Query Rewriting in $DL$-$Lite^{(\mathcal{HN})}_{horn}$

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Outline

1. Motivation
2. The DL $DL-Lite_{horn}^{(HN)}$
3. Knowledge Base Satisfiability
4. Query Answering
5. Conclusions
Motivation: Ontology-Based Data Access

- An ontology provides a high-level conceptual view of information.
- Data sources can be queried through ontologies.

Motivation: Ontology-Based Data Access

- Ontologies are used for accessing data.
- Query Rewriting in $DL-Lite^{(H.N)}_{horn}$
Motivation: Ontology-Based Data Access

- Ontologies are used for accessing data

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Query Answering by Rewriting

- We want to compute *certain answers* to a query
Query Answering by Rewriting

- We want to compute *certain answers* to a query
- Rewriting approach:
  1. **Rewrite** the query using the constraints in the ontology
  2. **Evaluate** the rewritten query over the database
Query Answering by Rewriting: Example

Ontology: $\mathcal{O} = \{\text{PhDStudent} \sqsubseteq \text{Student}\}$
Database: $\mathcal{DB}_A = \{\text{PhDStudent}(\text{john})\}$
Query: $q(x) \leftarrow \text{Student}(x)$
Query Answering by Rewriting: Example

Ontology: $\mathcal{O} = \{PhDStudent \sqsubseteq Student\}$

Database: $\mathcal{DB}_A = \{PhDStudent(john)\}$

Query: $q(x) \leftarrow Student(x)$

- The rewriting of $q$:
  
  $q_{ucq}(x) \leftarrow Student(x)$
  
  $q_{ucq}(x) \leftarrow PhDStudent(x)$

- By evaluating the rewriting over the ABox viewed as a DB:
  
  $\text{eval}(q_{ucq}, \mathcal{DB}_A) = \{john\} = \text{ans}(q, \langle \mathcal{O}, \mathcal{DB}_A \rangle)$
Such a rewriting approach can be applied only to FOL rewritable logics.
FOL Rewritable Logics

• Such a rewriting approach can be applied only to FOL rewritable logics.

• DL-Lite is a family of logics that has been shown to enjoy FOL rewritability:
  ▶ DL-Lite$_R$, DL-Lite$_F$, DL-Lite$_A$
• Such a rewriting approach can be applied only to FOL rewritable logics.

• *DL-Lite* is a family of logics that has been shown to enjoy FOL rewritability:
  - $DL$-Lite$_R$, $DL$-Lite$_F$, $DL$-Lite$_A$

• Extended *DL-Lite* family: additional constructs have been proposed
  - $DL$-Lite$_{horn}$ is the most interesting logic
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In this work we consider the logic $DL$-$Lite_{horn}^{(\mathcal{HN})}$:

- The most expressive tractable variant of $DL$-$Lite$ [ACKZ09].
In this work we consider the logic $\text{DL-Lite}^{(\mathcal{HN})}_{\text{horn}}$:

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In this work we consider the logic $DL-Lite^{(\mathcal{HN})}_{horn}$:

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    - $\text{hasConfPaper} \sqsubseteq \text{hasPublication}$
In this work we consider the logic $DL$-\textit{Lite}$^\text{(HN)}_{\text{horn}}$:

- The most expressive tractable variant of $DL$-\textit{Lite} [ACKZ09].
- Extends $DL$-\textit{Lite} with
  - role inclusions $\mathcal{H}$
    - $\text{hasConfPaper} \sqsubseteq \text{hasPublication}$
  - number restrictions $\mathcal{N}$
    - $\text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper}$
    - $\geq 2 \text{teaches}^- \sqsubseteq \bot$
In this work we consider the logic $DL$-Lite$^\text{(HN)}_{horn}$:

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    - $\geq 2 \text{teaches}^- \sqsubseteq \bot$
  - horn inclusions $horn$
    - $\text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}$
Questions Addressed by Our Work

For the logic $DL$-$Lite_{horn}^{(HN)}$:

- Can we check ontology satisfiability by relying on RDB technology?
- Can we answer queries by relying on RDB technology?
- Can we extend the practical algorithms developed for the simpler $DL$-$Lite$ logics?
- What is the complexity of such algorithms?
**Motivation**

