Overview of Part 3: Information integration

- Introduction to data integration
  - Basic issues in data integration
  - Logical formalization

- Query answering in the absence of constraints
  - Global-as-view (GAV) setting
  - Local-as-view (LAV) and GLAV setting

- Query answering in the presence of constraints
  - The role of integrity constraints
  - Global-as-view (GAV) setting
  - Local-as-view (LAV) and GLAV setting

- Concluding remarks

Outline

- Basic issues in data integration
- Data integration: Logical formalization
Basic issues in data integration

- The problem of data integration
- Variants of data integration
- Problems in data integration

Data integration: Logical formalization

Conceptual architecture of a data integration system

Relevance of data integration

Growing market

- One of the major challenges for the future of IT
- At least two contexts
  - Intra-organization data integration (e.g., EIS)
  - Inter-organization data integration (e.g., integration on the Web)
Basic issues in data integration

Data integration: Logical formalization

Variants of data integration

Architectures for integrated access to distributed data

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Part 3: Information Integration

KBDB – 2008/2009 (10/167)

Data integration: Available industrial efforts

- Distributed database systems
- Information on demand
- Tools for source wrapping
- Tools based on database federation, e.g., DB2 Information Integrator
- Distributed query optimization

Database federation tools: Characteristics

- **Physical transparency**, i.e., masking from the user the physical characteristics of the sources
- **Heterogeneity**, i.e., federating highly diverse types of sources
- **Extensibility**
- **Autonomy** of data sources
- **Performance**, through distributed query optimization

However, current tools do not (directly) support **logical (or conceptual) transparency**.

Logical transparency

Basic ingredients for achieving logical transparency:

- The global schema (ontology) provides a conceptual view that is independent from the sources.
- The global schema is described with a semantically rich formalism.
- The mappings are the crucial tools for realizing the independence of the global schema from the sources.
- Obviously, the formalism for specifying the mapping is also a crucial point.

All the above aspects are not appropriately dealt with by current tools. This means that data integration cannot be simply addressed on a tool basis.
Mediator based data integration

- Queries are expressed over a **global schema** (a.k.a. mediated schema, enterprise model, ...).
- Data are stored in a set of sources.
- **Wrappers** access the sources (provide a view in a uniform data model of the data stored in the sources).
- **Mediators** combine answers coming from wrappers and/or other mediators.

Peer-to-peer data integration

- Materialization of the global schema

\[ \text{Materialize} \]

\[ \text{Sources} \]

\[ \text{Global Schema} \]

Operations:
- \( \text{Answer}(Q, P_i) \)
- \( \text{Materialize}(P_i) \)
Main problems in data integration

- How to construct the global schema.
- (Automatic) source wrapping.
- How to discover mappings between sources and global schema.
- Limitations in mechanisms for accessing sources.
- Data extraction, cleaning, and reconciliation.
- How to process updates expressed on the global schema and/or the sources ("read/write" vs. "read-only" data integration).
- How to model the global schema, the sources, and the mappings between the two.
- How to answer queries expressed on the global schema.
- How to optimize query answering.

The modeling problem

Basic questions:
- How to model the global schema:
  - data model
  - constraints
- How to model the sources:
  - data model (conceptual and logical level)
  - access limitations
  - data values (common vs. different domains)
- How to model the mapping between global schemas and sources.
- How to verify the quality of the modeling process.

A word of caution: Data modeling (in data integration) is an art. Theoretical frameworks can help humans, not replace them.

The querying problem

- A query expressed in terms of the global schema must be reformulated in terms of (a set of) queries over the sources and/or materialized views.
- The computed sub-queries are shipped to the sources, and the results are collected and assembled into the final answer.
- The computed query plan should guarantee:
  - completeness of the obtained answers wrt the semantics;
  - efficiency of the whole query answering process;
  - efficiency in accessing sources.
- This process heavily depends on the approach adopted for modeling the data integration system.

