Description Logics

Structural Description Logics

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Description Logics

- A logical reconstruction and *unifying* formalism for the representation tools
  - Frame-based systems
  - Semantic Networks
  - Object-Oriented representations
  - Semantic data models
  - Ontology languages
  - ...

- A *structured* fragment of predicate logic

- Provide theories and systems for *expressing* structured information and for *accessing* and *reasoning* with it in a principled way.
Applications

Description logics based systems are currently in use in many applications.

- Configuration
- Conceptual Modeling
- Query Optimization and View Maintenance
- Natural Language Semantics
- I3 (Intelligent Integration of Information)
- Information Access and Intelligent Interfaces
- Terminologies and Ontologies
- Software Management
- Planning
A formalism

- Description Logics formalize many *Object-Oriented* representation approaches.

- As such, their purpose is to disambiguate many imprecise representations.
Frames or Objects

- Identifier
- Class
- Instance
- Slot (attribute)
  - Value
    - Identifier
    - Default
  - Value restriction
    - Type
    - Concrete Domain
    - Cardinality
    - Encapsulated method
Ambiguities: classes and instances

Person: AGE: Number,
      SEX: \(M, F\),
      HEIGHT: Number,
      WIFE: Person.

\[john\] : AGE: 29,
      SEX: \(M\),
      HEIGHT: 76,
      WIFE: \textit{mary}.
Ambiguities: incomplete information

\[ 29' \text{er}: \text{AGE}: 29, \]
\[ \text{SEX}: M, \]
\[ \text{HEIGHT}: \text{Number}, \]
\[ \text{WIFE}: \text{Person}. \]

\[ \text{john}: \text{AGE}: 29, \]
\[ \text{SEX}: M, \]
\[ \text{HEIGHT}: \text{Number}, \]
\[ \text{WIFE}: \text{Person}. \]
Ambiguities: is-a

Sub-class:

Person: AGE: Number,
       SEX: M, F,
       HEIGHT: Number,
       WIFE: Person.

Male: AGE: Number,
      SEX: M,
      HEIGHT: Number,
      WIFE: Female.
Ambiguities: is-a

Instance-of:

Male: AGE: Number,
SEX: M,
HEIGHT: Number,
WIFE: Female.

john: AGE: 35,
SEX: M,
HEIGHT: 76,
WIFE: mary.
Ambiguities: is-a

Instance-of:

29'er: AGE: 29,
    SEX: M,
    HEIGHT: Number,
    WIFE: Person.

john: AGE: 29,
    SEX: M,
    HEIGHT: Number,
    WIFE: Person.
Ambiguities: relations

Implicit relation:

\[
\begin{align*}
john & : \text{AGE} : 35, \\
& \quad \text{SEX} : M, \\
& \quad \text{HEIGHT} : 76, \\
& \quad \text{WIFE} : mary.
\end{align*}
\]

\[
\begin{align*}
mary & : \text{AGE} : 32, \\
& \quad \text{SEX} : F, \\
& \quad \text{HEIGHT} : 59, \\
& \quad \text{HUSBAND} : john.
\end{align*}
\]
Ambiguities: relations

Explicit relation:

john : AGE : 35,
SEX : M,
HEIGHT : 76.

mary : AGE : 32,
SEX : F,
HEIGHT : 59.

m-j-family : WIFE : mary,
HUSBAND : john.
Ambiguities: relations

Special relation:

\[
\begin{align*}
\text{Car} & \xrightarrow{\text{HAS-PART}} \text{Engine} \\
\text{Engine} & \xrightarrow{\text{HAS-PART}} \text{Valve}
\end{align*}
\]

\[
\Rightarrow
\]

\[
\begin{align*}
\text{Car} & \xrightarrow{\text{HAS-PART}} \text{Valve}
\end{align*}
\]
Ambiguities: relations

Normal relation:

\[
\begin{align*}
&\text{John} \quad \text{HAS-CHILD} \quad \text{Ronald} \\
&\text{Ronald} \quad \text{HAS-CHILD} \quad \text{Bill} \\
\end{align*}
\]
Ambiguities: default

The *Nixon* diamond:

President

\[\begin{array}{c}
\text{Quaker} \\
\text{Republican}
\end{array}\]

\[\begin{array}{c}
nixon
\end{array}\]

Quakers are pacifist, Republicans are not pacifist.

\[\implies\text{Is Nixon pacifist or not pacifist?}\]
Ambiguities: quantification

What is the exact meaning of:

Frog \text{HAS-COLOR} \text{Green}

Frogs are typically green, but there may be exceptions.
Ambiguities: quantification

What is the exact meaning of:

- Every frog is just green
Ambiguities: quantification

What is the exact meaning of:

- Every frog is just green
- Every frog is also green
Ambiguities: quantification

What is the exact meaning of:

- Every frog is just green
- Every frog is also green
- Every frog is of some green
Ambiguities: quantification

What is the exact meaning of:

\[\text{Frog} \xrightarrow{\text{HAS-COLOR}} \text{Green}\]

- Every frog is just green
- Every frog is also green
- Every frog is of some green
- There is a frog, which is just green
- ...
Ambiguities: quantification

What is the exact meaning of:

- Every frog is just green
- Every frog is also green
- Every frog is of some green
- There is a frog, which is just green
- ...  
- Frogs are typically green, but there may be exceptions
False friends

- The meaning of object-oriented representations is logically very ambiguous.

