Free University of Bozen-Bolzano – Faculty of Computer Science
Master of Science in Computer Science
Theory of Computing – A.A. 2004/2005
Final exam – 4/2/2005 – Part 1

Time: 90 minutes

Problem 1.1 [4.5 points] Decide which of the following statements is TRUE and which is FALSE. You must give a brief explanation of your answer to receive full credit.

(a) For all languages L_1 and L_2, it holds that $(L_1^* \cdot L_2^*)^* = (L_1 \cup L_2)^*$.

(b) If L_1 is regular and L_2 is non-regular, then $L_1 \cdot L_2$ must be non-regular.

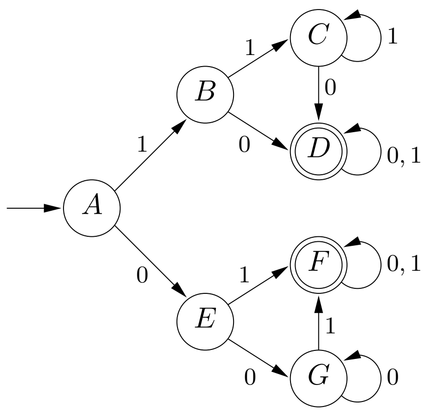
(c) There exists a language L such that L^* is not regular but $(L^*)^*$ is regular.

Problem 1.2 [1.5 points] Find a regular expression for the set of binary strings having 101 or 010 (or both) as substring.

Problem 1.3 [10 points] Describe in detail an algorithm to solve the following problem: Given a regular expression E with associated language $\mathcal{L}(E)$ over the alphabet Σ, construct a regular expression $\overline{E}$ such that $\mathcal{L}(\overline{E}) = \Sigma^* - \mathcal{L}(E)$. (Notice that set difference is not an operator that can be used in a regular expressions.)

Illustrate the algorithm on the regular expression $0^* \cdot 1^*$.

Problem 1.4 [6 points] Consider the following DFA A over $\{0,1\}$:



Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. Illustrate the steps of the algorithm you have followed to construct A_m.

Problem 1.5 [4 points] Show that the language $L = \{a^{k^2} \mid k \geqslant 0\}$ is not regular.

[*Hint*: Exploit the pumping lemma for regular languages.]

Problem 1.6 [4 points] Consider the grammar $G = (\{S\}, \{0,1\}, P, S)$, where P consists of the following productions

$$S \longrightarrow 0S1 \mid 1S0 \mid SS \mid \varepsilon$$

Prove that $\mathcal{L}(G) = \{w \in \{0,1\}^* \mid w \text{ has the same number of 0's and 1's}\}$.

[*Hint*: To show equality of the two languages you have to show inclusion in both directions. I.e., *(i)* each word of $\mathcal{L}(G)$ has the same number of 0's and 1's, and *(ii)* each word that has the same number of 0's and 1's is in $\mathcal{L}(G)$.]