The DL $DL-Lite_{horn}^{(HN)}$

**Knowledge Base Satisfiability**

**Query Answering**

**Conclusions**

---

**DL-Lite$_{horn}^{(HN)}$: Syntax**

Concept and role expressions

\[
B ::= \bot \mid A \mid \geq k R
\]

\[
R ::= P \mid P^-
\]

TBox assertions

\[
B_1 \sqcap \cdots \sqcap B_n \sqsubseteq B
\]

\[
R_1 \sqsubseteq R_2
\]

\[
\text{Dis}(R_1, R_2)
\]

---

Botoeva, Artale, Calvanese

Query Rewriting in $DL-Lite_{horn}^{(HN)}$

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**DL-Lite**$_{horn}$(HN) : Syntax

Concept and role expressions

\[ B ::= \bot \mid A \mid \geq k R \]
\[ R ::= P \mid P^- \]

TBox assertions

\[ B_1 \sqcap \cdots \sqcap B_n \sqsubseteq B \]
\[ R_1 \sqsubseteq R_2 \]
\[ \text{Dis}(R_1, R_2) \]

Restriction to ensure FOL rewritability:

if \( R \) has a proper sub-role, then \( \geq k R \) with \( k \geq 2 \) does not occur in the lhs of concept inclusions.
A $DL$-Lite$^{(\mathcal{HN})}_{\text{horn}}$ TBox

- basic concept inclusion

\[ \geq 1 \text{hasPublication} \sqsubseteq \text{Publication} \]
A $DL$-Lite$_{horn}^{(\mathcal{H},\mathcal{N})}$ TBox

- basic concept inclusion

$$\geq 1 \text{hasPublication} \sqsubseteq \text{Publication}$$

- role inclusion

$$\text{hasConfPaper} \sqsubseteq \text{hasPublication}$$
A $DL$-Lite$^\text{(HN)}_{\text{horn}}$ TBox

- basic concept inclusion
  \[ \geq 1 \text{hasPublication} \sqsubseteq \text{Publication} \]
- role inclusion
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  \geq 1 \text{hasPublication} \sqsubseteq \text{Publication}
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  \]
- number restrictions
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  \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper}
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- horn inclusion
  \[
  \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}
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A $DL$-$\text{Lite}_{horn}^{(HN)}$ TBox

- basic concept inclusion
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  \[ \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper} \]
- horn inclusion
  \[ \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent} \]
- local functionality assertion
  \[ \text{PhDStudent} \sqcap \geq 2 \text{teaches} \sqsubseteq \bot \]
A \( DL-Lite_{hor}^{(HN)} \) TBox

- basic concept inclusion
  \( \geq 1 \text{hasPublication}^- \sqsubseteq \text{Publication} \)

- role inclusion
  \( \text{hasConfPaper} \sqsubseteq \text{hasPublication} \)

- number restrictions
  \( \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper} \)

- horn inclusion
  \( \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent} \)

- local functionality assertion
  \( \text{PhDStudent} \sqcap \geq 2 \text{teaches} \sqsubseteq \bot \)

- global functionality assertion
  \( \geq 2 \text{teaches}^- \sqsubseteq \bot \)
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Motivation

The DL $DL-Lite^{(HN)}_{horn}$

Knowledge Base Satisfiability

Query Answering

Conclusions

Knowledge Base Satisfiability

Negative inclusions may lead to unsatisfiability:

$T: Student \sqcap Professor \sqsubseteq \bot$

$A: PhDStudent(john), Professor(john)$

$T: Dis(teaches, attends)$

$A: teaches(john, cl), attends(john, cl)$

$T: PhDStudent \sqsubseteq \geq 2 teaches \sqsubseteq \bot$

$A: PhDStudent(john), teaches(john, cl), teaches(john, db)$

$\Rightarrow$ We need to calculate closure of NIs w.r.t. PIs
Knowledge Base Satisfiability

Negative inclusions may lead to unsatisfiability:

- $\mathcal{T}$: Student ⊓ Professor ⊑ ⊥, PhDStudent ⊑ Student
- $\mathcal{A}$: PhDStudent(john), Professor(john)
Knowledge Base Satisfiability

Negative inclusions may lead to unsatisfiability:

- $\mathcal{T} : \text{Student} \sqcap \text{Professor} \sqsubseteq \bot$, $\text{PhDStudent} \sqsubseteq \text{Student}$
  
