When are Description Logic Knowledge Bases Indistinguishable?

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Query Entailment and Inseparability

Let $\mathcal{K}_1$ and $\mathcal{K}_2$ be knowledge bases: $\mathcal{K}_i = (\mathcal{T}_i, \mathcal{A}_i)$

$\mathcal{K}_1$ **Σ-query entails** $\mathcal{K}_2$ if for each **CQ** $\mathbf{q}(\vec{x})$ over Σ and each tuple $\vec{a}$ in ind($\mathcal{K}_2$)

$$\mathcal{K}_2 \models \mathbf{q}(\vec{a}) \text{ implies } \mathcal{K}_1 \models \mathbf{q}(\vec{a})$$

$\mathcal{K}_1$ and $\mathcal{K}_2$ are **Σ-query inseparable**, $\mathcal{K}_1 \equiv_\Sigma \mathcal{K}_2$, if

$\mathcal{K}_1$ Σ-query entails $\mathcal{K}_2$ and $\mathcal{K}_2$ Σ-query entails $\mathcal{K}_1$

Versioning

$\mathcal{K}_2$ is a Σ-query version of $\mathcal{K}_1$ if $\mathcal{K}_2 \equiv_\Sigma \mathcal{K}_1$

Modularisation

$\mathcal{K}' \subseteq \mathcal{K}$ is a Σ-module of $\mathcal{K}$ if $\mathcal{K}' \equiv_\Sigma \mathcal{K}$

Knowledge Exchange

$\mathcal{K}_2$ is a universal UCQ-solution for $\mathcal{K}_1$ under $\mathcal{T}_{12}$ if $\mathcal{K}_2 \equiv_{\Sigma_2} \mathcal{K}_1 \cup \mathcal{T}_{12}$

Forgetting

$\mathcal{K}'$ is the result of forgetting Σ in $\mathcal{K}$ if sig($\mathcal{K}'$) $\subseteq$ sig($\mathcal{K}$) $\setminus \Sigma$ and $\mathcal{K}' \equiv_{\text{sig}(\mathcal{K})\setminus\Sigma} \mathcal{K}$

