Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.Y. 2007/2008
Final exam – 30/6/2008 – Part 2

Time: 90 minutes

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID on every solution sheet. Good luck!

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE (or believed to be so, under certain assumptions, which you should state). You must give an explanation of your answer to receive full credit.

(a) $\mathrm{P}^{\mathrm{NP}} = \mathrm{NP}^{\mathrm{P}}$.

(b) Let L_1 and L_2 be languages. If $L_1 <_{poly} L_2$ and $L_2 \in \mathrm{NP}\textsc{space}$, then $L_1 \in \mathrm{P}\textsc{space}$.

(c) There is a language L such that $L \notin \textsc{Depth}(O(n))$.

Problem 2.2 [6 points] Consider a language L over an alphabet Σ, and let L be in NP.

(a) Show that L_1 is in NP, where

$$L_1 = \{\ w \mid \text{ there exists a decomposition } w = x{\cdot}y{\cdot}z \text{ such that } y \in L \text{ and } x, z \in \Sigma^*\ \}$$

[*Hint:* Use a non-deterministic TM M for L, and show how it can be used to construct a non-deterministic TM for L_1.]

(b) Show that L_2 is in coNP, where

$$L_2 = \{\ w \mid \text{ there exists a decomposition } w = x{\cdot}y{\cdot}z \text{ such that } y \notin L \text{ and } x, z \in \Sigma^*\ \}$$

[*Hint:* Consider the complement $\overline{L_2}$ of L_2, and show again how M can be used to construct a non-deterministic TM for $\overline{L_2}$.]

Problem 2.3 [6 points] Consider the proof of Cook's theorem that CSAT is NP-hard. Describe how in that proof the computation of a non-deterministic TM with running time $p(n)$, where $p(n)$ is a polynomial in n, is represented using propositional variables. Provide the CNF-formulas that encode those conditions holding between such propositional variables that depend on the actual transitions of the TM. How many clauses of which length are necessary to encode such conditions?

Problem 2.4 [6 points]

(a) Provide the definition of a circuit with n boolean input variables. Define the complexity measures for circuits.

(b) Give an example of a circuit for 3 inputs x_1, x_2, x_3. Describe the boolean function defined by your circuit.

(c) Describe briefly how circuits that compute boolean functions can be used as a computation model that is suitable for asymptotic analysis.

Problem 2.5 [6 points]

(a) Give the definition of when a language L belongs to $\textsc{Size}(s(n))$.

(b) Provide a sketch of the proof that for every language L, we have that $L \in \textsc{Size}(O(2^n))$.