NON-DETERMINISTIC FINITE AUTOMATA

EXERCISE 1
Pick out one of the DFA’s from exercise E2 (16/10/2008) and two strings of length at least five over the corresponding alphabet. Show whether the strings are accepted or not by using the extended transition function.

EXERCISE 2
Give a NFA accepting the following language over the alphabet \{a,b\}: the set of strings that end with ba, bb, or baa. Then show that the string baaab is not accepted by the NFA.

EXERCISE 3
Give NFA’s accepting the following languages:

a) the set of strings over \{0,1,...,9\} such that the final digit has appeared before;

b) the set of strings over \{0,1,...,9\} such that the final digit has not appeared before;

c) the set of strings over \{0,1\} such that there are two 0’s separated by a number of positions that is a multiple of four.

EXERCISE 4
Convert the following NFA to a DFA

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[9_0]</td>
<td>{9_0}</td>
<td>{9_0,9_1}</td>
</tr>
<tr>
<td>[9_1]</td>
<td>{9_1}</td>
<td>{9_0,9_2}</td>
</tr>
<tr>
<td>*[9_2]</td>
<td>{9_1,9_2}</td>
<td>{9_0,9_1,9_2}</td>
</tr>
</tbody>
</table>
1) We choose the DFA from exercise 2:

![DFA Diagram]

and show that: \( yxyxyx \) is accepted; \( xyyxxxy \) is not accepted.

In other words, we show that:
\[
\begin{align*}
\hat{\delta}(q_0, y) &= q_1 \\
\hat{\delta}(q_0, yy) &= q_4 \\
\hat{\delta}(q_0, yyy) &= q_5 \\
\hat{\delta}(q_0, yyxy) &= q_4 \\
\hat{\delta}(q_0, yyxxy) &= q_4 \\
\end{align*}
\]

\[
\begin{align*}
\hat{\delta}(q_0, x) &= q_3 \\
\hat{\delta}(q_0, xy) &= q_4 \\
\hat{\delta}(q_0, xyy) &= q_5 \\
\hat{\delta}(q_0, xyxx) &= q_3 \\
\hat{\delta}(q_0, xyxxx) &= q_4 \\
\end{align*}
\]

Note that we have underlined the recursive calls of the extended transition function \( \hat{\delta} \) in our calculations.
2) The NFA looks as follows:

We show that \( baab \) is not accepted, i.e. that 
\[
\hat{S}(q_0, baab) = \{q_0, q_1, q_2\} \] and thus \( q_3 \notin \hat{S}(q_0, baab) \).

\[
\hat{S}(q_0, \varepsilon) = \{q_0\} \\
\hat{S}(q_0, b) = \hat{S}(q_0, b) = \{q_0, q_2\} \\
\hat{S}(q_0, ba) = \hat{S}(q_0, a) \cup \hat{S}(q_2, a) = \{q_0\} \cup \{q_2, q_3\} = \{q_0, q_2, q_3\} \\
\hat{S}(q_0, baa) = \hat{S}(q_0, a) \cup \hat{S}(q_2, a) \cup \hat{S}(q_3, a) = \{q_0\} \cup \{q_3\} \cup \emptyset = \{q_0, q_3\} \\
\hat{S}(q_0, baab) = \hat{S}(q_0, b) \cup \hat{S}(q_3, b) = \{q_0, q_1, q_2\} \cup \emptyset = \{q_0, q_1\}
\]

The following graph might help to get a more intuitive understanding of what is going on.
3) The NFA's look as follows:

a)

b)

c)
Note that the NFA looks as follows:

The subset construction will be as follows:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
<td>$\emptyset$</td>
</tr>
<tr>
<td>${q_0}$</td>
<td>${q_0}$</td>
<td>${q_0, q_1}$</td>
</tr>
<tr>
<td>${q_1}$</td>
<td>${q_1}$</td>
<td>${q_0, q_2}$</td>
</tr>
<tr>
<td>${q_2}$</td>
<td>${q_1, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
</tr>
<tr>
<td>${q_0, q_1}$</td>
<td>${q_0, q_1}$</td>
<td>${q_0, q_1, q_2}$</td>
</tr>
<tr>
<td>${q_0, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
</tr>
<tr>
<td>${q_1, q_2}$</td>
<td>${q_1, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
</tr>
<tr>
<td>${q_0, q_1, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
<td>${q_0, q_1, q_2}$</td>
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</tbody>
</table>