4. Basics of Description Logics

Exercise 4.1 Translate the following DL expressions and axioms into first-order logic:

1. Father ⊓ ∀child. (Doctor ⊔ ∃managedBy ¬ (Company ⊓ (∈ 3 employs Doctor)))
2. Person ⊓ ∀child. HappyPerson ⊑ ∃child. ∀child. HappyPerson
3. Person ⊓ ∃child. HappyPerson ⊑ Happy ⊓ (Father ⊔ Mother)

Exercise 4.2 Translate the following sentences and first-order logic formulas into DL syntax, if possible:

1. Only humans have children that are humans.
2. A node cannot have two distinct $P$-successors, such that one is a $B$ and the second one is not a $B$.
3. $∀x_1, x_2, y_1, y_2. P(x_1, y_1) ∧ P(x_1, y_2) ∧ P(x_2, y_2) → x_1 = x_2 ∨ y_1 = y_2$
4. $∀x, y, z. P(x, y) ∧ P(y, z) ∧ P(z, x) → A(x)$
5. $∀x, y, z. P(x, y) ∧ Q(y, z) → R(x, z)$
6. $∀x, y, z. P(x, y) ∧ Q(y, z) → ∃w. R(x, w) ∧ S(w, z)$
7. $¬(∀x. A(x) → B(x)) ∨ (∀x. A(x) → C(x))$
8. $∃x. ∀y. R(x, y) ∨ S(x, y)$

Exercise 4.3 Compute the certain answers to the query $q$ over the KB $K = (T, A)$.

1. Let $q(x) = B(x), A = \{ A(a), B(b), C(c) \}$ and $T = \{ A ⊑ B, \ C ⊑ ∃R, \ ∃R − ⊑ B \}$.
2. Let $q() = ∃x.B(x)$ and $A = \{ A(a) \}$ and
   (a) $T = \{ A ⊑ ∃R, \ ∃R − ⊑ B \}$,
   (b) $T = \{ A ⊑ ∃R ∪ ∃S, \ ∃R − ⊑ B \}$,
   (c) $T = \{ A ⊑ ∃R ∩ (∃S ∪ ∃Q), \ ∃R − ⊑ B, \ ∃Q − ⊑ B \}$,
   (d) $T = \{ A ⊑ ∃R ∪ ∃S, \ ∃R − ⊑ B, \ ∃S − ⊑ ∃R ∪ ∃Q, \ ∃Q − ⊑ ∃R \}$,
3. Let $q(x) = ∃y.R(x, y), A = \{ A(a), R(b, c) \}$, and $T$ as above.
4. Let $q(x) = ∃y.R(x, y), A = \{ A(a), R(a, c) \}$, and $T$ as above.