EXERCISE 1

Give a DFA accepting the following language over the alphabet \{a,b\}: the set of all strings such that the second last symbol is b.

EXERCISE 2

Give a DFA accepting the following language over the alphabet \{x,y\}: the set of strings that either begin or end (or both) with \(yx\).

EXERCISE 3

Give a DFA accepting the following language over the alphabet \{0,1\}: the set of strings such that the number of 0's is divisible by five and the number of 1's is divisible by three.

EXERCISE 4

Give a DFA accepting the following language over the alphabet \{a,b,c,d\}: the set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.
1) The DFA looks as follows:

We show that it accepts bbaaba but not abbbbaa

- \((q_0, b) \rightarrow q_2, (q_1, b) \rightarrow q_2, (q_2, a) \rightarrow q_3, (q_3, a) \rightarrow q_0\)
- \((q_0, b) \rightarrow q_1, (q_1, a) \rightarrow q_2\)  \(q_2\) is a final state
- \((q_0, a) \rightarrow q_0, (q_0, b) \rightarrow q_1, (q_1, b) \rightarrow q_2, (q_2, b) \rightarrow q_2\)
- \((q_2, a) \rightarrow q_3, (q_3, a) \rightarrow q_0\)  \(q_0\) is not a final state

2) The DFA looks as follows:
3) The DFA looks as follows:

4) The DFA looks as follows:
NON-DETERMINISTIC FINITE AUTOMATA

EXERCISE 1

Pick out one of the DFA's from exercise E2 (16/10/2008) and two strings of length at least five over the corresponding alphabet. Show whether the strings are accepted or not by using the extended transition function.

EXERCISE 2

Give a NFA accepting the following language over the alphabet \{a,b\}: the set of strings that end with ba, bb, or baa. Then show that the string baab is not accepted by the NFA.

EXERCISE 3

Give NFA's accepting the following languages:

a) the set of strings over \{0,1,...,9\} such that the final digit has appeared before;
b) the set of strings over \{0,1,...,9\} such that the final digit has not appeared before;
c) the set of strings over \{0,1\} such that there are two 0's separated by a number of positions that is a multiple of four.
1) We choose the DFA from exercise 2

\[
\begin{align*}
\hat{\delta}(q_0, \varepsilon) &= q_0 \\
\hat{\delta}(q_0, y) &= \delta(\delta(q_0, \varepsilon), y) = \delta(q_0, y) = q_2 \\
\hat{\delta}(q_0, yy) &= \delta(\delta(q_0, y), y) = \delta(q_2, y) = q_4 \\
\hat{\delta}(q_0, yyx) &= \delta(\delta(q_0, yy), x) = \delta(q_4, x) = q_5 \\
\hat{\delta}(q_0, yyyy) &= \delta(\delta(q_0, yyyy), y) = \delta(q_5, y) = q_4 \\
\hat{\delta}(q_0, yyxy) &= \delta(\delta(q_0, yyx), y) = \delta(q_5, y) = q_4 \\
\hat{\delta}(q_0, yyyyy) &= \delta(\delta(q_0, yyyy), x) = \delta(q_4, x) = q_5
\end{align*}
\]

and show that: \( yyyxyx \) is accepted; \( xyyxxxyy \) is not accepted.

In other words, we show that:
\[
\begin{align*}
\hat{\delta}(q_0, yyyyxyy) &= q_5 \in F; \\
\hat{\delta}(q_0, xyyxxxyy) &= q_4 \notin F.
\end{align*}
\]

Note that we have underlined the recursive calls of the extended transition function \( \hat{\delta} \) in our calculations.
2) The NFA looks as follows:

We show that \( baab \) is not accepted, i.e. that 
\[ \hat{S}(q_0, baab) = \{q_0, q_1\} \] 
and thus \( q_3 \notin \hat{S}(q_0, baab) \).

\[ \hat{S}(q_0, \varepsilon) = \{q_0\} \]
\[ \hat{S}(q_0, b) = \hat{S}(q_0, b) = \{q_0, q_2\} \]
\[ \hat{S}(q_0, ba) = \hat{S}(q_0, a) \cup \hat{S}(q_2, a) = \{q_0\} \cup \{q_2, q_3\} = \{q_0, q_2, q_3\} \]
\[ \hat{S}(q_0, baa) = \hat{S}(q_0, a) \cup \hat{S}(q_2, a) \cup \hat{S}(q_3, a) = \{q_0\} \cup \{q_3\} \cup \emptyset = \{q_0, q_3\} \]
\[ \hat{S}(q_0, baab) = \hat{S}(q_0, b) \cup \hat{S}(q_3, b) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\} \]

The following graph might help to get a more intuitive understanding of what is going on.

![Graph showing transitions for \( baab \)]
3) The NFA's look as follows:

a)

b)

c)