- The appeal of the graphical nature of object-oriented representation tools has led to forms of reasoning that do not fall into standard logical categories, and are not yet very well understood.

- It is unfortunately much easier to develop some algorithm that appears to reason over structures of a certain kind, than to justify its reasoning by explaining what the structures are saying about the domain.
A structured logic

- Any (basic) Description Logic is a fragment of FOL.
- The representation is at the *predicate level*: no variables are present in the formalism.
- A Description Logic theory is divided in two parts:
  - the definition of predicates (*TBox*)
  - the assertion over constants (*ABox*)
- Any (basic) Description Logic is a subset of $\mathcal{L}_3$, i.e. the function-free FOL using only at most *three* variable names.
Why not FOL

If FOL is directly used without additional restrictions then

- the structure of the knowledge is destroyed, and it can not be exploited for driving the inference;

- the expressive power is too high for obtaining decidable and efficient inference problems;

- the inference power may be too low for expressing interesting, but still decidable theories.
Structured Inheritance Networks: KL-ONE

- Structured Descriptions
  - corresponding to the complex relational structure of objects,
  - built using a restricted set of epistemologically adequate constructs

- distinction between conceptual (terminological) and instance (assertional) knowledge;

- central role of automatic classification for determining the subsumption – i.e., universal implication – lattice;

- strict reasoning, no defaults.
Types of the TBox Language

- **Concepts** – denote *entities*
  (unary predicates, classes)

  *Example:* Student, Married

  \[
  \{ x \mid \text{Student}(x) \},
  \{ x \mid \text{Married}(x) \}
  \]

- **Roles** – denote *properties*
  (binary predicates, relations)

  *Example:* FRIEND, LOVES

  \[
  \{ \langle x, y \rangle \mid \text{FRIEND}(x, y) \},
  \{ \langle x, y \rangle \mid \text{LOVES}(x, y) \}
  \]
Concept Expressions

Description Logics organize the information in classes – concepts – gathering homogeneous data, according to the relevant common properties among a collection of instances.

Example:

\[ \text{Student} \sqcap \exists \text{FRIEND.Married} \]

\[
\{ x \mid \text{Student}(x) \land \\
\exists y. \text{FRIEND}(x, y) \land \text{Married}(y) \}\]
A note on $\lambda$’s

In general, $\lambda$ is an explicit way of forming names of functions:

$$\lambda x. f(x)$$ is the function that, given input $x$, returns the value $f(x)$

The $\lambda$-conversion rule says that:

$$(\lambda x. f(x))(a) = f(a)$$

Thus, $\lambda x. (x^2 + 3x - 1)$ is the function that applied to 2 gives 9:

$$(\lambda x. (x^2 + 3x - 1))(2) = 9$$

We can give a name to this function, so that:

$$f_{231} \equiv \lambda x. (x^2 + 3x - 1)$$

$$f_{231}(2) = 9$$
\( \lambda \) to define predicates

Predicates are special case of functions: they are *truth* functions. So, if we think of a formula \( P(x) \) as denoting a truth value which may vary as the value of \( x \) varies, we have:

\[ \lambda x. \ P(x) \] denotes a function from domain individuals to truth values.

In this way, as we have learned from FOL, \( P \) denotes exactly the set of individuals for which it is true. So, \( P(a) \) means that the individual \( a \) makes the predicate \( P \) true, or, in other words, that \( a \) is in the extension of \( P \).
For example, we can write for the **unary** predicate \( \text{Person} \):

\[
\text{Person} = \lambda x. \text{Person}(x)
\]

which is equivalent to say that \( \text{Person} \) denotes the **set** of persons:

\[
\begin{align*}
\text{Person} & \sim \{ x \mid \text{Person}(x) \} \\
\text{Person}^I & = \{ x \mid \text{Person}(x) \} \\
\text{Person}(john) & \iff john^I \in \text{Person}^I
\end{align*}
\]

In the same way for the **binary** predicate \( \text{FRIEND} \):

\[
\begin{align*}
\text{FRIEND} & = \lambda x, y. \text{FRIEND}(x, y) \\
\text{FRIEND}^I & = \{ (x, y) \mid \text{FRIEND}(x, y) \}
\end{align*}
\]
The functions we are defining with the $\lambda$ operator may be parametric:

$$\text{Student} \sqcap \text{Worker} = \lambda x. (\text{Student}(x) \land \text{Worker}(x))$$

$$(\text{Student} \sqcap \text{Worker})^\mathcal{I} = \{x \mid (\text{Student}(x) \land \text{Worker}(x))\}$$

$$(\text{Student} \sqcap \text{Worker})^\mathcal{I} = \text{Student}^\mathcal{I} \sqcap \text{Worker}^\mathcal{I}$$

