Query Rewriting in $DL\text{-}Lite_{horn}^{(\mathcal{HN})}$

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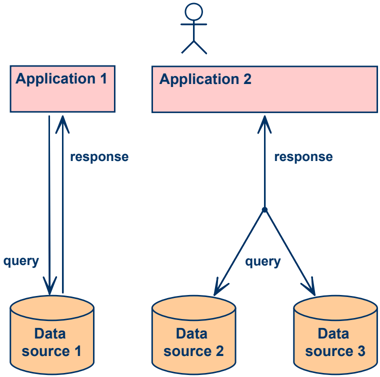
Outline

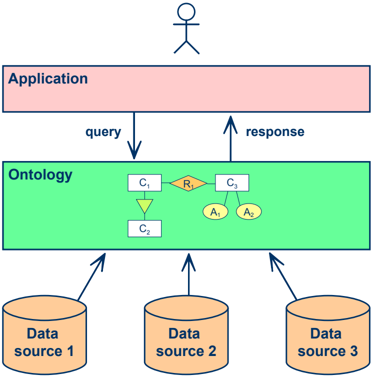
1. Motivation
2. The DL $DL\text{-}Lite_{horn}^{(HN)}$
3. Knowledge Base Satisfiability
4. Query Answering
5. Conclusions

Motivation: Ontology-Based Data Access



Motivation: Ontology-Based Data Access

- Ontologies are used for accessing data



- An ontology provides a high-level conceptual view of information sources
- Data sources can be queried through ontologies

Query Answering by Rewriting

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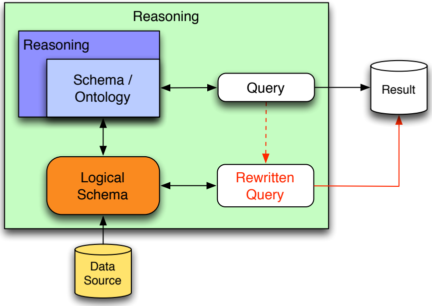
- Rewriting approach:
  1. Rewrite the query using the constraints in the ontology
  2. Evaluate the rewritten query over the database



Query Answering by Rewriting: Example

Ontology: $\mathcal{O} = \{PhDStudent \sqsubseteq Student\}$
Database: $\mathcal{DB}_{\mathcal{A}} = \{PhDStudent(\text{john})\}$
Query: $q(x) \leftarrow Student(x)$

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Ontology: $\mathcal{O} = \{PhDStudent \sqsubseteq Student\}$
Database: $\mathcal{DB}_\mathcal{A} = \{PhDStudent(\text{john})\}$
Query: $q(x) \leftarrow Student(x)$

- The rewriting of q:
  $q_{ucq}(x) \leftarrow Student(x)$
  $q_{ucq}(x) \leftarrow PhDStudent(x)$
- By evaluating the rewriting over the ABox viewed as a DB :
  $eval(q_{ucq},\ \mathcal{DB}_\mathcal{A}) = \{\text{john}\} = ans(q,\ \langle \mathcal{O}, \mathcal{DB}_\mathcal{A} \rangle)$

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  - $DL\text{-}Lite_{\mathcal{R}}$, $DL\text{-}Lite_{\mathcal{F}}$, $DL\text{-}Lite_{\mathcal{A}}$

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- Such a rewriting approach can be applied only to FOL rewritable logics.
- *DL-Lite* is a family of logics that has been shown to enjoy FOL rewritability:
  - $DL\text{-}Lite_{\mathcal{R}}$, $DL\text{-}Lite_{\mathcal{F}}$, $DL\text{-}Lite_{\mathcal{A}}$
- Extended *DL-Lite* family: additional constructs have been proposed
  - $DL\text{-}Lite_{horn}^{(\mathcal{HN})}$ is the most interesting logic

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In this work we consider the logic $DL\text{-}Lite_{horn}^{(\mathcal{HN})}$:

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    - $hasConfPaper \sqsubseteq hasPublication$

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    - $hasConfPaper \sqsubseteq hasPublication$
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    - $PhDStudent \sqsubseteq \geq 2\ hasConfPaper$
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    - $\geq 2\ teaches^{-} \sqsubseteq \bot$
  - horn inclusions *horn*
    - $Student \sqcap \geq 1\ teaches \sqsubseteq PhDStudent$

Questions Addressed by Our Work

For the logic $DL\text{-}Lite_{horn}^{(\mathcal{HN})}$:

- Can we check ontology satisfiability by relying on RDB technology?
- Can we answer queries by relying on RDB technology?
- Can we extend the practical algorithms developed for the simpler *DL-Lite* logics?
- What is the complexity of such algorithms?

$\mathit{DL\text{-}Lite}^{(\mathcal{HN})}_{horn}$: Syntax

Concept and role expressions

$$B ::= \bot \mid A \mid \geq k\, R$$
$$R ::= P \mid P^{-}$$

TBox assertions

$$B_1 \sqcap \cdots \sqcap B_n \sqsubseteq B$$
$$R_1 \sqsubseteq R_2$$
$$\mathrm{Dis}(R_1, R_2)$$

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Restriction to ensure FOL rewritability:

if R has a proper sub-role, then $\geq k\, R$ with $k \geq 2$ does not occur in the lhs of concept inclusions.

