Exercise 4

Consider the following problems:

1) Vertex-cover (VC)

Given an undirected graph \( G = (V, E) \) and an integer \( k \geq 2 \), is there a subset \( C \) of \( V \) with \( |C| \leq k \) such that \( C \) covers all edges of \( G \) (i.e., for each edge \( \{v_i, v_j\} \in E \) with \( v_i \neq v_j \), \( \{v_i, v_j\} \cap C \neq \emptyset \)).

2) Independent-set (IS)

Given an undirected graph \( G = (V, E) \) and an integer \( k \geq 2 \), is there a subset \( C \) of \( V \) with \( |C| \geq k \) such that for all \( v_i, v_j \in C \) with \( v_i \neq v_j \), \( \{v_i, v_j\} \notin E \).

3) Clique

Given an undirected graph \( G = (V, E) \) and an integer \( k \geq 2 \), is there a subset \( C \) of \( V \) with \( |C| \geq k \) such that for all \( v_i, v_j \in C \) with \( v_i \neq v_j \), \( \{v_i, v_j\} \in E \).

Show that VC, IS, and Clique can be reduced to each other in polynomial time.

N.B. In the definitions of VC, IS, and Clique we have ignored self-loops (since we required \( v_i \neq v_j \)).