Query-Based Entailment and Inseparability for $\mathcal{ALC}$ Ontologies

Elena Botoeva

Faculty of Computer Science, Free University of Bozen-Bolzano, Italy

joint work with Carsten Lutz, Vladislav Ryzhikov, Frank Wolter and Michael Zakharyaschev
Query Inseparability for Ontologies

By an ontology $\mathcal{O}$ we mean

- a knowledge base $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, or
- a TBox $\mathcal{T}$.

Query answering over ontologies is an important reasoning task.
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Ontologies \( \mathcal{O}_1 \) and \( \mathcal{O}_2 \) are **query-inseparable** when we **cannot distinguish** between them by means of queries.
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**Applications**

- extracting **modules**
- comparing **versions** of an ontology
- forgetting some symbols from an ontology
- exchanging **knowledge**
Query Inseparability for Knowledge Bases

Consider a class of queries $Q \in \{\text{CQ, UCQ}\}$, and a signature $\Sigma$ of concept and role names.

KBs $\mathcal{K}_1 = (\mathcal{T}_1, \mathcal{A}_1)$ and $\mathcal{K}_2 = (\mathcal{T}_2, \mathcal{A}_2)$ are $\Sigma$-inseparable, $\mathcal{K}_1 \equiv_{\Sigma}^Q \mathcal{K}_2$, if

\[
\mathcal{K}_1 \models q(a) \iff \mathcal{K}_2 \models q(a)
\]

for all $\Sigma$-queries $q \in Q$ and all individuals $a$ in $\mathcal{K}_1$ and $\mathcal{K}_2$. 
Query Inseparability for Knowledge Bases

Consider a class of queries \( Q \in \{ \text{CQ, UCQ} \} \), and a signature \( \Sigma \) of concept and role names.

KBs \( \mathcal{K}_1 = (T_1, A_1) \) and \( \mathcal{K}_2 = (T_2, A_2) \) are \( \Sigma - Q \) inseparable, \( \mathcal{K}_1 \equiv^Q_\Sigma \mathcal{K}_2 \), if

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for all \( \Sigma \)-queries \( q \in Q \) and all individuals \( a \) in \( \mathcal{K}_1 \) and \( \mathcal{K}_2 \).

\[ \Sigma = \{ \text{spots} \} \]
Examples

Query inseparability is different from logical equivalence:

\[ K_1 = ( \{ A \sqsubseteq B \}, \{ A(a) \} ) \quad \text{and} \quad K_2 = ( \emptyset, \{ A(a), B(a) \} ) \]

\[ K_1 \nleq K_2 \quad \text{but} \quad K_1 \equiv^{(U)CQ} \ K_2 \]
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UCQ-inseparability and CQ-inseparability are distinct:

\[ \mathcal{K}_1 = ( \{ A \sqsubseteq B \sqcup C \}, \{ A(a) \} ) \quad \text{and} \quad \mathcal{K}_2 = ( \emptyset, \{ A(a) \} ) \]
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Signature makes a difference:

\[ \mathcal{K}_1 \equiv^{UCQ}_{\{A\}} \mathcal{K}_2 \]
Consider signatures: $\Sigma_1$ for ABoxes and $\Sigma_2$ for queries.

TBoxes $\mathcal{T}_1$ and $\mathcal{T}_2$ are $(\Sigma_1, \Sigma_2)$-(U)CQ inseparable, $\mathcal{T}_1 \equiv^{(U)\text{CQ}}_{(\Sigma_1, \Sigma_2)} \mathcal{T}_2$, if

$$(\mathcal{T}_1, \mathcal{A}) \equiv^{(U)\text{CQ}}_{\Sigma_2} (\mathcal{T}_2, \mathcal{A})$$

for all $\Sigma_1$-ABoxes $\mathcal{A}$. 
Main Results

KBs:
- (rooted) CQ-inseparability **undecidable** for $\mathcal{ALC}$.
- (rooted) UCQ-inseparability **2ExpTime-complete**.

TBoxes:
- (rooted) CQ-inseparability **undecidable** for $\mathcal{ALC}$.
- CQ/UCQ-inseparability **2ExpTime-complete** for $\mathcal{Horn-ALC}$.
- rooted CQ/UCQ-inseparability **ExpTime-complete** for $\mathcal{Horn-ALC}$.
See you at the poster!