  $\mathcal{A} : \text{PhDStudent}(\text{john}), \text{Professor}(\text{john})$

- $\mathcal{T} : \text{Dis}(\text{teaches}, \text{attends})$
  
  $\mathcal{A} : \text{teaches}(\text{john}, \text{cl}), \text{attends}(\text{john}, \text{cl})$
Knowledge Base Satisfiability

Negative inclusions may lead to unsatisfiability:

- $\mathcal{T} : Student \sqcap Professor \sqsubseteq \bot, \ PhDStudent \sqsubseteq Student$
  $\mathcal{A} : PhDStudent(john), \ Professor(john)$

- $\mathcal{T} : \text{Dis}(teaches, \ attends)$
  $\mathcal{A} : teaches(john, \ cl), \ attends(john, \ cl)$

- $\mathcal{T} : PhDStudent \sqcap \geq 2 \ teaches \sqsubseteq \bot$
  $\mathcal{A} : PhDStudent(john), \ teaches(john, \ cl), \ teaches(john, \ db)$
Knowledge Base Satisfiability

Negative inclusions may lead to unsatisfiability:

- $\mathcal{T} : \text{Student} \sqcap \text{Professor} \sqsubseteq \bot$, $\text{PhDStudent} \sqsubseteq \text{Student}$
  $\mathcal{A} : \text{PhDStudent}(\text{john}), \text{Professor}(\text{john})$

- $\mathcal{T} : \text{Dis(teaches, attends)}$
  $\mathcal{A} : \text{teaches(}\text{john, cl}), \text{attends(}\text{john, cl})$

- $\mathcal{T} : \text{PhDStudent} \sqcap \geq 2 \text{ teaches} \sqsubseteq \bot$
  $\mathcal{A} : \text{PhDStudent}(\text{john}), \text{teaches(}\text{john, cl}), \text{teaches(}\text{john, db})$

$\Rightarrow$ We need to calculate closure of NIs w.r.t. PIs
Knowledge Base Satisfiability Algorithm

We reduce the problem to FOL query evaluation.
Knowledge Base Satisfiability Algorithm

We reduce the problem to FOL query evaluation.

Algorithm for checking KB satisfiability

1. Calculate the closure of NIs.
2. Translate the closure into a UCQ $q_{\text{unsat}}$ asking for violation of some NI.
3. Evaluate $q_{\text{unsat}}$ over the ABox (viewed as a DB).
   - if $\text{eval}(q_{\text{unsat}}, DB_A) = \emptyset$, then the KB is satisfiable;
   - otherwise the KB is unsatisfiable.
Closure of Negative Inclusions

Closure of NIs $\text{cln}(T)$ w.r.t. Pls

- every NI is in $\text{cln}(T)$. 
Closure of Negative Inclusions

Closure of NIs $\text{cln}(\mathcal{T})$ w.r.t. PIs

- every NI is in $\text{cln}(\mathcal{T})$.

- $\text{cln}(\mathcal{T}) : \begin{array}{l}
    \text{Professor} \sqcap \text{PhDStudent} \sqsubseteq \bot \\
    \mathcal{T} : \begin{array}{l}
        \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}
    \end{array}
\end{array}$ \Rightarrow
Closure of Negative Inclusions

Closure of NIs $\text{cln}(\mathcal{T})$ w.r.t. Pls

- every NI is in $\text{cln}(\mathcal{T})$.

- $\text{cln}(\mathcal{T}) : \text{Professor} \sqcap \text{PhDStudent} \sqsubseteq \bot$
- $\mathcal{T} : \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}$

$\Rightarrow$

add to $\text{cln}(\mathcal{T}) : \text{Professor} \sqcap \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \bot$
Closure of Negative Inclusions

Closure of NIs $cln(T)$ w.r.t. PIs

- every NI is in $cln(T)$.

- $cln(T) : \text{Professor} \sqcap \text{PhDStudent} \sqsubseteq \bot$
  
  $T : \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}$

  add to $cln(T) : \text{Professor} \sqcap \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \bot$

- $cln(T) : \text{PhDStudent} \sqcap \geq 2 \text{teaches} \sqsubseteq \bot$
  
  $T : \text{FullProfessor} \sqsubseteq \geq 3 \text{teaches}$

  $\Rightarrow$
Closure of Negative Inclusions

Closure of NIs $\text{cln}(\mathcal{T})$ w.r.t. PIs

- every NI is in $\text{cln}(\mathcal{T})$.

