$$

In general, it is also true that the location of a car is the same of its engine:

$$\texttt{Car} \sqsubseteq (\texttt{LOCATION} \doteq (\succeq \circ \texttt{LOCATION}))$$

We speak of *upward distributivity* when there are properties which the whole inherits from its parts: in many cases, for instance in the medical domain, you want to say that the whole is defective if one of its parts is defective.

The above mentioned downward and upward distributivity properties can be both referred to as *property inheritance through parts*. On the other hand, we can introduce *property refinement through parts*. This covers the case where an object related to a whole is also related to its parts. Stating, for example, that when an object is located in a region – say, r – then it is also located in any region that contains r, would validate the common-sense inference that a *fracture of the condyle of the femur* is a *fracture of the femur*, too. However, such principle of distribution is neither true for all roles nor globally true for the same role (for example, a *scoliosis of the thoracic spine* is not a *scoliosis of the spine*, and therefore the LOCATION role does not distribute). To address this problem, we can make use of the "$\rhd$" right distributive plural quantifier locally to a role:

$$(\rhd R)(a,b) \quad \text{iff} \quad \forall x.\ \succeq(b,x) \rightarrow R(a,x)$$

For instance, from the following definitions[2]:

$$\begin{aligned}&\texttt{Femur-Fracture} \doteq\\ &\quad \texttt{Fracture} \sqcap\\ &\quad \exists\texttt{LOCATION}.\texttt{Femur}\end{aligned}$$

$$\begin{aligned}&\texttt{Femur-Condyle-Fracture} \doteq\\ &\quad \texttt{Fracture} \sqcap\\ &\quad \exists(\rhd_{\preceq}\texttt{LOCATION}).(\texttt{Condyle} \sqcap \exists\preceq.\texttt{Femur})\end{aligned}$$

$$\begin{aligned}&\texttt{Spine-Scoliosis} \doteq\\ &\quad \texttt{Scoliosis} \sqcap\\ &\quad \exists\texttt{LOCATION}.\texttt{Spine}\end{aligned}$$

[2] We indicate with $\lhd_{\preceq}$ and $\rhd_{\preceq}$ the distributivity operators with respect to the role $\preceq$ (*is part of*).

$$\texttt{Thoracic-Scoliosis} \doteq \texttt{Scoliosis} \sqcap \exists\texttt{LOCATION}.(\texttt{Thoracic-Spine} \sqcap \exists\preceq.\texttt{Spine})$$

then we can derive that:

$$\texttt{Femur-Condyle-Fracture} \sqsubseteq \texttt{Femur-Fracture}$$

but not:

$$\texttt{Thoracic-Scoliosis} \sqsubseteq \texttt{Spine-Scoliosis}$$

The decidability and the complexity of $\mathcal{ALC}$ with the transitive "$\preceq$" relation and the left and right plural quantifiers is still an open problem, mainly because distributive plural quantifiers recall role-value-maps – even if in a simplified form [Schmidt-Schauß, 1989].

6 Conclusions

Part-whole relations have been extensively used in order to convey structural information. We have seen how their semantic peculiarities pose a number of modeling and reasoning problems, which require careful choices. Most of these problems are related to logical and computational properties which regard the notion of part-whole, like the transitivity property, and the interrelations among parts and whole. We must observe, however, that no one of the current systems is able to comprehensively cover the requirements we have discussed.

The most interesting and promising research direction to follow in the future is the use of a transitive role in the logic, without having the primitive direct part relation – i.e. the multimodal $K4_n$ logic. This research work is still at the beginning, and most of the interesting constructors are still missing, like complex non-transitive roles, functionality, inverse roles, and simplified role value maps.

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