EXERCISE 1

Write regular expressions for the following languages:

a) The set of all strings consisting of zero or more a's, followed by zero or more b's, followed by zero or more c's;

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times;

c) The set of strings that either begin or end (or both) with 01;

d) The set of strings over \{x,y,z\} such that the number of y's is divisible by three;

e) The set of strings over \{0,1\} such that at least one of the last ten positions is a 1;

f) The set of strings over \{0,1,\ldots,9\} such that the final digit has appeared before;

g) The set of strings over \{0,1,\ldots,9\} such that the final digit has not appeared before.

EXERCISE 2

Give English descriptions of the languages over the alphabet \{a,b,c\} defined by the following regular expressions:

a) \((a+b)(a+b)(a+b)\)  b) \((\varepsilon+a)\,b\,\varepsilon\,c\)

c) \((cb)^* + b(cb)^* + (cb)^*c + b(cb)^*c\)

EXERCISE 3

a) Show that for every regular language \(L\) we have \((L^*)^* = L^*\).

b) Show that for all regular languages \(L\) and \(M\) we have \((L^*M^*)^* = (L\cup M)^*\).  [Note: \((L\cup M)^* = L((L\cup M))^*\)]
SOLUTIONS (20/11/2008)

1) a) a * b * c *

1) b) (01)(01)* + (010)(010)* or (01)* + (010)*

1) c) (01)(0+1)* + (0+1)*(01)

Note: we assume that the strings are over {0, 1}.

1) d) ((x+r)* y (x+r)* y (x+r)* y (x+r)* y (x+r)* y (x+r)* y (x+r))

1) e) Let \( E_i = \frac{(0+1)^i \cdot (0+1)^i \cdot (0+1)^i}{\text{i times \ (i times}} \), \( i \in \{0, 1, \ldots, 9\} \).

Then \( E = (0+1)^* (E_0 + E_1 + \ldots + E_9) \).

1) f) Let \( E_d = 0+1+\ldots+9 \). Then \( E = E_d + E_d 0 + E_d 1 + \ldots + E_d 9 \).

1) g) Let \( E_0 = 1+2+\ldots+9 \), \( E_i = 0+\ldots+(i-1)+(i+1)+\ldots+9 \) \( (1 \leq i \leq 8) \), \( E_9 = 0+1+\ldots+8 \), and \( E_d = 0+1+\ldots+9 \).

Then \( E = E_d + E_d 0 + E_d 1 + \ldots + E_d 9 \). (Also: \( E = E_d 0 + E_d 1 + \ldots + E_d 9 \).)

2) a) The set of all strings of length three that do not contain the symbol c: \{aaa, aab, aba, abb, baa, bab, bba, bbb\}.

2) b) The set of all strings with exactly one b, eventually preceded by an a and/or followed by a c: \{b, ab, bc, abc\}.

2) c) The set of all strings consisting of alternating b's and c's. Alternative regular expressions for the language are:

- \((\varepsilon+c)(bc)^* (\varepsilon+b)\)
- \((bc)^* + (cb)^* + c(bc)^* + b(cb)^*\)
3a) We have to show that \( L^* \subseteq (L^*)^* \) and \((L^*)^* \subseteq L^* \).

\( L^* \subseteq (L^*)^* \)

Trivial since \((L^*)^* \) is defined as \( \{ \epsilon \} \cup \cup L^* L^* U \ldots \).

\( (L^*)^* \subseteq L^* \)

given \( w \in (L^*)^* \) we have to show that \( w \in L^* \).

If \( w \in (L^*)^* \) then there exists \( n \in \mathbb{N} \) such that \( w = w_1 \ldots w_n \) where \( w_i \in L^* \) (1 \( \leq \) i \( \leq \) n). Since, for all \( i \in \{1, 2, \ldots, n\} \), there exists \( m_i \in \mathbb{N} \) such that \( w_i = w_{i1} \ldots w_{in} \) where \( w_{ij} \in L \) (1 \( \leq \) j \( \leq \) \( m_i \)). We have that \( w = (w_{11} \ldots w_{1m_1}) \ldots (w_{nm} \ldots w_{nm}) \).

Thus \( w \in L^* \).

3b) We have to show that \((L^* M^*)^* \subseteq (L M^*)^* \) and \((L M^*)^* \subseteq (L^* M^*)^* \).

\( (L^* M^*)^* \subseteq (L M^*)^* \)

given \( w \in (L^* M^*)^* \) we have to show that \( w \in (L M^*)^* \).

If \( w \in (L^* M^*)^* \) then \( w = w_1 \ldots w_m \) where \( w_i \in L^* M^* \). Since, for all \( i \in \{1, 2, \ldots, n\} \), \( w_i = u_{i1} \ldots u_{im} \) where \( u_{ij} \in L \) and \( u_{ij} \in M \) we have that:

\[ w = (u_{11} \ldots u_{1m_1}) \ldots (u_{nm} \ldots u_{nm}) \]

Thus \( w \in (L M^*)^* \).

\( (L M^*)^* \subseteq (L^* M^*)^* \)

given \( w \in (L M^*)^* \) we have to show that \( w \in (L^* M^*)^* \).

If \( w \in (L M^*)^* \) then \( w = w_1 \ldots w_m \) where each \( w_i \) is in either \( L \) or \( M \). If \( w_i \) is in \( L \) then \( w_i \) is also in \( L^* \) and, since \( \epsilon \) is in \( M^* \), \( w_i = w_i \epsilon \) is in \( L^* M^* \).

Similarly, if \( w_i \) is in \( M \) then \( w_i \) is in \( L^* M^* \).

Thus \( w \in (L^* M^*)^* \).