- $\text{cln}(\mathcal{T}) : \begin{align*}
    & Professor \sqcap PhDStudent \sqsubseteq \bot \\
    \mathcal{T} : & Student \sqcap \geq 1 \text{ teaches} \sqsubseteq PhDStudent \\
    \end{align*} \Rightarrow \\
    \text{add to } \text{cln}(\mathcal{T}) : Professor \sqcap Student \sqcap \geq 1 \text{ teaches} \sqsubseteq \bot$

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Closure of Negative Inclusions

Closure of NIs \( cln(T) \) w.r.t. Pls

- every NI is in \( cln(T) \).

\[
\begin{align*}
cln(T) : & \quad \text{Professor} \sqcap \text{PhDStudent} \sqsubseteq \bot \\
T : & \quad \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent} \\
\Rightarrow & \quad \text{add to } cln(T) : \quad \text{Professor} \sqcap \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \bot
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\]

\[
\begin{align*}
cln(T) : & \quad \text{Professor} \sqcap \geq 1 \text{attends} \sqsubseteq \bot \\
T : & \quad \text{registeredTo} \sqsubseteq \text{attends} \\
\Rightarrow & \quad \text{add to } cln(T)
\end{align*}
\]
Closure of Negative Inclusions

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&\text{T} : \begin{cases}
\text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent} \\
\end{cases}
\end{align*} \Rightarrow

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Closure of Negative Inclusions

Closure of NIs $\text{cln} (\mathcal{T})$ w.r.t. PIs

- every NI is in $\text{cln} (\mathcal{T})$.

- $\text{cln} (\mathcal{T}) : \text{Professor} \sqcap \text{PhDStudent} \sqsubseteq \bot$
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  add to $\text{cln} (\mathcal{T}) : \text{Professor} \sqcap \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \bot$

- $\text{cln} (\mathcal{T}) : \text{PhDStudent} \sqcap \geq 2 \text{teaches} \sqsubseteq \bot$
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  add to $\text{cln} (\mathcal{T}) : \text{Professor} \sqcap \geq 1 \text{registeredTo} \sqsubseteq \bot$

- \ldots
Translation to FOL Queries

- $Professor \sqcap Student \sqsubseteq \bot \Rightarrow \exists x. Professor(x) \land Student(x)$. 
Translation to FOL Queries

- \( \text{Professor} \sqcap \text{Student} \sqsubseteq \bot \Rightarrow \exists x. \text{Professor}(x) \land \text{Student}(x) \).

- \( \geq 2 \text{teaches}^- \sqsubseteq \bot \Rightarrow \exists x_1, x_2, y. \text{teaches}(x_1, y) \land \text{teaches}(x_2, y) \land x_1 \neq x_2 \).
Translation to FOL Queries

- \( \text{Professor} \sqcap \text{Student} \sqsubseteq \bot \Rightarrow \exists x. \text{Professor}(x) \land \text{Student}(x). \)

- \( \geq 2 \text{ teaches} \sqsubseteq \bot \Rightarrow \exists x_1, x_2, y. \text{teaches}(x_1, y) \land \text{teaches}(x_2, y) \land x_1 \neq x_2. \)

- \( \text{Dis(attends, teaches)} \Rightarrow \exists x, y. \text{attends}(x, y) \land \text{teaches}(x, y). \)
KB Satisfiability: Complexity of the Algorithm

- **Optimal** data complexity: in $AC^0$ (follows from FOL rewritability)
**KB Satisfiability: Complexity of the Algorithm**

- **Optimal** data complexity: in $\mathcal{AC}^0$ (follows from FOL rewritability)

- Combined complexity: exponential
  - Worst case: the size of $\text{cln}(\mathcal{T})$ is **exponential** in the size of the TBox
    \[ \mathcal{T} = \{ A'_1 \sqsubseteq A_1, \ldots, A'_n \sqsubseteq A_n, A_1 \sqcap \cdots \sqcap A_n \sqsubseteq \bot \}. \]
Motivation

The DL DL-Lite\textsubscript{(HN)}

Knowledge Base Satisfiability

Query Answering

Conclusions

KB Satisfiability: Complexity of the Algorithm

- **Optimal** data complexity: in $AC^0$ (follows from FOL rewritability)

