Knowledge Base Exchange

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Description Logics Workshop
14 July 2011, Barcelona

Outline

1. Knowledge Base Exchange

2. Techniques for Deciding Knowledge Base Exchange

3. Conclusions

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Knowledge Base Exchange

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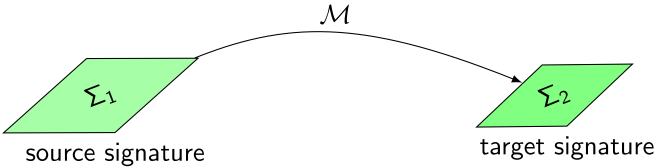

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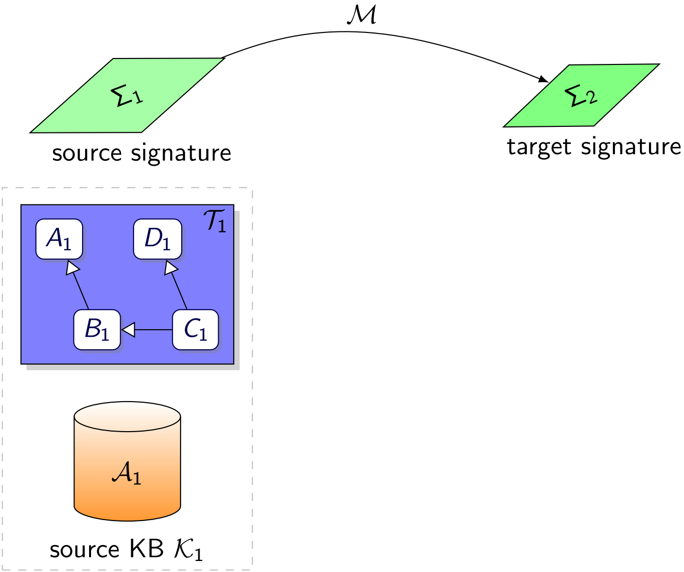

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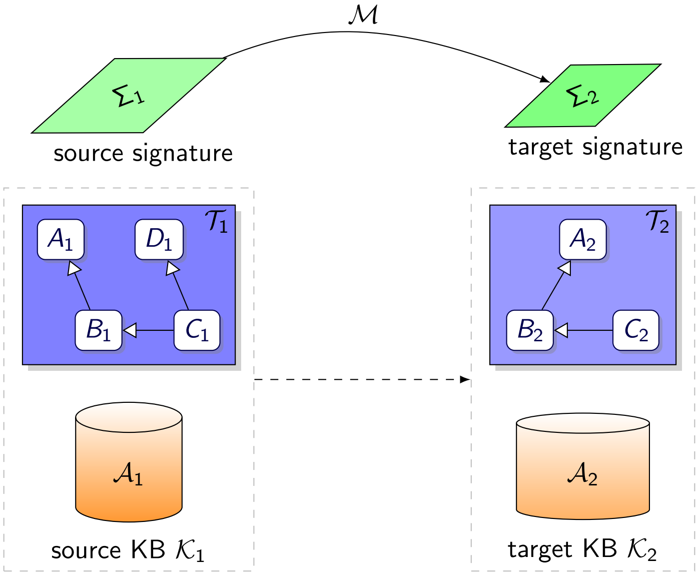

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- A *mapping* is a tuple $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$, where
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  - $\mathcal{T}_{12}$ is a TBox with assertions of the form
    - $C_1 \sqsubseteq C_2$, where C_1 is a concept over Σ_1, C_2 is a concept over Σ_2,
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- Let $\mathcal{I}$ be an interpretation of Σ_1 and $\mathcal{J}$ an interpretation of Σ_2. Then *$(\mathcal{I}, \mathcal{J})$ satisfies $\mathcal{M}$*, denoted $(\mathcal{I}, \mathcal{J}) \models \mathcal{M}$ if

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Solutions for Knowledge Base Exchange

Given an interpretation $\mathcal{I}$ of Σ_1 and a set $\mathcal{X}$ of interpretations of Σ_1, let

$$\begin{aligned} \textsc{Sat}_{\mathcal{M}}(\mathcal{I}) &= \{\mathcal{J} \mid (\mathcal{I}, \mathcal{J}) \models \mathcal{M}\}, \\ \textsc{Sat}_{\mathcal{M}}(\mathcal{X}) &= \bigcup_{\mathcal{I} \in \mathcal{X}} \textsc{Sat}_{\mathcal{M}}(\mathcal{I}). \end{aligned}$$

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Definition

Let $\mathcal{M}$ be a mapping, $\mathcal{K}_1$ a KB over Σ_1, and $\mathcal{K}_2$ a KB over Σ_2.

- $\mathcal{K}_2$ is a *solution* for $\mathcal{K}_1$ under $\mathcal{M}$ if:

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- $\mathcal{K}_2$ is a *universal solution* for $\mathcal{K}_1$ under $\mathcal{M}$ if:

$$\textsc{Mod}(\mathcal{K}_2) = \textsc{Sat}_{\mathcal{M}}(\textsc{Mod}(\mathcal{K}_1)).$$

Solutions for Knowledge Base Exchange: Example

Example

Let

$\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$

$$\begin{array}{ll} \mathcal{T}_1: & B_1 \sqsubseteq A_1 \\ \mathcal{A}_1: & B_1(b) \end{array}$$

and

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Moreover, $\mathcal{K}_2$ is a *universal solution* for $\mathcal{K}_1$ under $\mathcal{M}$,
while $\mathcal{K}_2'$ is not.