A $DL\text{-}Lite_{horn}^{(\mathcal{HN})}$ TBox

- basic concept inclusion

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- local functionality assertion

$$\mathit{PhDStudent} \sqcap \geq 2\, \mathit{teaches} \sqsubseteq \bot$$

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- global functionality assertion

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  $\mathcal{A}$: $\textit{teaches}(\text{john}, \text{cl}),\ \textit{attends}(\text{john}, \text{cl})$
- $\mathcal{T}$: $\textit{PhDStudent} \sqcap \geq 2\ \textit{teaches} \sqsubseteq \bot$
  $\mathcal{A}$: $\textit{PhDStudent}(\text{john}),\ \textit{teaches}(\text{john}, \text{cl}),\ \textit{teaches}(\text{john}, \text{db})$

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- $\mathcal{T}$: $\text{Dis}(teaches, attends)$
  $\mathcal{A}$: $teaches(\text{john}, \text{cl}), \ attends(\text{john}, \text{cl})$
- $\mathcal{T}$: $PhDStudent \sqcap \geq 2 \ teaches \sqsubseteq \bot$
  $\mathcal{A}$: $PhDStudent(\text{john}), \ teaches(\text{john}, \text{cl}), \ teaches(\text{john}, \text{db})$

$\Rightarrow$ We need to calculate closure of NIs w.r.t. PIs

Knowledge Base Satisfiability Algorithm

We reduce the problem to FOL query evaluation.

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Algorithm for checking KB satisfiability

1. Calculate the closure of NIs.
2. Translate the closure into a UCQ q_{unsat} asking for violation of some NI.
3. Evaluate q_{unsat} over the ABox (viewed as a DB).
   - if $eval(q_{unsat}, \mathcal{DB}_{\mathcal{A}}) = \emptyset$, then the KB is satisfiable;
   - otherwise the KB is unsatisfiable.

Closure of Negative Inclusions

Closure of NIs $cln(\mathcal{T})$ w.r.t. PIs

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- $\left.\begin{array}{ll} cln(\mathcal{T}): & PhDStudent \sqcap \geq 2\, teaches \sqsubseteq \bot \\ \mathcal{T}: & FullProfessor \sqsubseteq \geq 3\, teaches \end{array}\right\} \Rightarrow$

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- $cln(\mathcal{T})$: $PhDStudent \sqcap \geq 2\ teaches \sqsubseteq \bot$
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- $cln(\mathcal{T})$: $PhDStudent \sqcap \geq 2\ teaches \sqsubseteq \bot$
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  add to $cln(\mathcal{T})$: $PhDStudent \sqcap FullProfessor \sqsubseteq \bot$

- $cln(\mathcal{T})$: $Professor \sqcap \geq 1\ attends \sqsubseteq \bot$
  $\mathcal{T}$: $registeredTo \sqsubseteq attends$ $\Rightarrow$

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- $Professor \sqcap Student \sqsubseteq \bot \Rightarrow$
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- $Professor \sqcap Student \sqsubseteq \bot \Rightarrow$
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- $\mathrm{Dis}(attends, teaches) \Rightarrow$
  $\exists x, y.attends(x, y) \land teaches(x, y)$.

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Notice, that the problem is $\textsc{PTime}$ [ACKZ09].

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Query Answering: Example

$q(x) \leftarrow hasPublication(x, y) \wedge Publication(y)$

TBox $\mathcal{T} = \{$

$\geq 1\ hasPublication^{-} \sqsubseteq Publication$

$hasConfPaper \sqsubseteq hasPublication$

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$$\Downarrow\ \geq 1\ hasPublication^- \sqsubseteq Publication$$

$$q_2(x) \leftarrow hasPublication(x, y) \wedge E_1\, hasPublication^-(y)$$

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$\Downarrow$ unify the atoms

$$hasPublication(x, y)$$

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$$q_3(x) \leftarrow E_1\, hasPublication(x)$$

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$$q_3(x) \leftarrow E_1\, hasPublication(x)$$

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$$q_4(x) \leftarrow E_1 hasConfPaper(x)$$

$\Downarrow$ $\geq 2\ hasConfPaper \sqsubseteq \geq 1\ hasConfPaper$

$$q_5(x) \leftarrow E_2 hasConfPaper(x)$$

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$q_5(x) \leftarrow E_2\, hasConfPaper(x)$

$\Downarrow$ $PhDStudent \sqsubseteq \geq 2\ hasConfPaper$

$q_6(x) \leftarrow PhDStudent(x)$

$\Downarrow$ $Student \sqcap \geq 1\ teaches \sqsubseteq PhDStudent$

$q_7(x) \leftarrow Student(x) \wedge E_1\, teaches(x)$

Query Answering Algorithm

1. Compute the rewriting of the initial query, a UCQ.
   - Application of PIs to query atoms.

$$q(x) \leftarrow hasPublication(x,y) \wedge Publication(y)$$
$$\Downarrow \; \geq 1\, hasPublication^- \sqsubseteq Publication$$
$$q'(x) \leftarrow hasPublication(x,y) \wedge E_1\, hasPublication^-(y)$$

   - Unification of query atoms.

$$q(x) \leftarrow hasPublication(x,y) \wedge E_1\, hasPublication^-(y)$$
$$\Downarrow \text{ unify}$$
$$q'(x) \leftarrow hasPublication(x,y)$$

2. Evaluate the obtained UCQ over the ABox viewed as a DB.

What Is New?

Differences w.r.t. the algorithm for simpler variants of *DL-Lite*

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  - remove $E_{k'} R(\mathrm{z})$, if $E_k R(\mathrm{x})$ occurs in the query and $k > k'$

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Note that the size of the rewriting is exponential already w.r.t. the size of the TBox.

Outline

1. Motivation
2. The DL $DL\text{-}Lite_{horn}^{(HN)}$
3. Knowledge Base Satisfiability
4. Query Answering
5. Conclusions

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- The computational complexity of the algorithms is optimal w.r.t. data complexity:
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- Future work:
  - Implement the developed algorithms.
  - Study optimization techniques for the algorithm.
  - Extend the practical algorithm to positive existential queries.

Thank you
for your attention!