(Verify as exercise)
Concept Expressions

\[(\text{Student} \cap \exists \text{FRIEND. Married})^I\]

= 

\[(\text{Student})^I \cap (\exists \text{FRIEND. Married})^I\]

= 

\[\{x \mid \text{Student}(x)\} \cap \{x \mid \exists y. \text{FRIEND}(x, y) \land \text{Married}(y)\}\]

= 

\[\{x \mid \text{Student}(x) \land \exists y. \text{FRIEND}(x, y) \land \text{Married}(y)\}\]
Objects: classes

<table>
<thead>
<tr>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person</td>
</tr>
<tr>
<td>name: [String]</td>
</tr>
<tr>
<td>address: [String]</td>
</tr>
<tr>
<td>enrolled: [Course]</td>
</tr>
</tbody>
</table>

\[
\{ x \mid \text{Student}(x) \} = \{ x \mid \text{Person}(x) \land \\
(\exists y. \text{NAME}(x, y) \land \text{String}(y)) \land \\
(\exists z. \text{ADDRESS}(x, z) \land \text{String}(z)) \land \\
(\exists w. \text{ENROLLED}(x, w) \land \text{Course}(w)) \}
\]

\[
\text{Student} \vdash \text{Person} \Box \\
\exists \text{NAME}.\text{String} \Box \\
\exists \text{ADDRESS}.\text{String} \Box \\
\exists \text{ENROLLED}.\text{Course}
\]
## Objects: instances

<table>
<thead>
<tr>
<th>s1: Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>name: “John”</td>
</tr>
<tr>
<td>address: “Abbey Road...”</td>
</tr>
<tr>
<td>enrolled: cs415</td>
</tr>
</tbody>
</table>

\[
\text{Student}(s1) \wedge \\
\text{NAME}(s1, “john”) \wedge \text{String}(“john”) \wedge \\
\text{ADDRESS}(s1, “abbey-road”) \wedge \text{String}(“abbey-road”) \wedge \\
\text{ENROLLED}(s1, cs415) \wedge \text{Course}(cs415)
\]
Semantic Networks

∀x. Student(x) →
  ∃y. ENROLLED(x, y) ∧ Course(y)

Student ⊆ ∃ENROLLED.Course

∀x. Professor(x) →
  ∃y. TEACHES(x, y) ∧ Course(y)

Professor ⊆ ∃TEACHES.Course

∀x. Working-student(x) →
  Student(x) ∧ Professor(x)

Working-student ⊆ Student

Working-student ⊆ Professor
Quantification

\[
\begin{array}{c}
\text{Frog} \xrightarrow{\text{HAS-COLOR}} \text{Green}
\end{array}
\]

- \(\text{Frog} \subseteq \exists \text{HAS-COLOR.\text{Green}}\):
  Every frog is also green

- \(\text{Frog} \subseteq \forall \text{HAS-COLOR.\text{Green}}\):
  Every frog is just green

- \(\text{Frog} \subseteq \forall \text{HAS-COLOR.\text{Green}}\)
  \(\text{Frog}(x), \text{HAS-COLOR}(x, y)\):
  There is a frog, which is just green
Quantification: existential

Every frog is also green

\(\forall x. \text{Frog}(x) \rightarrow \exists y. (\text{HAS-COLOR}(x, y) \land \text{Green}(y))\)

**Exercise: is this a model?**

\(\text{Frog}(\text{oscar}), \text{Green}(\text{green}), \text{HAS-COLOR}(\text{oscar,green}), \text{Red}(\text{red}), \text{HAS-COLOR}(\text{oscar,red}).\)
Quantification: universal

Every frog is only green

Frog  ⊆  ∀HAS-COLOR.Green

∀x. Frog(x) →

∀y. (HAS-COLOR(x, y) → Green(y))

Exercise: is this a model? and this?

Frog(oscar), Green(green), Frog(sing),
HAS-COLOR(oscar,green), A G E N T (sing,oscar).
Red(red),
HAS-COLOR(oscar,red).
Analytic reasoning (intuition)

Person

*subsumes*

(Person *with every* male friend *is a* doctor)

*subsumes*

(Person *with every* friend *is a* 

(Doctor *with a* specialty *is surgery*)
Analytic reasoning (intuition)

Person

*subsumes*

(Person with *every* male friend *is a* doctor)

*subsumes*

(Person with *every* friend *is a*

  (Doctor with *a* specialty *is* surgery))

(Person with $\geq 2$ children)

*subsumes*

(Person with $\geq 3$ male children)
Analytic reasoning (intuition)

Person

\textit{subsumes}

(Person \textbf{with every} male friend \textbf{is a} doctor)

\textit{subsumes}

(Person \textbf{with every} friend \textbf{is a}

\hspace{1cm} (Doctor \textbf{with a} specialty \textbf{is} surgery))

(Person \textbf{with} \geq 2 children)

\textit{subsumes}

(Person \textbf{with} \geq 3 male children)

(Person \textbf{with} \geq 3 young children)

\textit{disjoint}

(Person \textbf{with} \leq 2 children)