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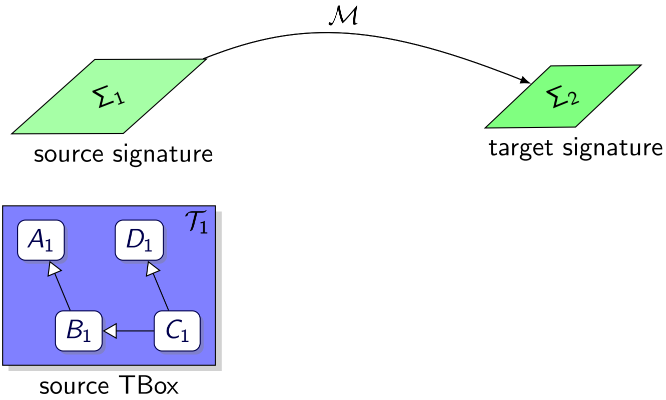
A New Problem: Representability



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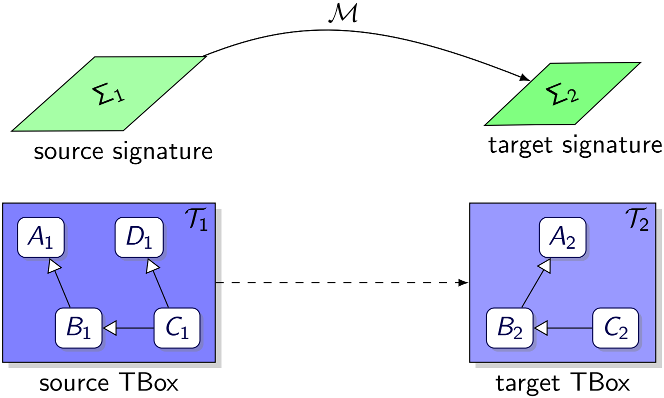

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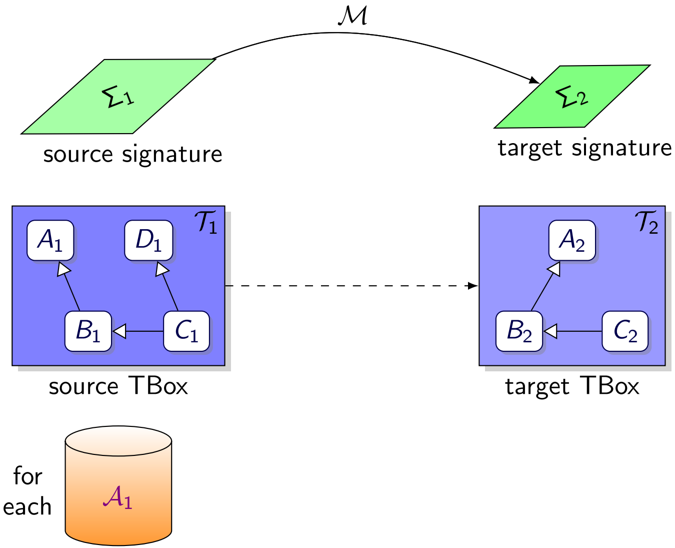

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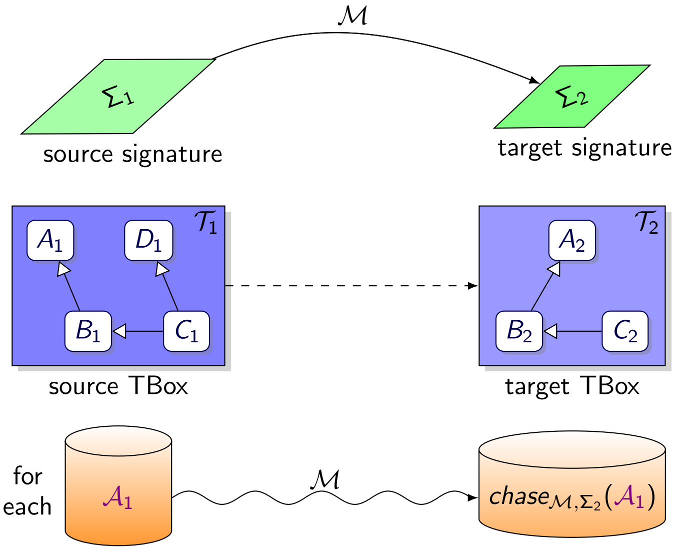

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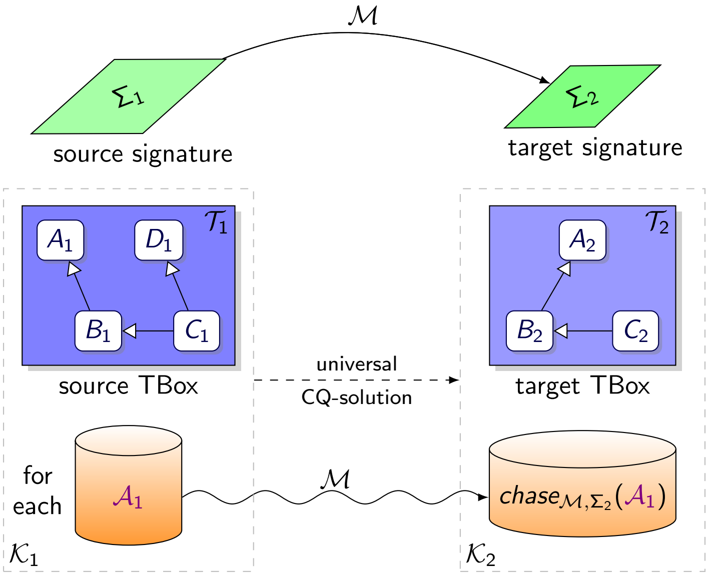
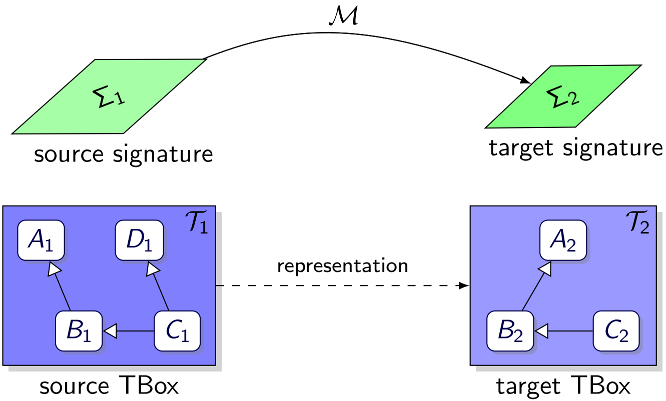


A New Problem: Representability contd



If such a $\mathcal{T}_2$ exists, we say that $\mathcal{T}_1$ is *representable* in $\mathcal{M}$.
$\mathcal{T}_2$ is called a *representation* of $\mathcal{T}_1$ in $\mathcal{M}$.