This is the problem that we want to address in this part of the course.

Outline

- Basic issues in data integration
- Data integration: Logical formalization
  - Semantics of a data integration system
  - Queries to a data integration system
  - Formalizing the mapping
  - Formalizing GAV data integration systems
  - Formalizing LAV data integration systems
  - Formalizing GLAV data integration systems
Def.: **Data integration system** $\mathcal{I}$

A data integration system is a triple $\mathcal{I} = (\mathcal{G}, \mathcal{S}, \mathcal{M})$, where:

- $\mathcal{G}$ is the global schema
  - *i.e., a logical theory over a relational alphabet $\mathcal{A}_G$.*
- $\mathcal{S}$ is the source schema
  - *i.e., simply a relational alphabet $\mathcal{A}_S$ disjoint from $\mathcal{A}_G$.*
- $\mathcal{M}$ is the mapping between $\mathcal{S}$ and $\mathcal{G}$.

We consider different approaches to the specification of mappings.

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**Basic issues in data integration**

The domain $\Delta$ is fixed, and we do not distinguish an element of $\Delta$ from the constant denoting it $\sim$ **standard names**.

Queries to $\mathcal{I}$ are relational calculus queries over the alphabet $\mathcal{A}_G$ of the global schema.

When "evaluating" $q$ over $\mathcal{I}$, we have to consider that for a given source database $\mathcal{D}$, there may be **many global databases** $\mathcal{B}$ in $\text{Sem}_I(\mathcal{D})$.

We consider those answers to $q$ that hold for all global databases in $\text{Sem}_I(\mathcal{D})$ $\sim$ **certain answers**.

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**Semantics of a data integration system**

Which are the dbs that satisfy $\mathcal{I}$, i.e., the logical models of $\mathcal{I}$?

- We refer only to dbs over a **fixed infinite domain** $\Delta$ of elements.
- We start from the data present in the sources: these are modeled through a source database $\mathcal{D}$ over $\Delta$ (also called source model), fixing the extension of the predicates of $\mathcal{A}_S$.
- The dbs for $\mathcal{I}$ are logical interpretations for $\mathcal{A}_G$, called **global dbs**.

**Def.: Semantics of a data integration system**

The set of databases for $\mathcal{A}_G$ that satisfy $\mathcal{I} = (\mathcal{G}, \mathcal{S}, \mathcal{M})$ relative to $\mathcal{D}$ is:

$$
\text{Sem}_I(\mathcal{D}) = \{ \mathcal{B} | \mathcal{B} \text{ is a global database that is legal wrt } \mathcal{G} \text{ and that satisfies } \mathcal{M} \text{ wrt } \mathcal{D} \}
$$

What it means to satisfy $\mathcal{M}$ wrt $\mathcal{D}$ depends on the nature of $\mathcal{M}$.

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**Queries to a data integration system $\mathcal{I}$**

- Query answering is **logical implication**.
- Complexity is measured mainly **the size of the source db $\mathcal{D}$**.
  - i.e., we consider **data complexity**.
- We consider the problem of deciding whether $\vec{c} \in \text{cert}(q, \mathcal{I}, \mathcal{D})$, for a given tuple $\vec{c}$ of constants.

**Semantics of queries to $\mathcal{I}$**

**Def.: Certain answers** in a data integration system

Given $q$, $\mathcal{I}$, and $\mathcal{D}$, the set of **certain answers** to $q$ wrt $\mathcal{I}$ and $\mathcal{D}$ is

$$
\text{cert}(q, \mathcal{I}, \mathcal{D}) = \{ (c_1, \ldots, c_n) \in q^{\mathcal{B}} | \text{ for all } \mathcal{B} \in \text{Sem}_I(\mathcal{D}) \}
$$
Databases with incomplete information, or knowledge bases

- **Traditional database**: one model of a first-order theory.
  Query answering means evaluating a formula in the model.

