Description Logics

Description Logics and Logics

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Tense Logic: *(point ontology)*

- Tense logic is a propositional modal logic, interpreted over temporal structure $\mathcal{T} = (\mathcal{P}, <)$, where $\mathcal{P}$ is a set of time points and $<$ is a strict partial order on $\mathcal{P}$.

- Mortal $\sqsubseteq$ LivingBeing $\sqcap \forall$LIVES-IN.Place $\sqcap$
  
  $\text{LivesIN} \cup_{\text{livingbeing}}^\cup (\text{livingbeing} \cup \circlearrowright \text{livingbeing}))$

- Satisfiability in $\mathcal{ALC}_{US}$ – the combination of tense logic with $\mathcal{K}_m$ – over a linear, unbounded, and discrete temporal structure has the same complexity as its base (PSPACE-complete).

- Satisfiability in $\mathcal{ALCQI}_{US}$ with ABox – the combination of tense logic with $\mathcal{ALCQI}$ with ABox – over a linear, unbounded, and discrete temporal structure has the same complexity as its base (EXPTIME-complete).
**HS**: Interval Temporal Propositional Modal Logic

- **HS** is a propositional modal logic interpreted over an interval set $T^*_<$, defined as the set of all closed intervals $[u, v] = \{x \in P \mid u \leq x \leq v, u \neq v\}$ in some temporal structure $T$.

- **HS** extends propositional logic with modal formulæ $\langle R \rangle \phi$ and $[R] \phi$ – where $R$ is a basic Allen’s algebra temporal relation:

  - before $(i, j)$
  - meets $(i, j)$
  - overlaps $(i, j)$
  - starts $(i, j)$
  - during $(i, j)$
  - finishes $(i, j)$

- Mortal $\models$ LivingBeing $\land \langle \text{after} \rangle \dot{\neg}$LivingBeing

- Satisfiability **HS** is undecidable for the most interesting classes of temporal structures.

- Therefore, **HS** $\cup$ **ALC** is undecidable.
Decidable Interval Temporal Description Logics

- $\mathcal{HS}^*$:
  - No universal quantification, or restricted to homogeneous properties:
    $\square(=, \text{starts}, \text{during}, \text{finishes}). \psi$
  - Allows for temporal variables:
    $\Diamond \vec{x} \ \text{TN}(\vec{x}). \psi$
    $\psi \@ x$
  - Global roles – denoting temporal *independent* properties.

- Logical implication in the combined language $\mathcal{HS}^* \cup \mathcal{ALC}$ is decidable (PSPACE-hard); satisfiability is PSPACE-complete.

- Logical implication in $\mathcal{HS}^* \cup \mathcal{F}$ is NP-complete.

- Useful for event representation and plan recognition.
The Block World Domain

Initial State

Grasp

Final State

Stack(OBJ1, OBJ2)

Clear-Block(OBJ1) Holding-Block(OBJ1) Clear-Block(OBJ1)

Clear-Block(OBJ2) ON(OBJ1, OBJ2)

Stack = \Diamond(x v y w) (\# finishes x)(\# meets y)(\# meets z)(v overlaps \#)(w finishes \#)(v meets w).

((\*OBJECT2 : Clear-Block)@x \\False
((\*OBJECT1 \circ ON = \*OBJECT2)@y \False
((\*OBJECT1 : Clear-Block)@v \\False
((\*OBJECT1 : Holding-Block)@w \\False
((\*OBJECT1 : Clear-Block)@z)
\[ \mathcal{L}^n \] FOL fragments

- \[ \mathcal{L}^n \] is the set of function-free FOL formulas with equality and constants, with only unary and binary predicates, and which can be expressed using at most \( n \) variable symbols.

- Satisfiability of \[ \mathcal{L}^3 \] formulas is undecidable.

- Satisfiability of \[ \mathcal{L}^2 \] formulas is NEXPTIME-complete.
**The DL description logic**

\[ ALCI + \text{ propositional calculus on roles,} \]

\[ + \text{ the concept } (R \subseteq S). \]

- The DL description logic and \( \hat{\mathcal{L}}^3 \) are equally expressive.
- The \( DL^- \) description logic (i.e., DL without the composition operator) and \( \hat{\mathcal{L}}^2 \) are equally expressive.
- Open problem: relation between DL including cardinalities and \( \hat{\mathcal{L}}^n \) – adding counting quantifiers to \( \hat{\mathcal{L}}^n \).
Guarded Fragments of FOL

The *guarded fragment* GF of FOL is defined as:

1. Every relational atomic formula is in GF
2. GF is propositionally closed
3. If \( x, y \) are tuples of variables, \( \alpha(x, y) \) is atomic, and \( \psi(x, y) \) is a formula in GF, such that \( \text{free}(\psi) \subseteq \text{free}(\alpha) = \{x, y\} \), then the following formulae are in GF:
   - \( \exists y. \alpha(x, y) \land \psi(x, y) \)
   - \( \forall y. \alpha(x, y) \rightarrow \psi(x, y) \)

The guarded fragment contains the modal fragment of FOL (and Description Logics); a weaker definition (LGF) is needed to include temporal logics.
Properties of GF

- GF has the finite model property
- GF and LGF have the tree model property
- Many important model theoretic properties which hold for FOL and the modal fragment, do hold also for GF and LGF
- Satisfiability is decidable for GF and LGF (deterministic double exponential time complete)
- Bounded-variable or bounded-arity fragments of GF and LGF (which include Description Logics) are in EXPTIME.
- GF with fix-points is decidable.