Representability: Example

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Let

$\mathcal{T}_1$:

$$B_1 \sqsubseteq A_1$$

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Then, $\mathcal{T}_1$ is *representable* in $\mathcal{M}$ and

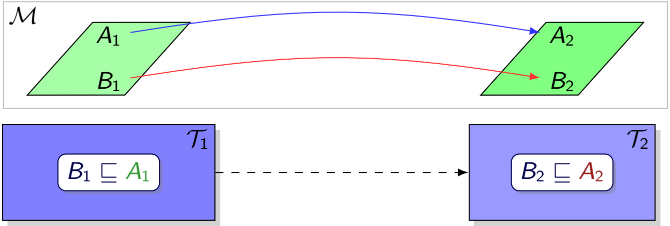
$\mathcal{T}_2$:

$$B_2 \sqsubseteq A_2$$

is a *representation* of $\mathcal{T}_1$ in $\mathcal{M}$.

In this example (and later for the *DL-Lite* setting) we exploit that certain answers are characterised in terms of chase.

Representability: Example contd



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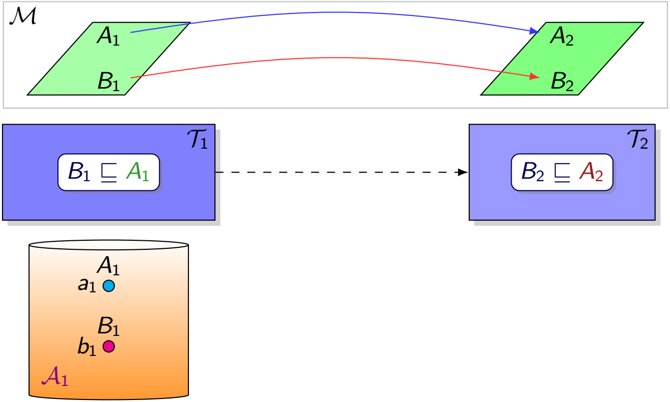

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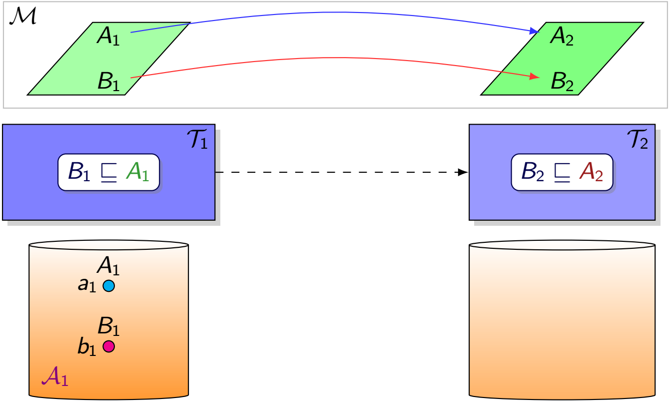

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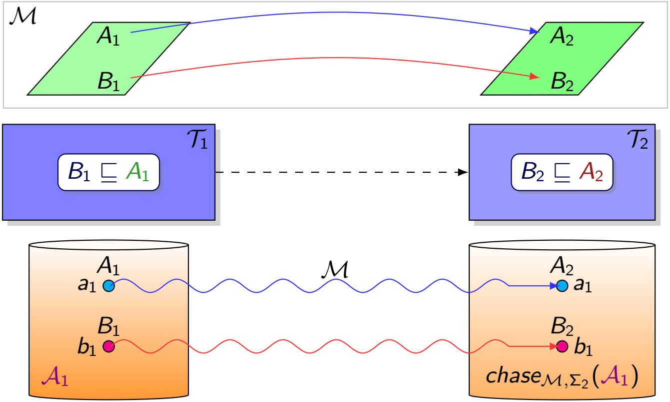

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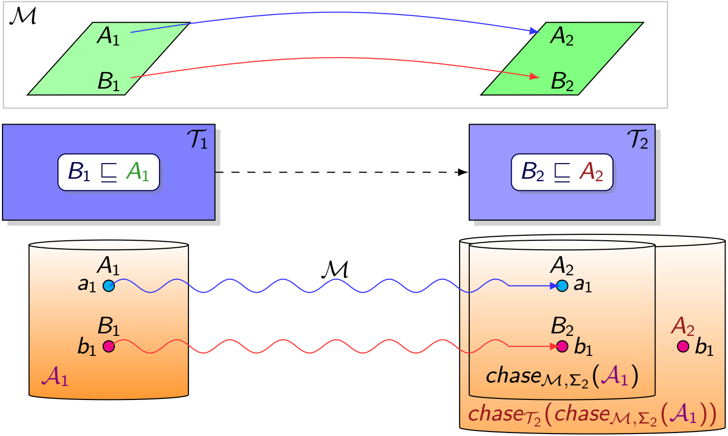

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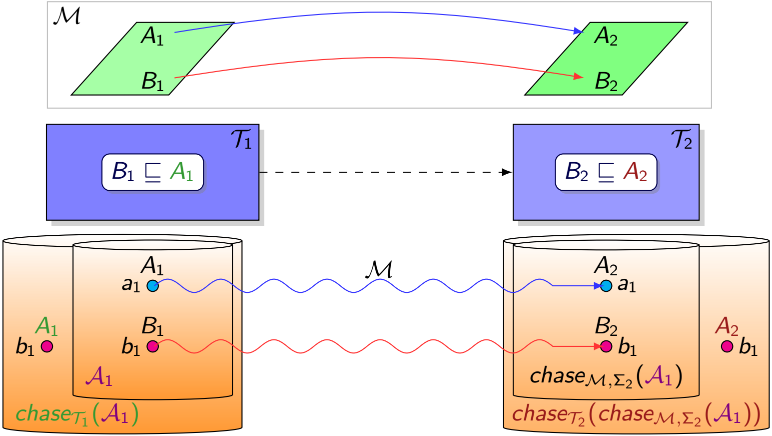

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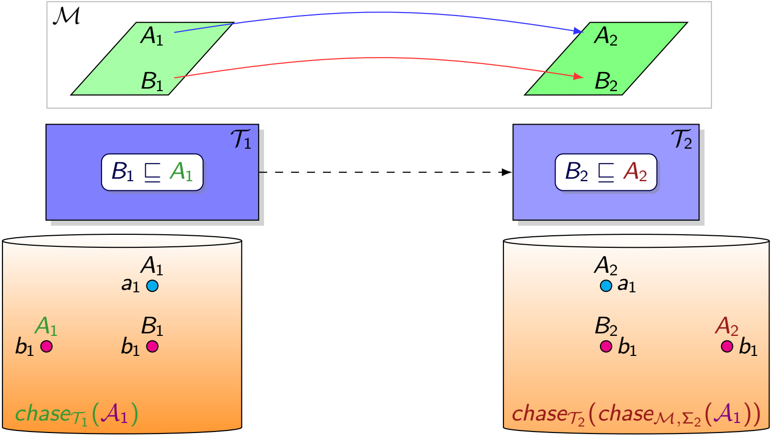