- Combined complexity: exponential
  - Worst case: the size of $cln(\mathcal{T})$ is exponential in the size of the TBox
  \[
  \mathcal{T} = \{ A_1' \sqsubseteq A_1, \ldots, A_n' \sqsubseteq A_n, A_1 \sqcap \cdots \sqcap A_n \sqsubseteq \bot \}. \]

Notice, that the problem is $\text{PTime}$ [ACKZ09].
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Query Answering: Example

$q(x) ← hasPublication(x, y) \land Publication(y)$

$\text{TBox } \mathcal{T} = \{ $

$\geq 1 \text{hasPublication} \sqsubseteq \text{Publication}$

$\text{hasConfPaper} \sqsubseteq \text{hasPublication}$

$\text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper}$

$\text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}$

$\}$
Query Answering: Example

\[ q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \]

\[
\geq 1 \text{hasPublication} \sqsubseteq \text{Publication} \\
\text{hasConfPaper} \sqsubseteq \text{hasPublication} \\
\text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper} \\
\text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}
\]
Query Answering: Example

\[ q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \]

\[ \downarrow \geq 1 \text{hasPublication}^{-} \sqsubseteq \text{Publication} \]

\[ q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1\text{hasPublication}^{-}(y) \]
Query Answering: Example

\[ q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \]
\[ \Downarrow \quad \geq 1\text{hasPublication}^- \sqsubseteq \text{Publication} \]

\[ q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1\text{hasPublication}^-(y) \]
\[ \Downarrow \quad \text{unify the atoms} \]
\[ \text{hasPublication}(x, y) \]

\[ \geq 1\text{hasPublication}^- \sqsubseteq \text{Publication} \]
\[ \text{hasConfPaper} \sqsubseteq \text{hasPublication} \]
\[ \text{PhDStudent} \sqsubseteq \geq 2\text{hasConfPaper} \]
\[ \text{Student} \sqcap \geq 1\text{teaches} \sqsubseteq \text{PhDStudent} \]

Motivation  The DL $DL-Lite_{\text{horn}}^{(HN)}$  Knowledge Base Satisfiability  Query Answering  Conclusions
Query Answering: Example

\[ q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \]
\[ \downarrow \quad \geq 1 \text{hasPublication}^- \sqsubseteq \text{Publication} \]

\[ q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1 \text{hasPublication}^-(y) \]
\[ \downarrow \quad \text{unify the atoms} \]
\[ \text{hasPublication}(x, y) \]
\[ \downarrow \quad \text{remove unbound variables} \]

\[ q_3(x) \leftarrow E_1 \text{hasPublication}(x) \]

\[ \geq 1 \text{hasPublication}^- \sqsubseteq \text{Publication} \]
\[ \text{hasConfPaper} \sqsubseteq \text{hasPublication} \]
\[ \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper} \]
\[ \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent} \]
Query Answering: Example

\[ q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \]
\[ \downarrow \geq 1 \text{hasPublication} \sqsubseteq \text{Publication} \]

\[ q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1\text{hasPublication}^-(y) \]
\[ \downarrow \text{unify the atoms} \]
\[ \text{hasPublication}(x, y) \]
\[ \downarrow \text{remove unbound variables} \]

\[ q_3(x) \leftarrow E_1\text{hasPublication}(x) \]
\[ \downarrow \text{hasConfPaper} \sqsubseteq \text{hasPublication} \]

\[ q_4(x) \leftarrow E_1\text{hasConfPaper}(x) \]
Query Answering: Example

$q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y)$

$\downarrow \geq 1 \text{hasPublication}^{-} \sqsubseteq \text{Publication}$

$q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1 \text{hasPublication}^{-}(y)$

