

Computational Logic

Assignment 4

Due: 22/03/2006

1. Consider the following formula:

$$(\forall X)p(X, f(X)) \wedge (\forall Y)\neg p(Y, Y) \wedge (\forall X, Y, Z)(p(X, Y) \wedge p(Y, Z) \Rightarrow p(X, Z))$$

- (a) Find a model for this formula
 - (b) Show that there exists no model for this formula with a finite domain.
2. Prove with natural deduction that $(\exists X)(F \Rightarrow G) \Rightarrow ((\forall X)F \Rightarrow G)$ is valid for all formulas F, G such that G does not contain a free occurrence of X
 3. Use resolution to find the validity of the following arguments
 - (a) Some students are anxious. Some students study. If a student is anxious he will not pass his examination unless he studies. Therefore, no student will pass the examinations.
 - (b) Some students are anxious. Some students study. If a student is anxious he will not pass his examination unless he studies. Therefore, some students will pass the examinations.
 - (c) Some students are anxious. All students study. If a student is anxious he will not pass his examination unless he studies. Therefore, all students will pass the examinations.
 - (d) All students are anxious. Some students study. If a student is anxious he will not pass his examination unless he studies. Therefore, some students will pass the examinations.
 4. Determine if the argument is valid using tableaux
 - (a) All fruit is tasty if it is not cooked. This apple is cooked. Therefore, it is not tasty.
 - (b) All lecturers are determined. Anyone who is determined and intelligent will give satisfactory service. Clare is an intelligent lecturer. Therefore, Clare will give satisfactory service.

- (c) All those who honour both parents are blessed. If anyone dislikes any of his siblings he does not honour his parents. Jack likes his sister Jill. Therefore, Jack is blessed.
- (d) Some lecturers are imaginative but poor communicators. Only good students are lecturers. Good students are not imaginative. Every artist is imaginative. Therefore not every good student is an artist.
- (e) Dilly loves all and only those who love Milly. Milly loves all and only those who do not love Dilly. Dilly loves herself. Therefore, Milly loves herself.