- **Database with incomplete information, or knowledge base**: set of models (specified, for example, as a restricted first-order theory).
  Query answering means computing the tuples that satisfy the query in all the models in the set.

There is a **strong connection** between query answering in data integration and query answering in databases with incomplete information under constraints (or, query answering in knowledge bases).

Query answering with incomplete information

- [Rei84]: relational setting, databases with incomplete information modeled as a first order theory
- [Var86]: relational setting, complexity of reasoning in closed world databases with unknown values
- Several approaches both from the DB and the KR community
- [vdM98]: survey on logical approaches to incomplete information in databases

The mapping

How is the mapping $M$ between $S$ and $G$ specified?

- Are the sources defined in terms of the global schema? Approach called **source-centric**, or **local-as-view**, or **LAV**.
- Is the global schema defined in terms of the sources? Approach called **global-schema-centric**, or **global-as-view**, or **GAV**.
- A mixed approach? Approach called **GLAV**.

GAV vs. LAV – Example

**Global schema**:
- `movie(Title, Year, Director)`
- `european(Director)`
- `review(Title, Critique)`

**Source 1**:
- $r_1(Title, Year, Director)$ since 1960, european directors

**Source 2**:
- $r_2(Title, Critique)$ since 1990

**Query**: Title and critique of movies in 1998
- $q(t, r) \leftarrow \exists d. \text{movie}(t, 1998, d) \land \text{review}(t, r)$, in Datalog notation
- $q(t, r) \leftarrow \text{movie}(t, 1998, d), \text{review}(t, r)$
Formalization of GAV

In GAV (with sound sources), the mapping $M$ is a set of assertions:

$$\phi_S \leadsto g$$

one for each element $g$ in $A_S$, with $\phi_S$ a query over $S$ of the arity of $g$.

Given a source db $D$, a db $B$ for $G$ satisfies $M$ wrt $D$ if for each $g \in G$:

$$\phi^B \subseteq g^B$$

In other words, the assertion means:

$$\forall \vec{x}. \phi_S(\vec{x}) \rightarrow g(\vec{x})$$.

Given a source database, $M$ provides direct information about which data satisfy the elements of the global schema.

Relations in $G$ are views, and queries are expressed over the views. Thus, it seems that we can simply evaluate the query over the data satisfying the global relations (as if we had a single db at hand).

GAV – Example

**Global schema:**
- movie$(Title, Year, Director)$
- european$(Director)$
- review$(Title, Critique)$

**GAV:** to each relation in the global schema, $M$ associates a view over the sources:

$$q_1(t,y,d) \leftarrow r_1(t,y,d) \leadsto movie(t,y,d)$$
$$q_2(d) \leftarrow r_2(y,d) \leadsto european(d)$$
$$q_3(t,r) \leftarrow r_2(t,r) \leadsto review(t,r)$$

Logical formalization:

$$\forall t,y,d. \ r_1(t,y,d) \rightarrow movie(t,y,d)$$
$$\forall d. (\exists t, y. r_1(t,y,d)) \rightarrow european(d)$$
$$\forall t, r. r_2(t,r) \rightarrow review(t,r)$$

GAV – Example of query processing

The query

$$q(t,r) \leftarrow movie(t,1998,d), \ review(t,r)$$

is processed by means of unfolding, i.e., by expanding each atom according to its associated definition in $M$, so as to come up with source relations.

