Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2005/2006

Final exam – 15/6/2006 – Part 2

Time: 90 minutes

Problem 2.1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, if L_1 is in P and L_2 is in NP, then $L_1 \cap L_2$ is in P.

(b) For all languages L_1 and L_2, if L_2 is in NP and $L_1 <_{poly} L_2$, then L_1 is in P.

(c) The class NP is closed under union.

(d) There exists a language L such that both L and $\overline{L}$ are recursively enumerable, but neither L nor $\overline{L}$ are recursive.

Problem 2.2 [6 points] Consider the context free grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$ where P consists of the following productions:

$$\begin{array}{rcl} S & \longrightarrow & A \mid ABa \mid AbA \\ A & \longrightarrow & Aa \mid \varepsilon \\ B & \longrightarrow & Bb \mid BC \\ C & \longrightarrow & CB \mid CA \mid bB \end{array}$$

convert G into Chomsky Normal Form. Illustrate the various steps of the algorithm.

Problem 2.3 [6 points] Let $M = (Q, \Sigma, \Gamma, \delta, q_0, \mathtt{B}, F)$ be a standard Turing Machine that accepts a language L, i.e., $\mathcal{L}(M) = L$. Informally, but precisely describe how to construct, from M a new Turing Machine M' that accepts a string $w \in \Sigma^*$ if and only if there is a substring of w in L. [*Hint*: Make use of standard TM constructions and extensions of the basic TM model, e.g., with non-determinism.]

Problem 2.4 [6 points] Let $L_e \subseteq \{0, 1\}^*$ be the language of binary words that have the same number of 0's and 1's. Construct a Turing Machine M_e that decides L_e, i.e., such that M_e always halts and $\mathcal{L}(M_e) = L_e$. Show the sequence of IDs of M_e on the accepted input string 1010 and on the non-accepted input string 1011.

Problem 2.5 [6 points] For a Turing Machine M with input alphabet $\Sigma = \{a, b\}$, let $\mathcal{E}(M)$ denote the encoding of M, and $\langle \mathcal{E}(M), w \rangle$ denote the encoding of M together with an input word w. Consider the language $L = \{\langle \mathcal{E}(M), w \rangle \mid M$, when started on an input word w, eventually prints the symbol a on two consecutive transitions $\}$.

(a) Show that L is recursively enumerable. [*Hint*: Make use of a universal TM.]

(b) Show that L is not recursive. [*Hint*: Exploit a reduction from the halting problem.]