$\downarrow$ unify the atoms

$\text{hasPublication}(x, y)$

$\downarrow$ remove unbound variables

$q_3(x) \leftarrow E_1 \text{hasPublication}(x)$

$\downarrow \text{hasConfPaper} \sqsubseteq \text{hasPublication}$

$q_4(x) \leftarrow E_1 \text{hasConfPaper}(x)$

$\downarrow \geq 2 \text{hasConfPaper} \sqsubseteq \geq 1 \text{hasConfPaper}$

$q_5(x) \leftarrow E_2 \text{hasConfPaper}(x)$
Query Answering: Example

\[ q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \]
\[ \downarrow \ \geq 1 \text{hasPublication} \sqsubseteq \text{Publication} \]

\[ q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1\text{hasPublication}(y) \]
\[ \downarrow \text{unify the atoms} \]
\[ \text{hasPublication}(x, y) \]
\[ \downarrow \text{remove unbound variables} \]

\[ q_3(x) \leftarrow E_1\text{hasPublication}(x) \]
\[ \downarrow \text{hasConfPaper} \sqsubseteq \text{hasPublication} \]

\[ q_4(x) \leftarrow E_1\text{hasConfPaper}(x) \]
\[ \downarrow \geq 2 \text{hasConfPaper} \sqsubseteq \geq 1 \text{hasConfPaper} \]

\[ q_5(x) \leftarrow E_2\text{hasConfPaper}(x) \]
\[ \downarrow \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper} \]

\[ q_6(x) \leftarrow \text{PhDStudent}(x) \]

\[ \geq 1 \text{hasPublication} \sqsubseteq \text{Publication} \]
\[ \text{hasConfPaper} \sqsubseteq \text{hasPublication} \]
\[ \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper} \]
\[ \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent} \]
Query Answering: Example

\(q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y)\)
\[\downarrow \geq 1 \text{hasPublication}^{-} \sqsubseteq \text{Publication}\]

\(q_2(x) \leftarrow \text{hasPublication}(x, y) \land E_1 \text{hasPublication}^{-}(y)\)
\[\downarrow \text{unify the atoms}\]
\(\text{hasPublication}(x, y)\)
\[\downarrow \text{remove unbound variables}\]

\(q_3(x) \leftarrow E_1 \text{hasPublication}(x)\)
\[\downarrow \text{hasConfPaper} \sqsubseteq \text{hasPublication}\]

\(q_4(x) \leftarrow E_1 \text{hasConfPaper}(x)\)
\[\downarrow \geq 2 \text{hasConfPaper} \sqsubseteq \geq 1 \text{hasConfPaper}\]

\(q_5(x) \leftarrow E_2 \text{hasConfPaper}(x)\)
\[\downarrow \text{PhDStudent} \sqsubseteq \geq 2 \text{hasConfPaper}\]

\(q_6(x) \leftarrow \text{PhDStudent}(x)\)
\[\downarrow \text{Student} \sqcap \geq 1 \text{teaches} \sqsubseteq \text{PhDStudent}\]

\(q_7(x) \leftarrow \text{Student}(x) \land E_1 \text{teaches}(x)\)
Motivation

The DL DL-Lite$_{horn}$(HN)

Knowledge Base Satisfiability

Query Answering

Conclusions

Query Answering Algorithm

1. Compute the rewriting of the initial query, a UCQ.
   - Application of PIs to query atoms.
     \[
     q(x) \leftarrow \text{hasPublication}(x, y) \land \text{Publication}(y) \\
     \Downarrow \geq 1 \text{hasPublication}^{-} \sqsubseteq \text{Publication} \\
     q'(x) \leftarrow \text{hasPublication}(x, y) \land E_1\text{hasPublication}^{-}(y)
     \]
   - Unification of query atoms.
     \[
     q(x) \leftarrow \text{hasPublication}(x, y) \land E_1\text{hasPublication}^{-}(y) \\
     \Downarrow \text{unify} \\
     q'(x) \leftarrow \text{hasPublication}(x, y)
     \]

2. Evaluate the obtained UCQ over the ABox viewed as a DB.
What Is New?

Differences w.r.t. the algorithm for simpler variants of $DL-Lite$
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Differences w.r.t. the algorithm for simpler variants of $DL$-$Lite$

- Number restrictions imply new inclusions:
What Is New?

Differences w.r.t. the algorithm for simpler variants of DL-Lite

- Number restrictions imply new inclusions: extend the TBox
  - $\geq k R \sqsubseteq \geq k' R$, where $k > k'$
  - $\geq k R \sqsubseteq \geq k R'$ for each subrole $R$ of $R'$
What Is New?