In this case:

$$q(t,r) \leftarrow movie(t,1998,d), \ review(t,r)$$

unfolding

$$q(t,r) \leftarrow r_1(t,1998,d), \ r_2(t,r)$$

GAV – Example of constraints

**Global schema containing constraints:**
- movie$(Title, Year, Director)$
- european$(Director)$
- review$(Title, Critique)$
- european_movie_60s$(Title, Year, Director)$

$$\forall t, y, d. \ european_movie_60s(t,y,d) \rightarrow movie(t,y,d)$$
$$\forall d, \exists t, y. \ european_movie_60s(t,y,d) \rightarrow european(d)$$

**GAV mappings:**

$$q_1(t,y,d) \leftarrow r_4(t,y,d) \leadsto european_movie_60s(t,y,d)$$
$$q_2(d) \leftarrow r_4(t,y,d) \leadsto european(d)$$
$$q_3(t,r) \leftarrow r_2(r) \leadsto review(t,r)$$
Formalization of LAV

In LAV (with sound sources), the mapping $M$ is a set of assertions:

\[ s \sim \phi_g \]

one for each source element $s$ in $A_S$, with $\phi_g$ a query over $G$.

Given a source db $D$, a db $B$ for $G$ satisfies $M$ wrt $D$ if for each $s \in S$:

\[ s \in D, s \subseteq \phi_g \in B \]

In other words, the assertion means:

\[ \forall \vec{x}, s(\vec{x}) \rightarrow \phi_g(\vec{x}). \]

The mapping $M$ and the source database $D$ do not provide direct information about which data satisfy the global schema.

Sources are views, and we have to answer queries on the basis of the available data in the views.

GAV and LAV – Comparison

**GAV:** (e.g., Carnot, SIMS, Tsimmis, IBIS, Momis, Mastro, . . .)

- Quality depends on how well we have compiled the sources into the global schema through the mapping.
- Whenever a source changes or a new one is added, the global schema needs to be reconsidered.
- Query processing can be based on some sort of unfolding (query answering looks easier – without constraints).

**LAV:** (e.g., Information Manifold, DWQ, Picssel)

- Quality depends on how well we have characterized the sources.
- High modularity and extensibility (if the global schema is well designed, when a source changes, only its definition is affected).
- Query processing needs reasoning (query answering complex).

LAV – Example

**Global schema:**

- $movie(Title, Year, Director)$
- $european(Director)$
- $review(Title, Critique)$

**LAV:** to each source relation, $M$ associates a view over the global schema:

- $r_1(t, y, d) \sim q_1(t, y, d) \leftarrow movie(t, y, d), european(d), y \geq 1960$
- $r_2(t, r) \sim q_2(t, r) \leftarrow movie(t, y, d), review(t, r), y \geq 1990$

The query $q(t, r) \leftarrow movie(t, 1998, d), review(t, r)$ is processed by means of an inference mechanism that aims at re-expressing the atoms of the global schema in terms of atoms at the sources. In this case:

\[ q(t, r) \leftarrow r_2(t, r), r_1(t, 1998, d), r_1(t, 1999, d). \]

Beyond GAV and LAV: GLAV

In GLAV (with sound sources), the mapping $M$ is a set of assertions:

\[ \phi_S \sim \phi_g \]

with $\phi_S$ a query over $S$, and $\phi_g$ a query over $G$ of the same arity as $\phi_S$.

Given a source db $D$, a db $B$ for $G$ satisfies $M$ wrt $D$ if for each $\phi_S \sim \phi_g$ in $M$:

\[ \phi_S \subseteq \phi_g \in B \]

In other words, the assertion means:

\[ \forall \vec{x}, \phi_S(\vec{x}) \rightarrow \phi_g(\vec{x}). \]

As in LAV, the mapping $M$ does not provide direct information about which data satisfy the global schema.

To answer a query $q$ over $G$, we have to infer how to use $M$ in order to access the source database $D$. 
GLAV – A technical observation

In GLAV (with sound sources), the mapping $M$ is constituted by a set of assertions:

$$\phi_S \leadsto \phi_G$$

Each such assertion can be rewritten wlog by introducing a new predicate $r$ of the same arity as the two queries and replace the assertion with the following two:

$$\phi_S \leadsto r$$

$$r \leadsto \phi_G$$

In other words, we replace $\forall \vec{x}. \phi_S(\vec{x}) \rightarrow \phi_G(\vec{x})$ with $\forall \vec{x}. \phi_S(\vec{x}) \rightarrow r(\vec{x})$ and $\forall \vec{x}. r(\vec{x}) \rightarrow \phi_G(\vec{x})$.