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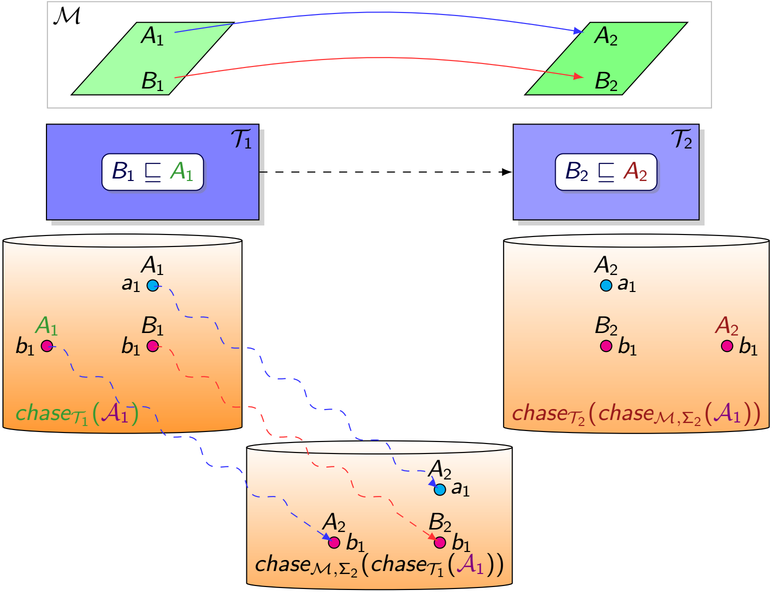

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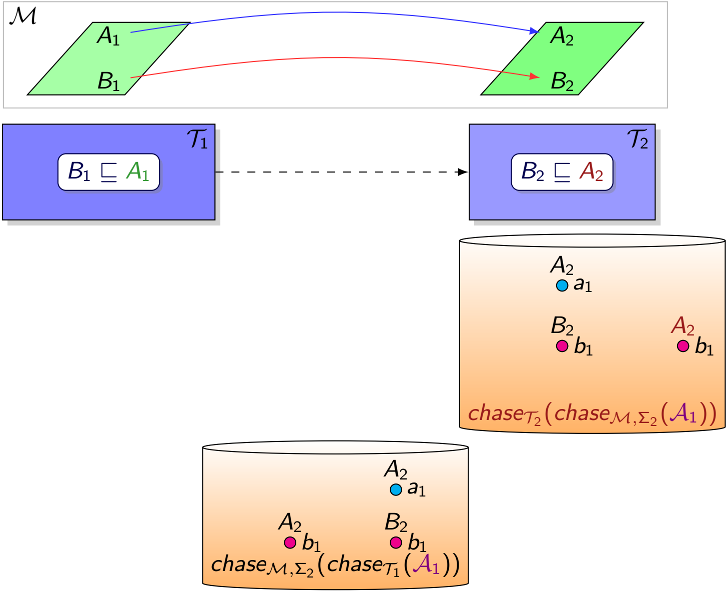

Our Setting: Definite Inclusions

In this paper we tackle the problems for definite inclusions.

- A $DL\text{-}Lite_{\mathcal{R}}$ inclusion is called *definite* if its right-hand side is an atomic concept or an atomic role.
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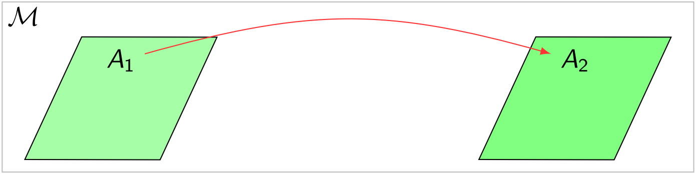

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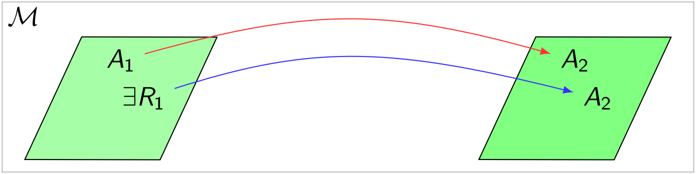

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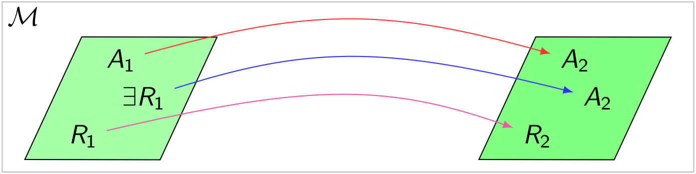

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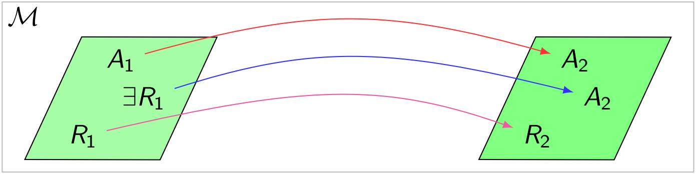

- and *DL-Lite*$_{RDFS}$ KBs.
  - We call *DL-Lite*$_{RDFS}$ the fragment of *DL-Lite*$_\mathcal{R}$ obtained by considering only definite *DL-Lite*$_\mathcal{R}$ TBoxes.

Outline

Computing (Universal) (CQ-)Solutions

Proposition

Let $\mathcal{M}$ be a definite mapping and $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ a *DL-Lite*$_{RDFS}$ KB over Σ_1. Then $\langle \emptyset, \mathit{chase}_{\mathcal{M},\Sigma_2}(\mathit{chase}_{\mathcal{T}_1}(\mathcal{A}_1)) \rangle$ is a universal solution for $\mathcal{K}_1$ under $\mathcal{M}$.

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Theorem

For definite mappings and DL-Lite$_{RDFS}$ *KBs, the problems of computing (universal) (CQ-)solutions can be solved in polynomial time.*

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Output: Yes, if $\mathcal{T}_2$ is a *representation* of $\mathcal{T}_1$ in $\mathcal{M}$,

NO, otherwise.

