STATE MINIMIZATION

EXERCISE 1

Minimize the following DFA.

EXERCISE 2

Minimize the following DFA.

EXERCISE 3

Minimize the following DFA.
We start with the following initial partition: $C_1 = F = \{q_6, q_7\}$ and $C_2 = \Delta - F = \{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$.

**Step 1**

- $\{q_6, q_7\}$
- $\{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$

- $\{q_3, q_4\}$ $\{q_0, q_1, q_2, q_5, q_8\}$
- $\{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$

**Step 2**

- $\{q_3, q_4\}$ $\{q_0, q_1, q_2, q_5, q_8\}$

- $\{q_3, q_4\}$ $\{q_0, q_1, q_2, q_5, q_8\}$

- $\{q_0, q_1, q_2, q_5, q_8\}$ $\{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$

**Step 3**

- $\{q_0, q_1, q_2, q_5, q_8\}$ $\{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$

- $\{q_0, q_1, q_2, q_5, q_8\}$ $\{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$

**Final partition:** $\{q_3, q_4\}$ $\{q_0, q_1, q_2, q_3, q_4, q_5, q_8\}$

The minimized automaton looks as follows:

**Note:** $q_4$ is not reachable, thus it has been omitted from the minimized automaton.
2) Initial partition:
\[ C_1 = \{ q_0, q_1, q_2, q_3 \}, \quad C_2 = \{ q_4 \} \]

Step 0
\[ \{ q_0, q_1, q_2, q_3 \} \rightarrow \{ q_4 \} \]
\[ a, b, c \]

Step 1
\[ \{ q_0, q_1, q_2 \} \rightarrow \{ q_3 \} \rightarrow \{ q_4 \} \]
\[ a, c \rightarrow b \]
\[ \{ q_0, q_1, q_2, q_3, q_4 \} \rightarrow \{ q_0, q_1, q_2, q_3 \} \rightarrow \{ q_4 \} \]

Final partition: \[ \{ q_0, q_2 \} \rightarrow \{ q_3 \} \rightarrow \{ q_4 \} \]

The minimized automaton looks as follows:

3) Initial partition:
\[ C_1 = \{ q_0, q_2, q_6, q_8 \}, \quad C_2 = \{ q_1, q_3, q_4, q_5, q_7 \} \]

Step 0
\[ \{ q_0, q_2, q_6, q_8 \} \rightarrow \{ q_7 \} \rightarrow \{ q_5, q_7 \} \rightarrow \{ q_1, q_3, q_4, q_5, q_7 \} \]
\[ 0 \rightarrow 1 \]
\[ \{ q_0, q_2, q_6, q_8 \} \rightarrow \{ q_3, q_4 \} \rightarrow \{ q_5 \} \rightarrow \{ q_1, q_4, q_7 \} \]

Final partition: \[ \{ q_0, q_2, q_6, q_8 \} \rightarrow \{ q_7 \} \rightarrow \{ q_4 \} \]
3) can't

The minimized automaton looks as follows:

```
[Q0] --0--> [Q1]  
|      |      |      |
|      v      v      v
| 1 --1 --> 0 1 1

[Q3] --0--> [Q4]
```

Alternative solution:

One could first minimize the automata $A_2$ and $A_3$ of the last exercise in E8 (12/12/2008) and then apply the so-called product construction.