Automata With ε-Transitions

Exercise 1
Design ε-NFA's for the following languages:

a) The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

b) The set of all strings consist of either of repeated one or more times, or 010 repeated one or more times.

Exercise 2
Consider the following ε-NFA:

\[
\begin{array}{c|cccc}
 & \epsilon & a & b & c \\
\hline
\rho & \{q,r\} & \emptyset & \{q\} & \{r\} \\
q & \emptyset & \{p\} & \{r\} & \{p,q\} \\
\ast r & \emptyset & \emptyset & \emptyset & \emptyset \\
\end{array}
\]

Ε-NFA 1

a) Compute the ε-closure of each state.
b) Which of the following strings are accepted by the automata:

\[\text{bb, bba, a, a, ab, a} \]
c) Convert the automata to a DFA.
d) Repeat steps a), b), c) for the following ε-NFA:

\[
\begin{array}{c|cccc}
 & \epsilon & a & b & c \\
\hline
\rho & \emptyset & \{p\} & \{q\} & \{r\} \\
q & \{p\} & \{q\} & \{r\} & \emptyset \\
\ast r & \{q\} & \{r\} & \emptyset & \{p\} \\
\end{array}
\]

Ε-NFA 2

Exercise 3
Convert the following NFA's to a DFA's:

Ε-NFA 1

Ε-NFA 2
2) a) \[ \text{Remember that:} \begin{align*}
q & \in \text{ECLOSE}(q) \\
p \in \text{ECLOSE}(q) \land r \in S(p, e) \Rightarrow r \in \text{ECLOSE}(q)
\end{align*} \]

\[ \text{E-NFA: } \text{ECLOSE}(p) = \{p, q, r\}, \text{ECLOSE}(q) = \{q\}, \text{ECLOSE}(r) = \{r\} \]

2) b) \[bb \not\in bba \times a \not\in \varepsilon \not\in aba \not\in \varepsilon\]

2) c) \[\text{From E-NFA to NFA} \]

\[\text{Remember that } S_N(9, a) = \hat{S}_E(9, a) = \text{ECLOSE}\left( \bigcup_{p \in \text{ECLOSE}(q)} S(p, a) \right)\]

The calculation of E-NFA go a follow:

\[\begin{align*}
\hat{S}_E(p, a) &= \text{ECLOSE}(S(p, a)) = \text{ECLOSE}(\{p\} \cup \emptyset) = \{p, q, r\} \\
\hat{S}_E(p, b) &= \text{ECLOSE}(S(p, b)) = \text{ECLOSE}(\{q\} \cup \{r\}) = \{q, r\} \\
\hat{S}_E(p, c) &= \text{ECLOSE}(S(p, c)) = \text{ECLOSE}(\{q\} \cup \emptyset) = \{q, r\} \]

\[\hat{S}_E(9, a) = \text{ECLOSE}(S(9, a)) = \text{ECLOSE}(\{p\}) = \{p, q, r\} \]

\[\hat{S}_E(9, b) = \text{ECLOSE}(S(9, b)) = \text{ECLOSE}(\{r\}) = \{r\} \]

\[\hat{S}_E(9, c) = \text{ECLOSE}(S(9, c)) = \text{ECLOSE}(\{p, q\}) = \{p, q, r\} \]

\[\hat{S}_E(r, a) = \text{ECLOSE}(S(r, a)) = \text{ECLOSE}(\emptyset) = \emptyset \]

\[\hat{S}_E(r, b) = \text{ECLOSE}(S(r, b)) = \text{ECLOSE}(\emptyset) = \emptyset \]

\[\hat{S}_E(r, c) = \text{ECLOSE}(S(r, c)) = \text{ECLOSE}(\emptyset) = \emptyset \]
we get the following NFA:

<table>
<thead>
<tr>
<th>S_N</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>{q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>q</td>
<td>{q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>* r</td>
<td>\emptyset</td>
<td>\emptyset</td>
<td>\emptyset</td>
</tr>
</tbody>
</table>

Note that \( \varepsilon \cdot \text{NFA}_1 \) accepts the empty string \( \varepsilon \).