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i.e., for each $\mathcal{A}_1$, $\langle \mathcal{T}_2, \mathit{chase}_{\mathcal{M},\Sigma_2}(\mathcal{A}_1) \rangle$ is a universal CQ-solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$.

NO, otherwise.

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Input: a definite mapping $\mathcal{M}$,
a *DL-Lite*$_{RDFS}$ TBox $\mathcal{T}_1$ over Σ_1,
a *DL-Lite*$_{RDFS}$ TBox $\mathcal{T}_2$ over Σ_2.

Output: Yes, if $\mathcal{T}_2$ is a *representation* of $\mathcal{T}_1$ in $\mathcal{M}$,

i.e., for each $\mathcal{A}_1$, $\langle \mathcal{T}_2, \mathit{chase}_{\mathcal{M},\Sigma_2}(\mathcal{A}_1) \rangle$ is a universal CQ-solution for $\langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$.

NO, otherwise.

We base our technique on the notion of the *translation set* $\mathsf{M}(\alpha, \mu)$.

Translation Set $\mathsf{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathsf{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathsf{M}(\alpha, \mu)$.

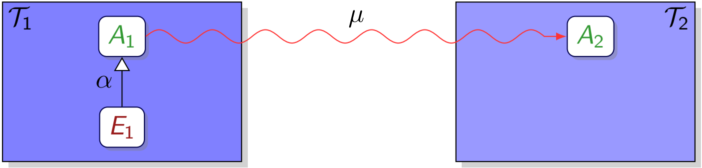
α	μ	ν	β
$E_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$E_1 \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$
$\exists R_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$\exists R_1 \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$
		$R_1 \sqsubseteq R_2$	$\exists R_2 \sqsubseteq A_2$
$R_1 \sqsubseteq S_1$	$S_1 \sqsubseteq S_2$	$R_1 \sqsubseteq R_2$	$R_2 \sqsubseteq S_2$
	$\exists S_1 \sqsubseteq A_2$	$\exists R_1 \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$
		$R_1 \sqsubseteq R_2$	$\exists R_2 \sqsubseteq A_2$
	$\exists S_1^- \sqsubseteq A_2$	$\exists R_1^- \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$
		$R_1 \sqsubseteq R_2$	$\exists R_2^- \sqsubseteq A_2$

Translation Set $\mathsf{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathsf{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathsf{M}(\alpha, \mu)$.

α	μ	ν	β
$E_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$		

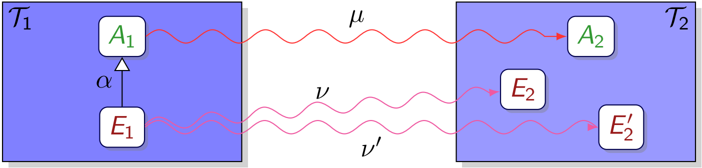


Translation Set $\mathsf{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathsf{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathsf{M}(\alpha, \mu)$.

α	μ	ν	β
$E_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$E_1 \sqsubseteq E_2$	

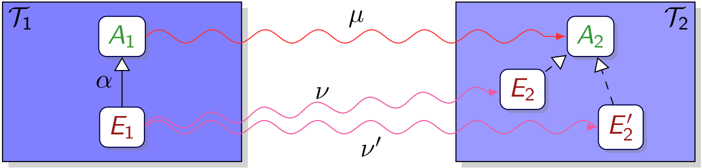


Translation Set $\mathsf{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathsf{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathsf{M}(\alpha, \mu)$.

α	μ	ν	β
$E_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$E_1 \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$

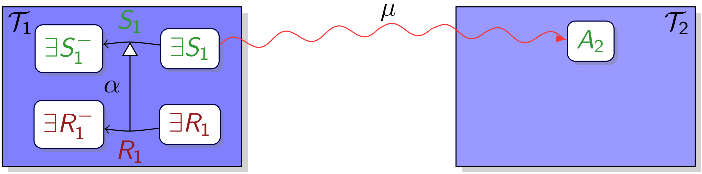


Translation Set $\mathsf{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathsf{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathsf{M}(\alpha, \mu)$.

α	μ	ν	β
$R_1 \sqsubseteq S_1$			
	$\exists S_1 \sqsubseteq A_2$		

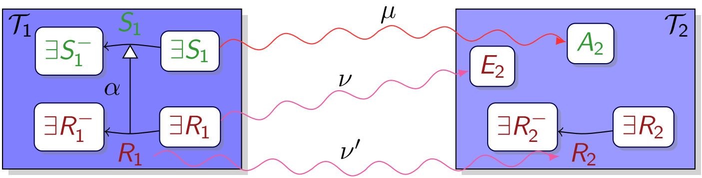


Translation Set $\mathsf{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathsf{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathsf{M}(\alpha, \mu)$.

α	μ	ν	β
$R_1 \sqsubseteq S_1$			
	$\exists S_1 \sqsubseteq A_2$	$\exists R_1 \sqsubseteq E_2$	
		$R_1 \sqsubseteq R_2$	

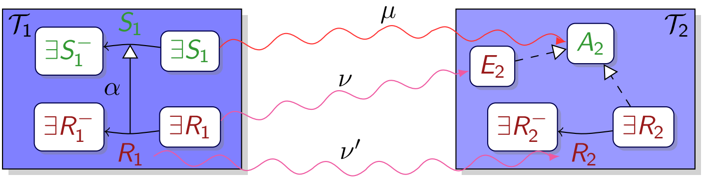


Translation Set $\mathrm{M}(\alpha, \mu)$

Let α be a *DL-Lite*$_{RDFS}$ inclusion over Σ_1, and $\mu \in \mathcal{M}$.

Then $\mathrm{M}(\alpha, \mu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_2 such that, if there exists an inclusion $\nu \in \mathcal{M}$ as in the table, then $\beta \in \mathrm{M}(\alpha, \mu)$.

α	μ	ν	β
$R_1 \sqsubseteq S_1$			
	$\exists S_1 \sqsubseteq A_2$	$\exists R_1 \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$
		$R_1 \sqsubseteq R_2$	$\exists R_2 \sqsubseteq A_2$



Reverse Translation Set $\mathsf{M}^{-}(\beta, \nu)$

Let β be a *DL-Lite*$_{RDFS}$ inclusion over Σ_2, and $\nu \in \mathcal{M}$.

