Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2005/2006
Final exam – 15/6/2006 – Part 1

Time: 90 minutes

Problem 1.1 [6 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

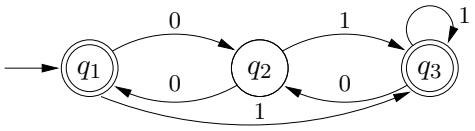
(a) For all languages L_1 and L_2, it holds that $(L_1^* \cdot L_2^*)^+ = (L_1^+ \cdot L_2^+)^*$.

(b) For all languages L_1 and L_2, if L_1 is non-regular and $L_1 \subseteq L_2$, then L_2 is non-regular.

(c) If L_1 and L_2 are both non-regular, then $L_1 \cup L_2$ could be regular.

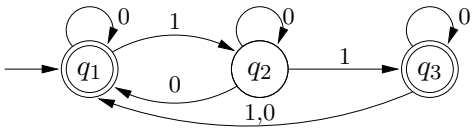
(d) There exists a language L with $\varepsilon \notin L$ such that $L = L^*$.

Problem 1.2 [6 points] Consider the following DFA A over $\{0, 1\}$:



Construct a regular expression E such that $\mathcal{L}(E) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct E.

Problem 1.3 [6 points] Consider the following NFA N over $\{0, 1\}$:



Construct a DFA A such that $\mathcal{L}(A) = \mathcal{L}(N)$. Illustrate the steps of the algorithm you have followed to construct A.

Problem 1.4 [6 points] Show that the language $L = \{a^n b^m \mid n \geqslant m\}$ is not regular.
[*Hint*: Exploit the pumping lemma for regular languages.]

Problem 1.5 [6 points] Let L_1 be the set of strings over $\{a, b\}$ that do *not* have aab as a substring. Let further L_2 be the language over $\{a, b\}$ inductively defined as follows:

1. ε is in L_2;
2. for every w in L_2, also wa, bw, and abw are in L_2;
3. nothing else is in L_2.

(a) Prove that $L_2 \subseteq L_1$, making all steps of the proof explicit. [*Hint:* use structural induction on the rules used to define L_2.]

(b) Prove that $L_1 \subseteq L_2$, making all steps of the proof explicit. [*Hint:* use induction on the length of a string in L_1.]