Exchanging OWL 2 QL Knowledge Bases

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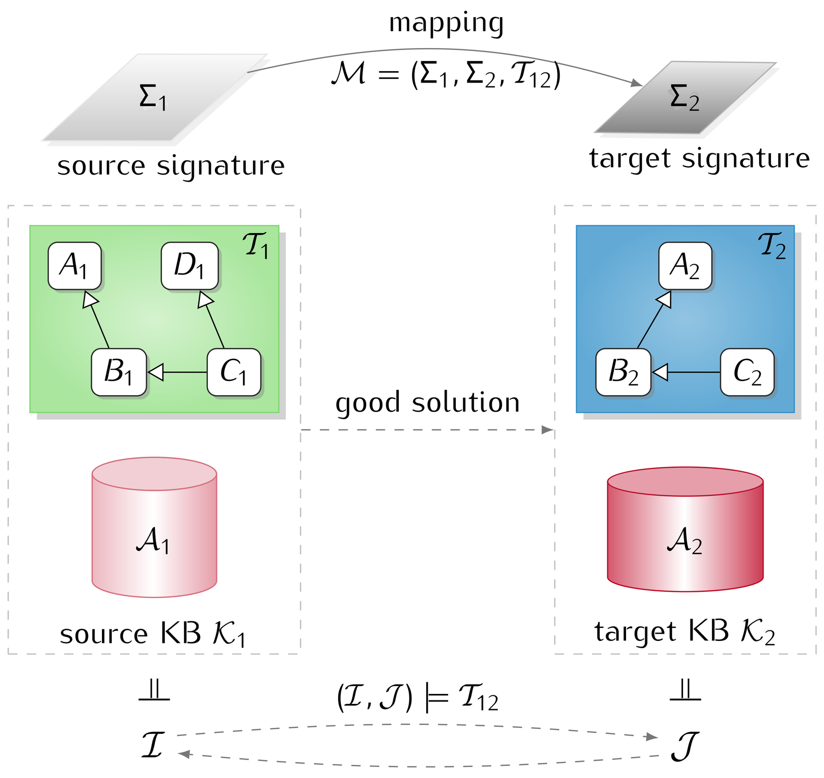
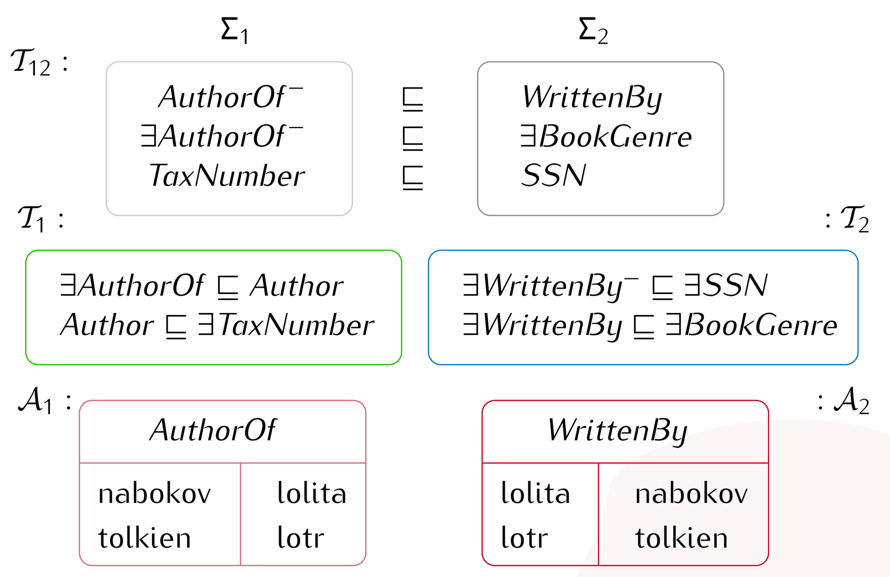
The Knowledge Base Exchange Framework