Then $\mathsf{M}^{-}(\beta, \nu)$, is the set of *DL-Lite*$_{RDFS}$ inclusions over Σ_1 such that, if there exists an inclusion $\mu \in \mathcal{M}$ as in the table, then $\alpha \in \mathsf{M}^{-}(\beta, \nu)$.

α	μ	ν	β
$E_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$E_1 \sqsubseteq E_2$	$E_2 \sqsubseteq A_2$
$\exists R_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$\exists R_1 \sqsubseteq E_2$	
$R_1 \sqsubseteq S_1$	$\exists S_1 \sqsubseteq A_2$		
$R_1 \sqsubseteq S_1$	$\exists S_1^{-} \sqsubseteq A_2$	$\exists R_1^{-} \sqsubseteq E_2$	
$\exists R_1 \sqsubseteq A_1$	$A_1 \sqsubseteq A_2$	$R_1 \sqsubseteq R_2$	$\exists R_2 \sqsubseteq A_2$
$R_1 \sqsubseteq S_1$	$\exists S_1 \sqsubseteq A_2$		
$R_1 \sqsubseteq S_1$	$\exists S_1^{-} \sqsubseteq A_2$	$R_1 \sqsubseteq R_2$	$\exists R_2^{-} \sqsubseteq A_2$
$R_1 \sqsubseteq S_1$	$S_1 \sqsubseteq S_2$	$R_1 \sqsubseteq R_2$	$R_2 \sqsubseteq S_2$

Checking Representability

We get the following characterisation of representations.

Proposition

Let $\mathcal{M}$ be a definite mapping, $\mathcal{T}_1$ a *DL-Lite*$_{RDFS}$ TBox over Σ_1, and $\mathcal{T}_2$ a *DL-Lite*$_{RDFS}$ TBox over Σ_2. Then $\mathcal{T}_2$ is a *representation* of $\mathcal{T}_1$ in $\mathcal{M}$ if and only if

- for each inclusion α, s.t. $\mathcal{T}_1 \models \alpha$, and for each inclusion $\mu \in \mathcal{M}$ left-compatible with *rhs*(α), there exists $\beta \in \mathsf{M}(\alpha, \mu)$, s.t. $\mathcal{T}_2 \models \beta$, and
- for each inclusion β, s.t. $\mathcal{T}_2 \models \beta$, and for each inclusion $\nu \in \mathcal{M}$ right-compatible with *lhs*(β), there exists $\alpha \in \mathsf{M}^{-}(\beta, \nu)$, s.t. $\mathcal{T}_1 \models \alpha$.

Deciding Representability

Theorem

Let $\mathcal{M}$ be a definite mapping and $\mathcal{T}_1$ a DL-Lite$_{RDFS}$ TBox over Σ_1. Then we can check whether $\mathcal{T}_1$ is representable in $\mathcal{M}$ in polynomial time.

Proof.

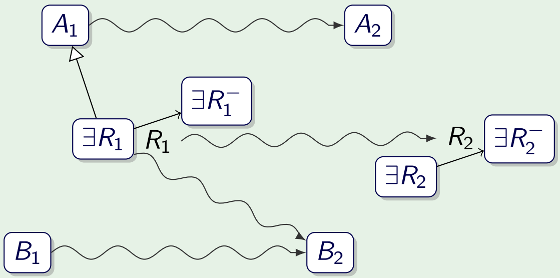
1. Take $\mathrm{M}(\mathcal{T}_1, \mathcal{M}) = \bigcup \mathrm{M}(\alpha, \mu)$, where the union ranges over all α, s.t. $\mathcal{T}_1 \models \alpha$, and $\mu \in \mathcal{M}$ is left-compatible with $rhs(\alpha)$;
2. Remove from $\mathrm{M}(\mathcal{T}_1, \mathcal{M})$ every β s.t. there exists an inclusion $\nu \in \mathcal{M}$ right-compatible with $lhs(\beta)$ and for each $\alpha \in \mathrm{M}^-(\beta, \nu)$, $\mathcal{T}_1 \not\models \alpha$. Let the resulting TBox be denoted with $\mathcal{T}_2 = \mathrm{Rep}(\mathcal{T}_1, \mathcal{M})$.
3. Check whether $\mathcal{T}_2$ is a representation of $\mathcal{T}_1$ in $\mathcal{M}$.
   - If the check succeeds, then $\mathcal{T}_1$ is representable in $\mathcal{M}$.
   - Otherwise, $\mathcal{T}_1$ is not representable in $\mathcal{M}$.

□

Constructing Representations: Example

Example

$$\mathcal{T}_1: \quad \exists R_1 \sqsubseteq A_1 \qquad\qquad \mathcal{M}: \quad \begin{array}{rcl} \exists R_1 \sqsubseteq B_2 & & A_1 \sqsubseteq A_2 \\ B_1 \sqsubseteq B_2 & & R_1 \sqsubseteq R_2 \end{array}$$



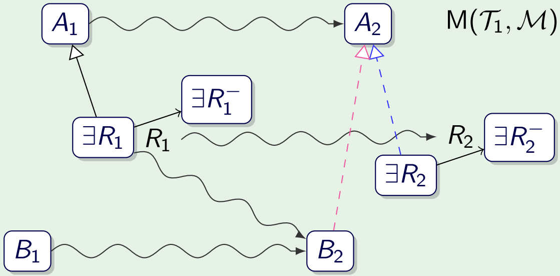
Constructing Representations: Example

Example

$\mathcal{T}_1 : \quad \exists R_1 \sqsubseteq A_1$

$\mathcal{M} : \quad \exists R_1 \sqsubseteq B_2 \qquad A_1 \sqsubseteq A_2$

$\qquad B_1 \sqsubseteq B_2 \qquad R_1 \sqsubseteq R_2$



$\mathsf{M}(\mathcal{T}_1, \mathcal{M}) : \quad \exists R_2 \sqsubseteq A_2$

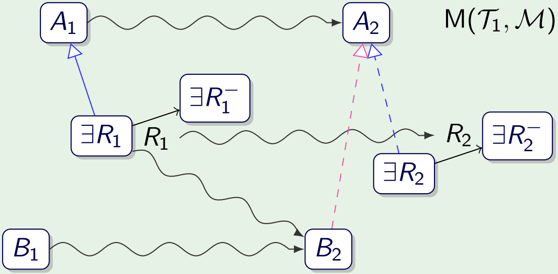
$\qquad B_2 \sqsubseteq A_2$

Constructing Representations: Example

Example

$\mathcal{T}_1 : \quad \exists R_1 \sqsubseteq A_1$

$\mathcal{M} : \quad \exists R_1 \sqsubseteq B_2 \qquad A_1 \sqsubseteq A_2$
$\qquad B_1 \sqsubseteq B_2 \qquad R_1 \sqsubseteq R_2$