From NFA to DFA

The subset construction yields:

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>{}</td>
<td>{}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>* {p}</td>
<td>{p, q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>* {9}</td>
<td>{p, q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>* {r}</td>
<td>\emptyset</td>
<td>\emptyset</td>
<td>\emptyset</td>
</tr>
<tr>
<td>* {p, q}</td>
<td>{p, q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>{p, q}</td>
<td>{p, q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>{q, r}</td>
<td>{p, q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>* {p, q, r}</td>
<td>{p, q, r}</td>
<td>{9, r}</td>
<td>{p, q, r}</td>
</tr>
</tbody>
</table>

Attention: The states: \{q\}, \{p, q\}, \{p, r\} are not reachable from the initial state, so they are excluded from the NFA diagram.
2) (d) 

a) $E \cdot NFA_2$: $ECLOSE(p) = \{p\}$, $ECLOSE(q) = \{p, q\}$, $ECLOSE(r) = \{p, q, r\}$

b) $bb \vee bba \vee aX \vee aX ab aX$

c) $E \cdot NFA \rightarrow NFA$

\[
\begin{array}{c|ccc}
S_N & a & b & c \\
\hline
\rightarrow p & \{p\} & \{p, q\} & \{p, q, r\} \\
p & \{p, q\} & \{p, q, r\} & \{p, q, r\} \\
q & \{p, q, r\} & \{p, q, r\} & \{p, q, r\} \\
\star r & \{p, q, r\} & \{p, q, r\} & \{p, q, r\} \\
\end{array}
\]

$NFA \rightarrow DFA$

The subset construction yields:

\[
\begin{array}{c|ccc}
\emptyset & a & b & c \\
\hline
\emptyset & \emptyset & \emptyset & \emptyset \\
\rightarrow \emptyset & \emptyset & \emptyset & \emptyset \\
\{p\} & \{p\} & \{p, q\} & \{p, q, r\} \\
\{q\} & \{p, q\} & \{p, q, r\} & \{p, q, r\} \\
\{r\} & \{p, q, r\} & \{p, q, r\} & \{p, q, r\} \\
\{p, q\} & \{p, q\} & \{p, q, r\} & \{p, q, r\} \\
\{p, r\} & \{p, r\} & \{p, q, r\} & \{p, q, r\} \\
\{q, r\} & \{p, q, r\} & \{p, q, r\} & \{p, q, r\} \\
\star \{p, q, r\} & \{p, q, r\} & \{p, q, r\} & \{p, q, r\} \\
\end{array}
\]

Other states are not reachable from the initial state.
3) $\varepsilon$-NFA$_1$

From $\varepsilon$-NFA$_1$ to NFA$_1$ (as above):

<table>
<thead>
<tr>
<th></th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow$</td>
<td>$p$</td>
<td>{p,q,r}</td>
<td>{q,r}</td>
</tr>
<tr>
<td>q</td>
<td>$\emptyset$</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>r</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>{r}</td>
</tr>
</tbody>
</table>

From NFA$_1$ to DFA$_1$ (subset construction):

<table>
<thead>
<tr>
<th>S0</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>$\downarrow$ {p}</td>
<td>{p,q,r}</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>{q}</td>
<td>$\emptyset$</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>r</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>{r}</td>
</tr>
<tr>
<td>{p,q}</td>
<td>{p,q,r}</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>r</td>
<td>$\emptyset$</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>{p,r}</td>
<td>{p,q,r}</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>{q,r}</td>
<td>$\emptyset$</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
<tr>
<td>{p,q,r}</td>
<td>{p,q,r}</td>
<td>{q,r}</td>
<td>{r}</td>
</tr>
</tbody>
</table>

$\varepsilon$-NFA$_2$

we only provide the final DFA:

[Diagram of DFA]