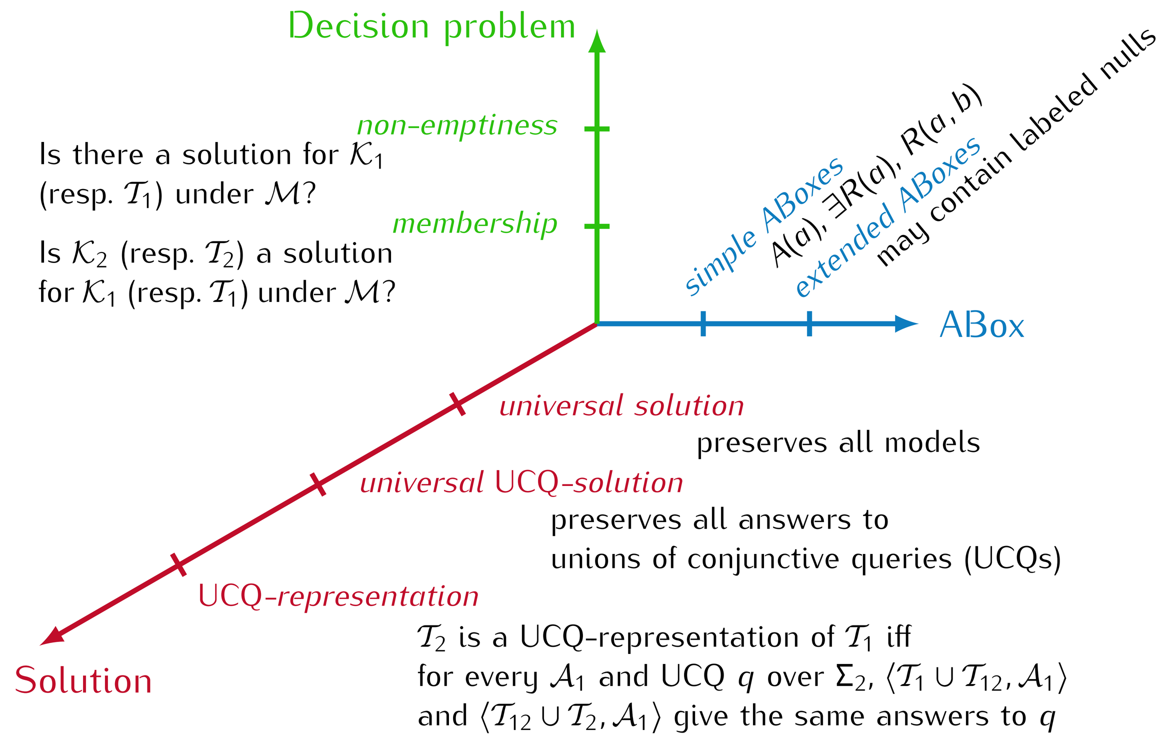
We study the knowledge base (KB) exchange problem for OWL 2 QL KBs.



$\mathcal{K}_2 = \langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a desired solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$:



We consider computational problems along three dimensions:



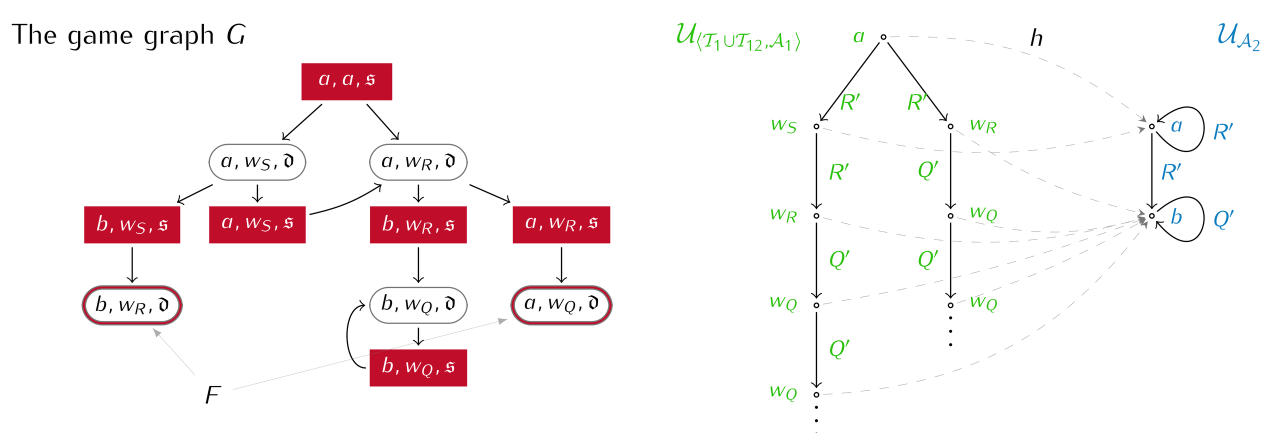
The *canonical model* of $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$, denoted by $\mathcal{U}_\mathcal{K}$, is a model of $\mathcal{K}$ that corresponds to the chase of $\mathcal{A}$ w.r.t. $\mathcal{T}$. For example, for $\mathcal{A} = \{A(a)\}$ and $\mathcal{T} = \{A \sqsubseteq \exists R, \exists R^- \sqsubseteq \exists R\}$:

Let $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$. We use the following characterizations:

- $\mathcal{K}_2 = \langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ $\Leftrightarrow$ $\mathcal{T}_2 = \emptyset$ and $\mathcal{U}_{\mathcal{A}_2}$ is homomorphically equivalent to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ on the target symbols (Σ_2);
- $\mathcal{K}_2 = \langle \mathcal{T}_2, \mathcal{A}_2 \rangle$ is a universal UCQ-solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M}$ $\Leftrightarrow$ $\mathcal{U}_{\langle \mathcal{T}_2, \mathcal{A}_2 \rangle}$ is finitely homomorphically equivalent to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ on the target symbols;
- $\mathcal{T}_2$ is a UCQ-representation of $\mathcal{T}_1$ under $\mathcal{M}$ $\Leftrightarrow$ for each ABox $\mathcal{A}_1$, $\mathcal{U}_{\langle \mathcal{T}_{12} \cup \mathcal{T}_2, \mathcal{A}_1 \rangle}$ is homomorphically equivalent to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ on the target symbols.

It is easy to check the homomorphism from $\mathcal{U}_{\mathcal{A}_2}$ to $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$. For the opposite direction, we employ the technique of reachability games on graphs known to be PTime-complete.

Let $\mathcal{T}_{12} = \{R \sqsubseteq R', S \sqsubseteq R', Q \sqsubseteq Q'\}$, $\mathcal{T}_1 = \{\exists S^- \sqsubseteq \exists R, \exists R^- \sqsubseteq \exists Q, \exists Q^- \sqsubseteq \exists Q\}$, $\mathcal{A}_1 = \{\exists R(a), \exists S(a)\}$, and $\mathcal{A}_2 = \{R'(a, a), R'(a, b), Q'(b, b)\}$.

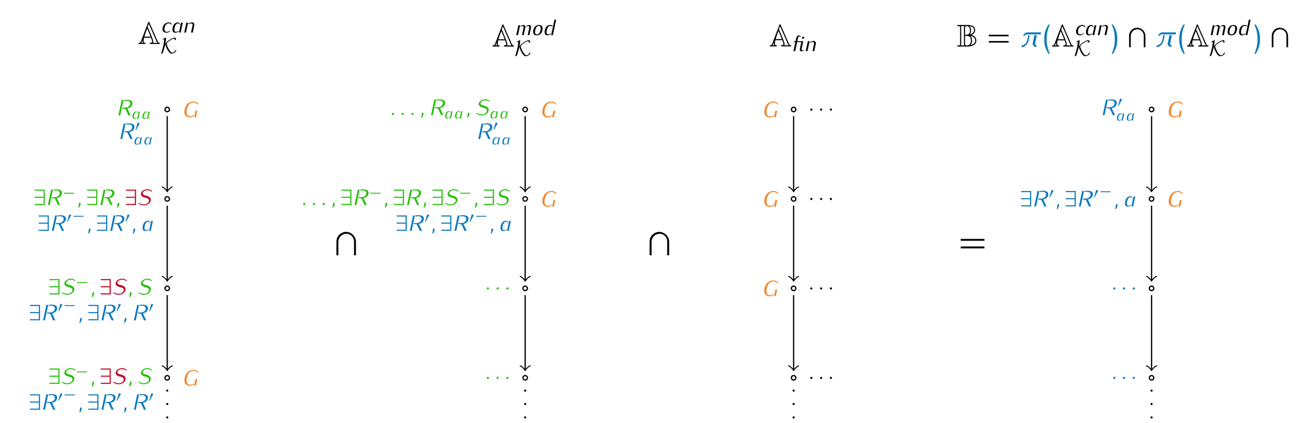


There exists a homomorphism from $\mathcal{U}_{\langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle}$ to $\mathcal{U}_{\mathcal{A}_2}$ iff Duplicator has a strategy in $\mathcal{G}$ from a, a, s against Spoiler to avoid F.

The upper bound is obtained by using two-way alternating tree automata (2ATA):

- 2ATA $\mathbb{A}^{can}_\mathcal{K}$ accepts $\mathcal{U}_\mathcal{K}$ arbitrary labeled with a reserved symbol G;
- 2ATA $\mathbb{A}^{mod}_\mathcal{K}$ accepts tree models of $\mathcal{K}$ labeled with G; and
- TA $\mathbb{A}_{fin}$ accepts trees with a finite prefix labeled with G.

Let $\mathcal{T}_{12} = \{R \sqsubseteq R', S \sqsubseteq R'\}$, $\mathcal{T}_1 = \{\exists S^- \sqsubseteq \exists S\}$ and $\mathcal{A}_1 = \{R(a, a), \exists S(a)\}$.



There exists a universal solution for $\mathcal{K}_1 = \langle \mathcal{T}_1, \mathcal{A}_1 \rangle$ under $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$ iff the language of the automaton $\mathbb{B} = \pi(\mathbb{A}^{can}_\mathcal{K}) \cap \pi(\mathbb{A}^{mod}_\mathcal{K}) \cap \mathbb{A}_{fin}$ is non-empty, for $\mathcal{K} = \langle \mathcal{T}_1 \cup \mathcal{T}_{12}, \mathcal{A}_1 \rangle$.