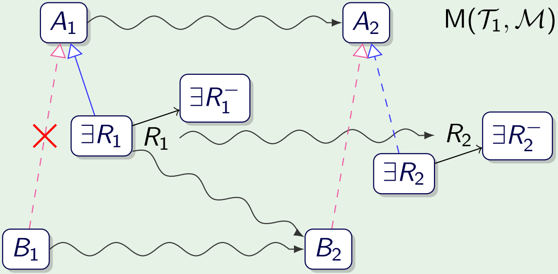
$\mathsf{M}(\mathcal{T}_1, \mathcal{M}) : \quad \exists R_2 \sqsubseteq A_2$
$\qquad B_2 \sqsubseteq A_2$

Constructing Representations: Example

Example

$\mathcal{T}_1 : \quad \exists R_1 \sqsubseteq A_1$

$\mathcal{M} : \quad \exists R_1 \sqsubseteq B_2 \qquad A_1 \sqsubseteq A_2$
$\qquad B_1 \sqsubseteq B_2 \qquad R_1 \sqsubseteq R_2$



$\mathsf{M}(\mathcal{T}_1, \mathcal{M}) : \quad \exists R_2 \sqsubseteq A_2$
$\qquad B_2 \sqsubseteq A_2$

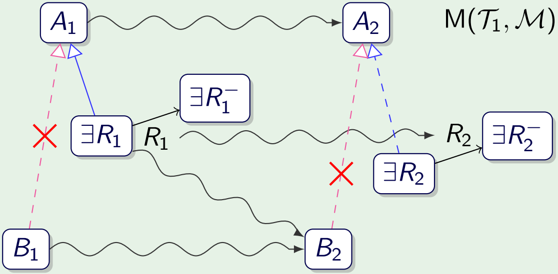
Constructing Representations: Example

Example

$\mathcal{T}_1 : \quad \exists R_1 \sqsubseteq A_1$

$\mathcal{M} : \quad \exists R_1 \sqsubseteq B_2 \qquad A_1 \sqsubseteq A_2$

$\qquad B_1 \sqsubseteq B_2 \qquad R_1 \sqsubseteq R_2$



$\mathsf{M}(\mathcal{T}_1, \mathcal{M}) : \quad \exists R_2 \sqsubseteq A_2$

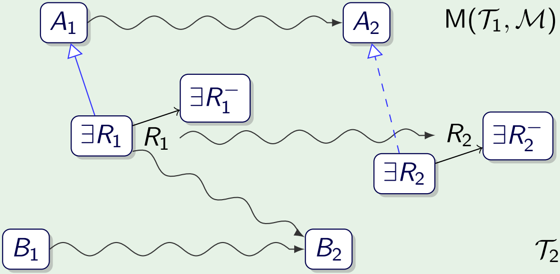
$B_2 \sqsubseteq A_2$

Constructing Representations: Example

Example

$\mathcal{T}_1: \quad \exists R_1 \sqsubseteq A_1$

$\mathcal{M}: \quad \exists R_1 \sqsubseteq B_2 \qquad A_1 \sqsubseteq A_2$
$\qquad B_1 \sqsubseteq B_2 \qquad R_1 \sqsubseteq R_2$



$\mathrm{M}(\mathcal{T}_1, \mathcal{M}): \quad \exists R_2 \sqsubseteq A_2$
$B_2 \sqsubseteq A_2$

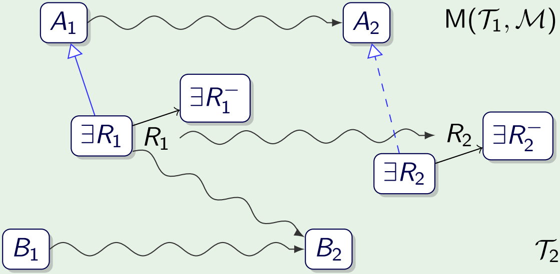
$\mathcal{T}_2: \quad \exists R_2 \sqsubseteq A_2$

Constructing Representations: Example

Example

$\mathcal{T}_1: \quad \exists R_1 \sqsubseteq A_1$

$\mathcal{M}: \quad \exists R_1 \sqsubseteq B_2 \quad A_1 \sqsubseteq A_2$
$\quad B_1 \sqsubseteq B_2 \quad R_1 \sqsubseteq R_2$

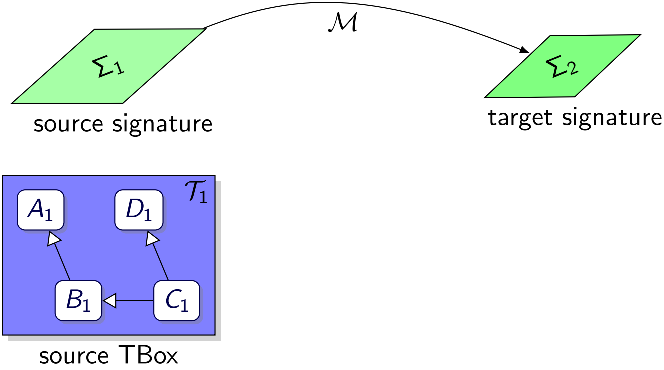


$\mathsf{M}(\mathcal{T}_1, \mathcal{M}): \quad \exists R_2 \sqsubseteq A_2$
$\quad B_2 \sqsubseteq A_2$

$\mathcal{T}_2: \quad \exists R_2 \sqsubseteq A_2$

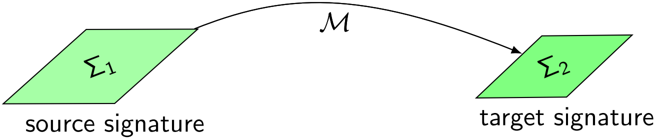
$\mathcal{T}_1$ is *representable* in $\mathcal{M}$ and $\mathcal{T}_2$ is a representation of $\mathcal{T}_1$ in $\mathcal{M}$.