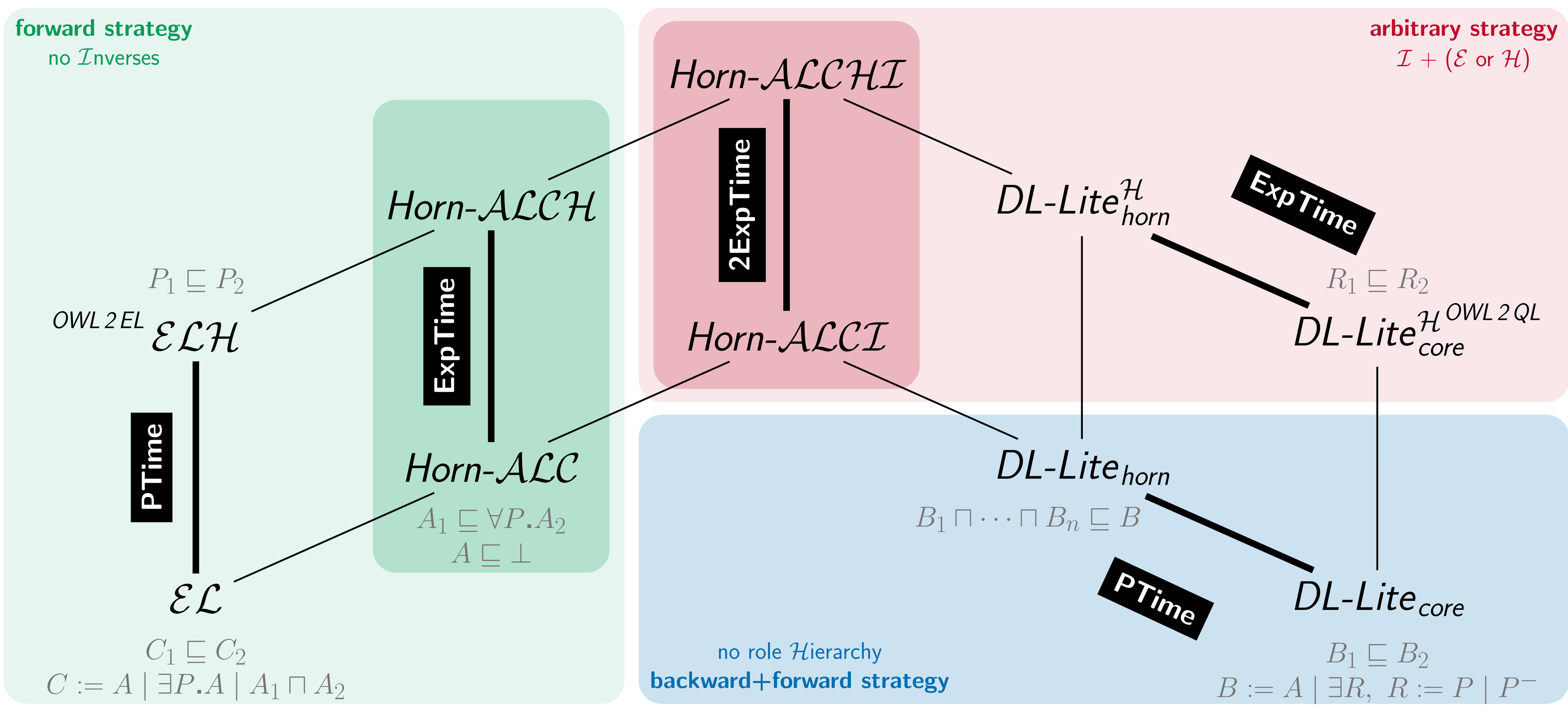
The Landscape of Logics

Horn-$\mathcal{ALCHI}$ normal form:

$C \sqsubseteq A$,
$C \sqsubseteq \forall R.A$,
$A \sqsubseteq \exists R.C$,
$\exists R.C \sqsubseteq A$,
$A_1 \sqcap A_2 \sqsubseteq A$,
$R_1 \sqsubseteq R_2$,

where

A, A_1, A_2 are concept names
C is a concept name or $\top$
R, R_1, R_2 are role names or their inverses



Query Entailment

Let $\mathcal{K}_1 = (\mathcal{T}_1, \{A(a)\})$ and $\mathcal{K}_2 = (\mathcal{T}_2, \{A(a)\})$

$$\mathcal{T}_1 = \left\{ \begin{array}{c} A \sqsubseteq \exists S.(\exists R.A \sqcap \exists T.\exists W.\exists Q) \\ \exists Q^- \sqsubseteq \exists Q,\ S \sqsubseteq S_1,\ T \sqsubseteq T_1,\ W \sqsubseteq W_1 \end{array} \right\}$$

$$\mathcal{T}_2 = \left\{ \begin{array}{c} A \sqsubseteq \exists P.B,\ \exists Q^- \sqsubseteq \exists Q \\ B \sqsubseteq \exists R^-.(\exists S^-.B \sqcap \exists T) \\ \exists T^- \sqsubseteq \exists W.(\exists W_1^-.\exists T_1^-.\exists S_1^- \sqcap \exists Q) \end{array} \right\}$$

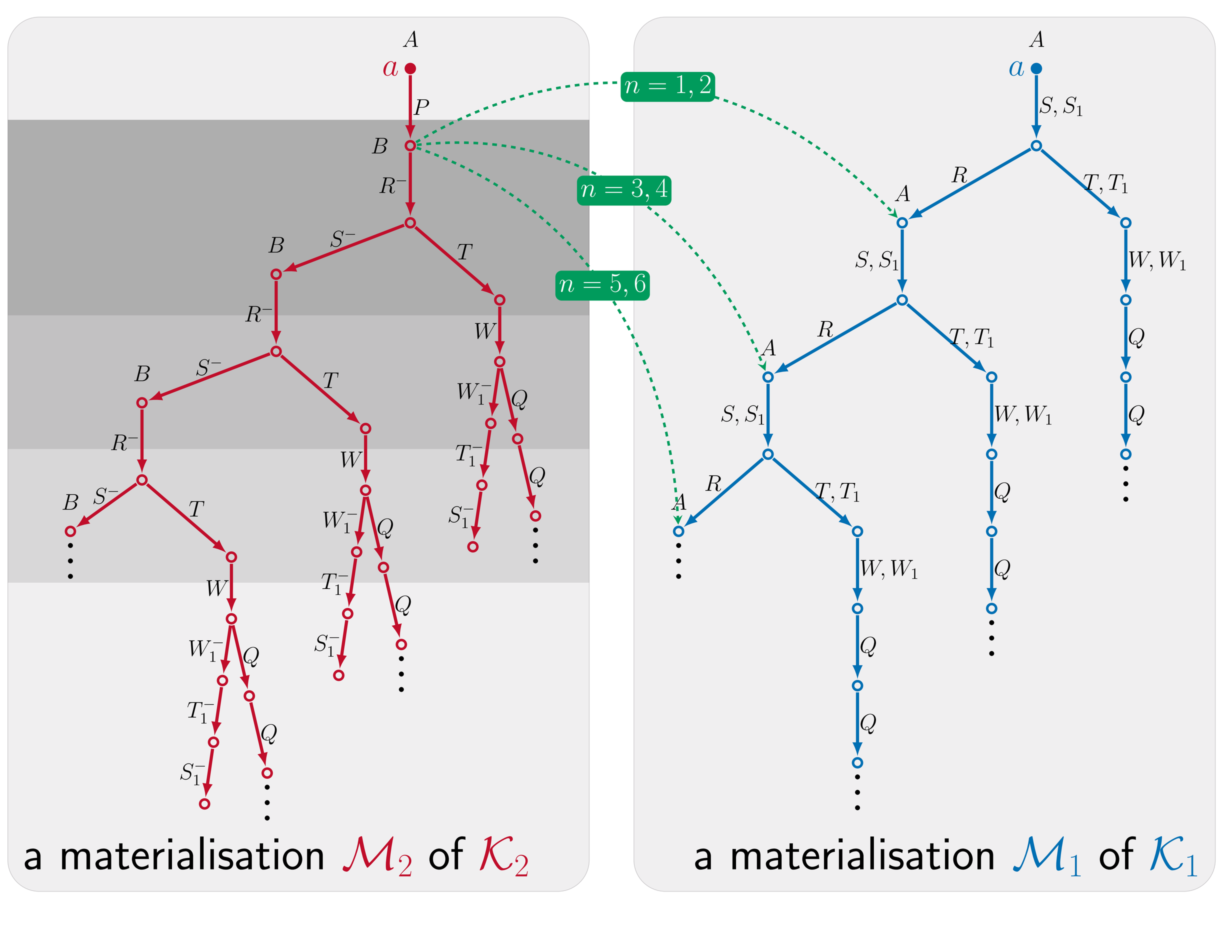
$$\Sigma = \{ R, S, S_1, T, T_1, Q, W, W_1 \}$$