Summary of the Results

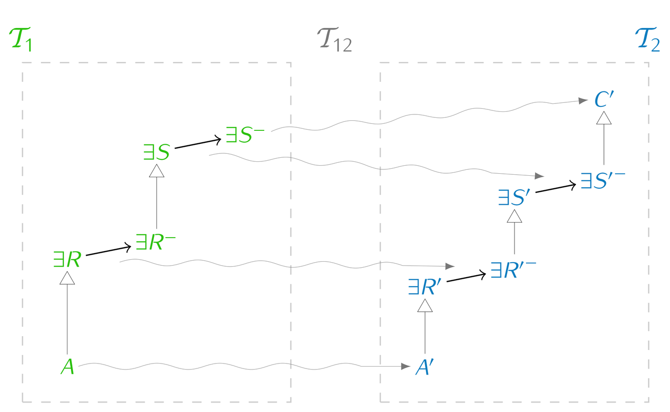
Membership	Simple ABoxes	Extended ABoxes	Non-emptiness	Simple ABoxes	Extended ABoxes
Universal solutions	PTime*	NP-complete	Universal solutions	PTime*	PSpace-hard, in ExpTime
UCQ-representations	NLogSpace-complete		UCQ-representations	NLogSpace-complete	

The membership problem for universal UCQ-solutions with simple ABoxes is PSpace-hard.

*new result

We provide a number of conditions on $\mathcal{T}_1$, $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$, and $\mathcal{T}_2$.

Consider
$$\mathcal{T}_{12} = \{A \sqsubseteq A', R \sqsubseteq R', S \sqsubseteq S', \exists S^- \sqsubseteq C'\},$$
$$\mathcal{T}_1 = \{A \sqsubseteq \exists R, \ \exists R^- \sqsubseteq \exists S\},$$
$$\mathcal{T}_2 = \{A' \sqsubseteq \exists R', \exists R'^- \sqsubseteq \exists S', \exists S'^- \sqsubseteq C'\}.$$



In particular, these conditions are satisfied:

$\mathcal{T}_1 \cup \mathcal{T}_{12} \models A \sqsubseteq \exists R'$ $\Leftrightarrow$ $\mathcal{T}_{12} \cup \mathcal{T}_2 \models A \sqsubseteq \exists R'$
$\mathcal{T}_1 \cup \mathcal{T}_{12} \models \exists S^- \sqsubseteq C'$ $\Leftrightarrow$ $\mathcal{T}_{12} \cup \mathcal{T}_2 \models \exists S^- \sqsubseteq C'$
$\mathcal{T}_1 \cup \mathcal{T}_{12} \models A \neq \emptyset \to C' \neq \emptyset$ $\Rightarrow$ $\mathcal{T}_{12} \cup \mathcal{T}_2 \models A \neq \emptyset \to C' \neq \emptyset$
$\mathcal{T}_1 \models A \sqsubseteq \exists R$ and $\mathcal{T}_{12} \models R \sqsubseteq R'$ $\Leftarrow$ $\mathcal{T}_{12} \cup \mathcal{T}_2 \models A \sqsubseteq \exists R'$ and $\mathcal{T}_2 \models \exists R'^- \sqsubseteq \exists S'$

Hence $\mathcal{T}_2$ is a UCQ-representation for $\mathcal{T}_1$ under $\mathcal{M}$.

We provide a set of conditions on $\mathcal{T}_1$ and $\mathcal{M} = (\Sigma_1, \Sigma_2, \mathcal{T}_{12})$.

Let $\mathcal{T}_1 = \{A \sqsubseteq B\}$, $B \sqsubseteq B' \in \mathcal{T}_{12}$, and

$A \sqsubseteq A' \in \mathcal{T}_{12}$	or	$A \sqsubseteq A' \in \mathcal{T}_{12}$, $A \sqsubseteq A''$	or	$A \sqsubseteq A' \in \mathcal{T}_{12}$, $C \sqsubseteq A'$	or	$A \sqsubseteq A' \in \mathcal{T}_{12}$, $A \sqsubseteq A''$, $C \sqsubseteq A'$

$D' \to A'$	$D' \to A'$ or A''	$D' \to \emptyset$	$D' \to A''$

There exists a UCQ-representation of $\mathcal{T}_1$ under $\mathcal{M}$ iff there exists $D' \in \Sigma_2$ s.t. $A \sqsubseteq D' \in \mathcal{T}_{12}$, and for every D: $\mathcal{T}_1 \cup \mathcal{T}_{12} \models D \sqsubseteq D'$ implies $\mathcal{T}_1 \cup \mathcal{T}_{12} \models D \sqsubseteq B'$.