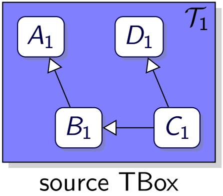
A New Problem: Weak Representability



A New Problem: Weak Representability

$\mathcal{M}^*$ s.t. $\mathcal{M} \subseteq \mathcal{M}^*$ and $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$





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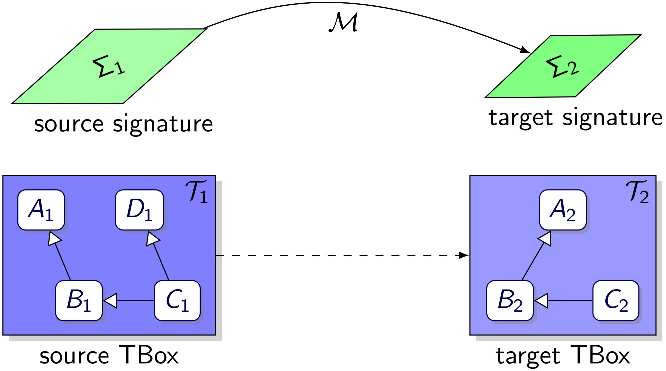
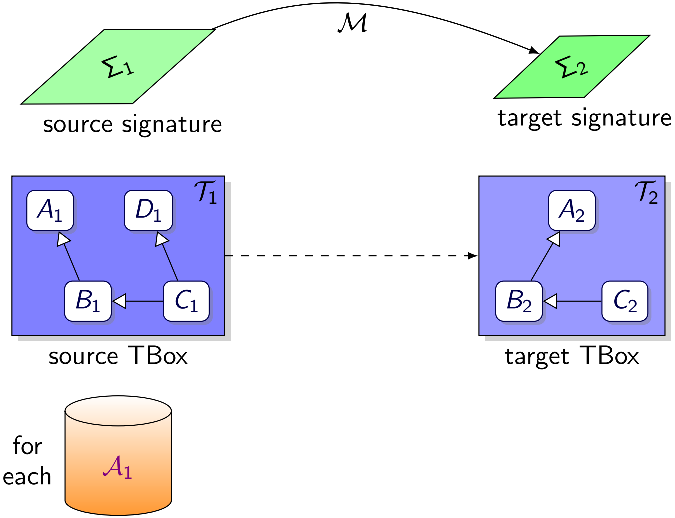


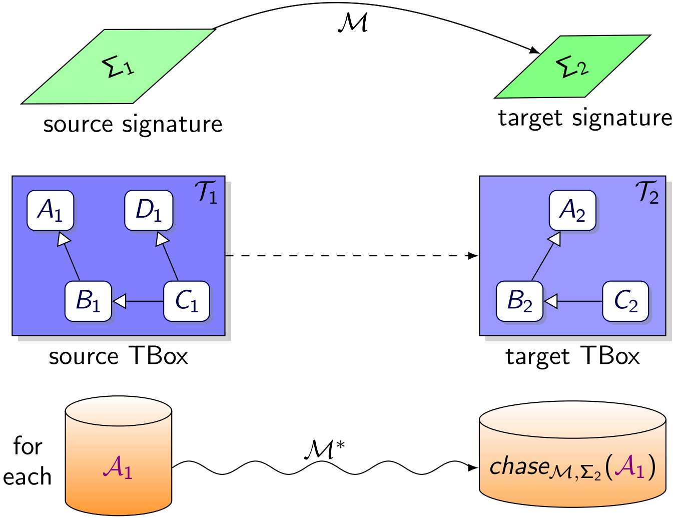
A New Problem: Weak Representability

$\mathcal{M}^*$ s.t. $\mathcal{M} \subseteq \mathcal{M}^*$ and $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$



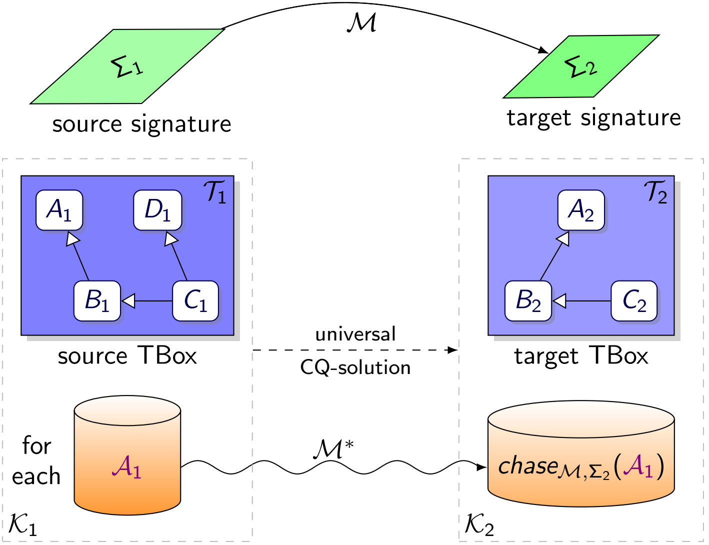
A New Problem: Weak Representability

$\mathcal{M}^*$ s.t. $\mathcal{M} \subseteq \mathcal{M}^*$ and $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$



A New Problem: Weak Representability

$\mathcal{M}^*$ s.t. $\mathcal{M} \subseteq \mathcal{M}^*$ and $\mathcal{T}_1 \cup \mathcal{M} \models \mathcal{M}^*$



Deciding Weak Representability

Theorem

Let $\mathcal{M}$ be a definite mapping and $\mathcal{T}_1$ a DL-Lite$_{RDFS}$ TBox over Σ_1.
Then $\mathcal{T}_1$ is weakly representable in $\mathcal{M}$.

Outline

Conclusions and Future Work

- We have specialised the framework for KB exchange to the case of DLs.
- We have defined new reasoning tasks: representability and weak representability of a TBox in a mapping.
- We have shown the following results for definite mappings and $\mathit{DL\text{-}Lite}_{\mathit{RDFS}}$ KBs:
  - the problems of computing (universal) (CQ-)solutions can be solved in polynomial time.
  - the problem of representability of a TBox in a mapping is decidable in polynomial time.
  - every $\mathit{DL\text{-}Lite}_{\mathit{RDFS}}$ TBox is weakly representable in a definite mapping.
- We plan to extend the results to the case of full $\mathit{DL\text{-}Lite}_{\mathcal{R}}$.
  The issues to explore:
  - labelled nulls in the chase
  - disjointness constraints

Thank you
for your attention!