Differences w.r.t. the algorithm for simpler variants of \textit{DL-Lite}

- Number restrictions imply new inclusions: extend the TBox
  - $\geq k \, R \sqsubseteq \geq k' \, R$, where $k > k'$
  - $\geq k \, R \sqsubseteq \geq k \, R'$ for each subrole $R$ of $R'$

- Introduce new predicates $E_k R(x)$ to handle inequalities implied by number restrictions $\geq k \, R$
What Is New?

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• Introduce new predicates $E_k R(x)$ to handle inequalities implied by number restrictions $\geq k R$

• Unification for newly introduced predicates
  ▶ $P(x,y)$ unifies with $P(z,w)$, $E_1 P(z)$, or $E_1 P^-(w)$
  ▶ $E_k R(x)$ unifies with $E_1 R^-(\_)$
    Notice that $E_1 R^-(\_)$ stands for $R(\_,\_)$
What Is New?

Differences w.r.t. the algorithm for simpler variants of \textit{DL-Lite}:

- Number restrictions imply new inclusions: extend the TBox
  \[ \geq k \, R \sqsubseteq \geq k' \, R, \text{ where } k > k' \]
  \[ \geq k \, R \sqsubseteq \geq k' \, R' \text{ for each subrole } R \text{ of } R' \]

- Introduce new predicates \( E_k R(x) \) to handle inequalities implied by number restrictions \( \geq k \, R \)

- Unification for newly introduced predicates
  \[ P(x,y) \text{ unifies with } P(z,w), \ E_1 P(z), \text{ or } E_1 P^-(w) \]
  \[ E_k R(x) \text{ unifies with } E_1 R^-(\_\_) \]
  Notice that \( E_1 R^-(\_\_) \) stands for \( R(\_,\_\_) \)

- Horn inclusions increase the length of the query
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• Number restrictions imply new inclusions: extend the TBox
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• Horn inclusions increase the length of the query
  ▶ remove duplicated atoms
What Is New?

Differences w.r.t. the algorithm for simpler variants of \( DL-Lite \)

- Number restrictions imply new inclusions: extend the TBox
  - \( \geq k \, R \sqsubseteq \geq k' \, R \), where \( k > k' \)
  - \( \geq k \, R \sqsubseteq \geq k \, R' \) for each subrole \( R \) of \( R' \)

- Introduce new predicates \( E_k \, R(x) \) to handle inequalities implied by number restrictions \( \geq k \, R \)

- Unification for newly introduced predicates
  - \( P(x,y) \) unifies with \( P(z,w) \), \( E_1 \, P(z) \), or \( E_1 \, P^-(w) \)
  - \( E_k \, R(x) \) unifies with \( E_1 \, R^-(\_\_) \)
    Notice that \( E_1 \, R^-(\_\_) \) stands for \( R(\_,\_) \)

- Horn inclusions increase the length of the query
  - remove duplicated atoms
  - remove \( E_{k'} \, R(z) \), if \( E_k \, R(x) \) occurs in the query and \( k > k' \)
Query Answering: Complexity of the Algorithm

- **Optimal** data complexity: in $\text{AC}^0$
Query Answering: Complexity of the Algorithm

- **Optimal** data complexity: in $AC^0$

- Combined complexity: in $NP$

Note that the size of the rewriting is exponential already w.r.t. the size of the TBox.
Outline

1. Motivation
2. The DL $DL-Lite_{horn}^{HN}$
3. Knowledge Base Satisfiability
4. Query Answering
5. Conclusions
Conclusion

- We reduced knowledge satisfiability and query answering in $DL-Lite_{horn}^{(\mathcal{HN})}$ to FOL evaluation.
  - Practically implementable algorithms.
  - We can rely on relational database technology for managing the data and query evaluation.
Conclusion

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  - Practically implementable algorithms.
  - We can rely on relational database technology for managing the data and query evaluation.

- The computational complexity of the algorithms is optimal w.r.t. data complexity:
  - in $AC^0$. 
Conclusion

- We reduced knowledge satisfiability and query answering in \( DL-Lite_{\text{horn}}^{(\mathcal{HN})} \) to FOL evaluation.
  - Practically implementable algorithms.
  - We can rely on relational database technology for managing the data and query evaluation.

- The computational complexity of the algorithms is optimal w.r.t. data complexity:
  - in \( AC^0 \).

- Future work:
  - Implement the developed algorithms.
  - Study optimization techniques for the algorithm.
  - Extend the practical algorithm to positive existential queries.
Thank you for your attention!