$\mathcal{K}_1$ Σ-query entails $\mathcal{K}_2$

Σ-Homomorphisms

$\mathcal{M}_2$ is finitely Σ-homomorphically embeddable into $\mathcal{M}_1$

(each **finite** subinterpretation of $\mathcal{M}_2$ can be Σ-homomorphically mapped to $\mathcal{M}_1$)



a materialisation $\mathcal{M}_2$ of $\mathcal{K}_2$ a materialisation $\mathcal{M}_1$ of $\mathcal{K}_1$

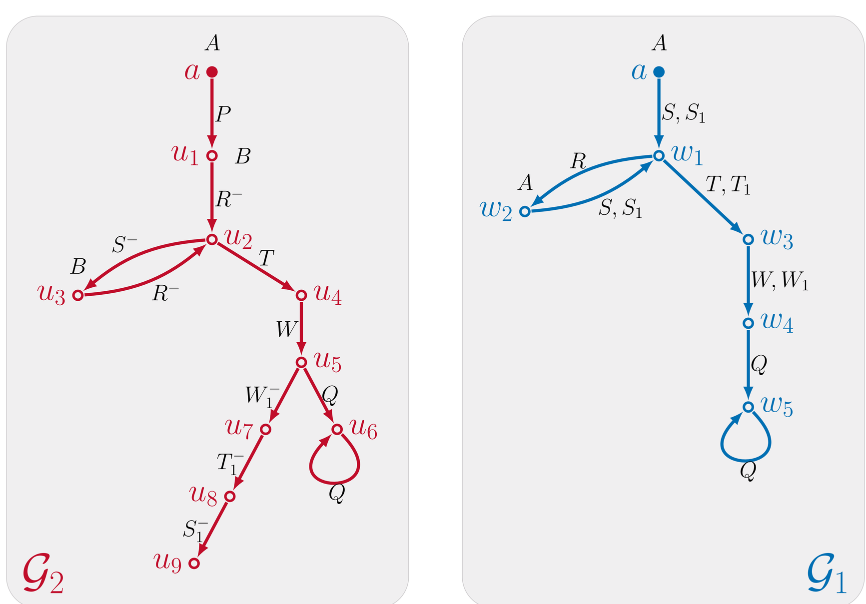
$\mathcal{M}$ is a **materialisation** of $\mathcal{K}$ if for each $\mathbf{q}(\vec{x})$ and each $\vec{a}$

$$\mathcal{K} \models \mathbf{q}(\vec{a}) \quad \text{iff} \quad \mathcal{M} \models \mathbf{q}(\vec{a})$$

Games and Generating Structures

$\mathcal{G}_i$ is a **generating structure** for $\mathcal{M}_i$ if $\mathcal{M}_i$ is the unravelling of $\mathcal{G}_i$

(used in the combined approach to query answering)



