EXERCISE 1

Design $\varepsilon$-NFA's for the following languages.

a) The set of strings consisting of zero or more a's, followed by zero or more b's, followed by zero or more c's.

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

EXERCISE 2

Consider the following $\varepsilon$-NFA's.

\[
\begin{array}{c|ccc}
   & \varepsilon & a & b & c \\
\hline
P & \{q_1\} & \emptyset & \{q_1\} & \{q_1\} \\
Q & \emptyset & \{p_1\} & \{r_1\} & \{p_4\} \\
R & \emptyset & \emptyset & \emptyset & \emptyset \\
\end{array}
\]

$\varepsilon$-NFA_1

\[
\begin{array}{c|ccc}
   & \varepsilon & a & b & c \\
\hline
P & \emptyset & \{p_1\} & \{q_1\} & \{r_1\} \\
Q & \{p_1\} & \{q_1\} & \{r_1\} & \emptyset \\
R & \emptyset & \emptyset & \emptyset & \{p_1\} \\
\end{array}
\]

$\varepsilon$-NFA_2

a) Compute the $\varepsilon$-closure of each state.

b) Give all the strings of length three or less accepted by the automata.

c) Convert the automata to DFA's.

EXERCISE 3

Convert the following $\varepsilon$-NFA's to a DFA's.

- $\varepsilon$-NFA_1
- $\varepsilon$-NFA_2
1) a) 

2) a) Remember that: \( q \in \text{ECLOSE}(q) \) 
   \[ p \in \text{ECLOSE}(q) \land r \in S(p, \varepsilon) \Rightarrow r \in \text{ECLOSE}(q) \]

- \( \varepsilon \)-NFA_1: \( \text{ECLOSE}(p) = \{ p, q, r \} \), \( \text{ECLOSE}(q) = \{ q \} \), \( \text{ECLOSE}(r) = \{ r \} \)
- \( \varepsilon \)-NFA_2: \( \text{ECLOSE}(p) = \{ p \} \), \( \text{ECLOSE}(q) = \{ p, q \} \), \( \text{ECLOSE}(r) = \{ p, q, r \} \)

2) b) \( \varepsilon \)-NFA_1
   All strings except for: bba, bbb, bbc

- \( \varepsilon \)-NFA_2
   All strings except for: \( \varepsilon, a, b, aa, ab, ba, aaa, aab, aba, baa \)

2) c) From \( \varepsilon \)-NFA to NFA
   Remember that \( S_N(q_1a) = \delta_N(q_1a) = \text{ECLOSE}(U_{p \in \text{ECLOSE}(q_1)} S(p, a)) \)
The calculations for ε-NFA₁ go as follows:

\[
\begin{align*}
\delta_ε(p,a) &= ECLOSE(\delta(p,a) \cup \delta(qₐ,a) \cup \delta(qₐ,a)) = ECLOSE(\emptyset \cup \{p\} \cup \emptyset) = \{p,q₁r\}
\delta_ε(p,b) &= ECLOSE(\delta(p,b) \cup \delta(q₂,b) \cup \delta(qₐ,b)) = ECLOSE(\emptyset \cup \{q₂\} \cup \emptyset) = \{q₂\}
\delta_ε(p,c) &= ECLOSE(\delta(p,c) \cup \delta(qₐ,c) \cup \delta(qₐ,c)) = ECLOSE(\emptyset \cup \{p,qₐ\} \cup \emptyset) = \{p,qₐ\}
\delta_ε(qₐ,a) &= ECLOSE(\delta(qₐ,a)) = ECLOSE(\{p\}) = \{p,q₁r\}
\delta_ε(qₐ,b) &= ECLOSE(\delta(qₐ,b)) = ECLOSE(\{qₐ\}) = \{qₐ\}
\delta_ε(qₐ,c) &= ECLOSE(\delta(qₐ,c)) = ECLOSE(\{p,qₐ\}) = \{p,qₐ\}
\delta_ε(q₂,a) &= ECLOSE(\delta(q₂,a)) = ECLOSE(\emptyset) = \emptyset
\delta_ε(q₂,b) &= ECLOSE(\delta(q₂,b)) = ECLOSE(\emptyset) = \emptyset
\delta_ε(q₂,c) &= ECLOSE(\delta(q₂,c)) = ECLOSE(\emptyset) = \emptyset
\end{align*}
\]

By doing the same calculations for ε-NFA₂ we get the following NFA's:

<table>
<thead>
<tr>
<th>SN</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>→ P</td>
<td>{p,q₁r}</td>
<td>{q₁, r}</td>
<td>{p,q₁r}</td>
</tr>
<tr>
<td>P</td>
<td>{p,q₁r}</td>
<td>{q₁, r}</td>
<td>{p,q₁r}</td>
</tr>
<tr>
<td>r</td>
<td>∅</td>
<td>∅</td>
<td>∅</td>
</tr>
<tr>
<td>→ P</td>
<td>{p}</td>
<td>{p,q₁}</td>
<td>{p,q₁r}</td>
</tr>
<tr>
<td>P</td>
<td>{p,q₁}</td>
<td>{p,q₁r}</td>
<td>{p,q₁r}</td>
</tr>
<tr>
<td>r</td>
<td>{p,q₁r}</td>
<td>{p,q₁r}</td>
<td>{p,q₁r}</td>
</tr>
</tbody>
</table>

(NFA₁
(note that ε-NFA₁ accepts the empty string ε))

NFA₂

From NFA to DFA

This was already done in exercise E4 (06/12/2008)
3) \( e-NFA_2 \)

From \( e-NFA_2 \) to NFA (as above)

<table>
<thead>
<tr>
<th>( s_0 )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p \to p )</td>
<td>( p,q,r )</td>
<td>( q,r )</td>
<td>( r )</td>
</tr>
<tr>
<td>( p )</td>
<td>( \phi )</td>
<td>( q,r )</td>
<td>( r )</td>
</tr>
<tr>
<td>( r )</td>
<td>( \phi )</td>
<td>( \phi )</td>
<td>( r )</td>
</tr>
</tbody>
</table>

From NFA (subset construction)

<table>
<thead>
<tr>
<th>( s_D )</th>
<th>( x )</th>
<th>( y )</th>
<th>( z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = \phi )</td>
<td>( A )</td>
<td>( A )</td>
<td>( A )</td>
</tr>
<tr>
<td>( B = { p } )</td>
<td>( H )</td>
<td>( G )</td>
<td>( D )</td>
</tr>
<tr>
<td>( C = { q } )</td>
<td>( A )</td>
<td>( G )</td>
<td>( D )</td>
</tr>
<tr>
<td>( D = { r } )</td>
<td>( A )</td>
<td>( A )</td>
<td>( D )</td>
</tr>
<tr>
<td>( E = { p,q } )</td>
<td>( H )</td>
<td>( G )</td>
<td>( D )</td>
</tr>
<tr>
<td>( F = { p,r } )</td>
<td>( H )</td>
<td>( G )</td>
<td>( D )</td>
</tr>
<tr>
<td>( G = { q,r } )</td>
<td>( A )</td>
<td>( G )</td>
<td>( D )</td>
</tr>
<tr>
<td>( H = { p,q,r } )</td>
<td>( H )</td>
<td>( G )</td>
<td>( D )</td>
</tr>
</tbody>
</table>

\( e-NFA_2 \)

We only provide the final DFA