Forward strategies (no inverse roles):
Player 1 can only move forward in $\mathcal{M}_1$

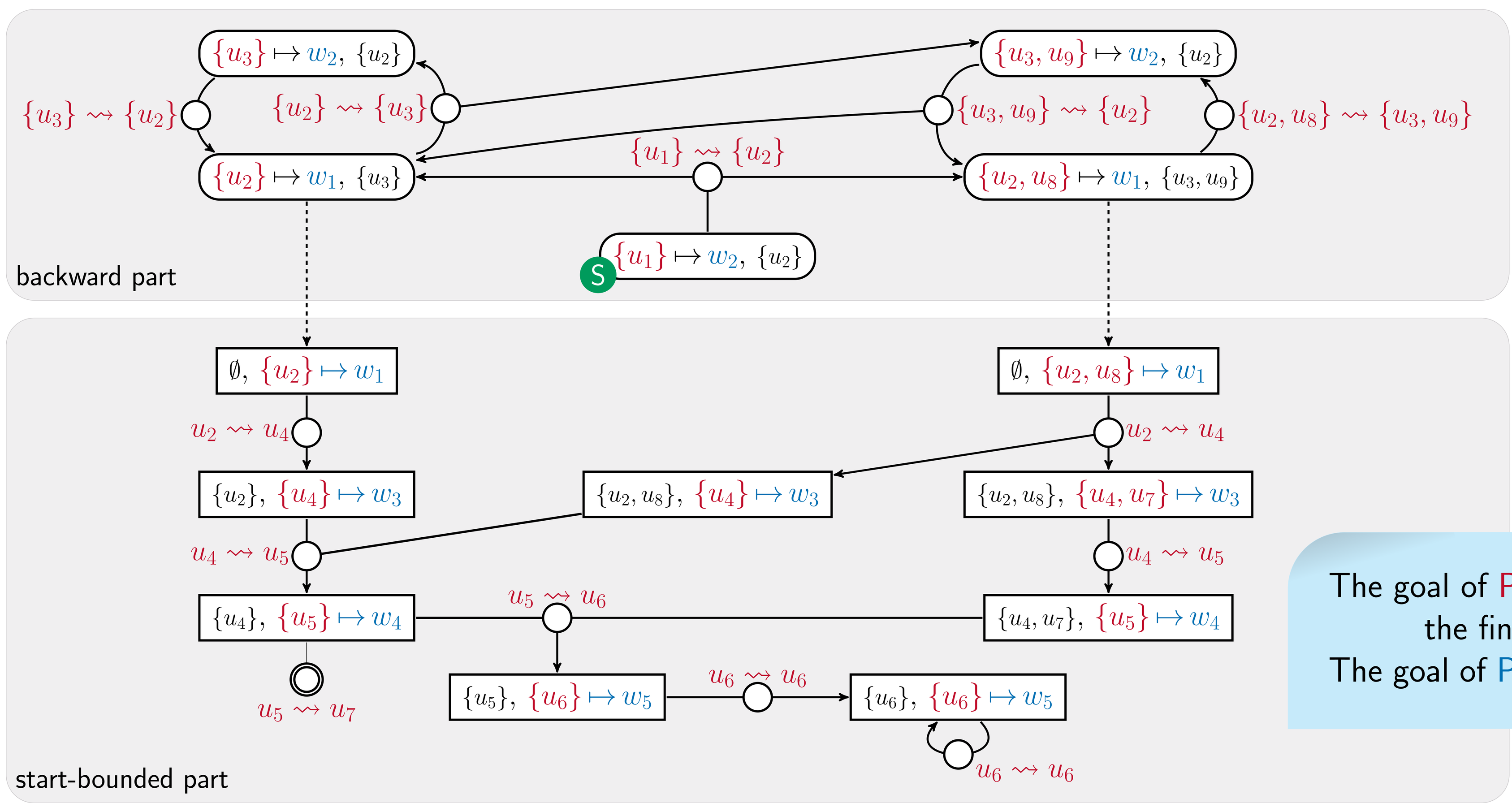
General strategies are combinations of
a **backward** strategy
Player 1 can only move backwards in $\mathcal{M}_1$
and a number of **start-bounded** strategies
Player 1 cannot move in $\mathcal{M}_1$ closer to the ABox than the initial response

Player 2 has no winning strategy against Player 1 in an elaborate game on $\mathcal{G}_2$ and $\mathcal{G}_1$

$\Updownarrow$

Player 2 has no winning strategy against Player 1 in the naive game on $\mathcal{M}_2$ and $\mathcal{M}_1$

Let's play?



The goal of Player 2 is to reach the final state (double circle)
The goal of Player 1 is to avoid it