Note: The new relations $r$ can considered to be part of $G$ (but should not appear in user queries). Hence, $\phi_G \leadsto r$ is like a GAV mapping assertion, while $r \leadsto \phi_G$ is a form of constraint on $G$.

Outline

- Query answering in GAV without constraints
- Query answering in (G)LAV without constraints
Query answering in different approaches

The problem of query answering comes in different forms, depending on several parameters:

- **Global schema**
  - without constraints (i.e., empty theory)
  - with constraints
- **Mapping**
  - GAV
  - LAV (or GLAV)
- **Queries**
  - user queries
  - queries in the mapping

Conjunctive queries

We recall the following definition:

**Definition:** A conjunctive query (CQ) is a query of the form

\[ q(\vec{x}) \leftarrow \exists \vec{y}_1. r_1(\vec{x}_1, \vec{y}_1) \land \cdots \land r_m(\vec{x}_m, \vec{y}_m) \]

where

- \( \vec{x} \) is the union of the \( \vec{x}_i \)'s, called the distinguished variables;
- \( \vec{y} \) is the union of the \( \vec{y}_i \)'s, called the non-distinguished variables;
- \( r_1, \ldots, r_m \) are relation symbols (not built-in predicates).

Unless otherwise specified, we consider conjunctive queries, both as user queries and as queries in the mapping.

Incompleteness and inconsistency

Query answering heavily depends upon whether incompleteness/inconsistency shows up:

<table>
<thead>
<tr>
<th>Constraints in ( \mathcal{G} )</th>
<th>Type of mapping</th>
<th>Incompleteness</th>
<th>Inconsistency</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>GAV</td>
<td>yes / no</td>
<td>no</td>
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<tr>
<td>no</td>
<td>(G)LAV</td>
<td>yes</td>
<td>no</td>
</tr>
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<td>yes</td>
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Outline

- Query answering in GAV without constraints
- Retrieved global database
- Query answering via unfolding
- Query answering in (G)LAV without constraints
GAV data integration systems without constraints

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<td>(G)LAV</td>
<td>yes</td>
<td>yes</td>
</tr>
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</table>

Def.: **Retrieved global database**

Given a source database $D$, we call retrieved global database, denoted $\mathcal{M}(D)$, the global database obtained by “applying” the queries in the mapping, and “transferring” to the elements of $\mathcal{G}$ the corresponding retrieved tuples.

**Example of Retrieved Global Database**

Consider $I = (\mathcal{G}, S, M)$, with

Global schema $\mathcal{G}$:  
- student($\text{Code}$, $\text{Name}$, $\text{City}$)  
- university($\text{Code}$, $\text{Name}$)  
- enrolled($\text{Scode}$, $\text{Ucode}$)

Source schema $S$: relations $s_1$($\text{Scode}$, $\text{Sname}$, $\text{City}$, $\text{Age}$), $s_2$($\text{Ucode}$, $\text{Uname}$), $s_3$($\text{Scode}$, $\text{Ucode}$)

Mapping $M$:

$q_1(c, n, ci) \leftarrow s_1(c, n, ci, a) \sim \text{student}(c, n, ci)$
$q_2(c, n) \leftarrow s_2(c, n) \sim \text{university}(c, n)$
$q_3(s, u) \leftarrow s_3(s, u) \sim \text{enrolled}(s, u)$

Example of source database $D$ and corresponding retrieved global database $\mathcal{M}(D)$. 

Example of retrieved global database $\mathcal{M}(D)$.
GAV – Minimal model

GAV mapping assertions $\phi_S \rightarrow g$ have the logical form:

$$\forall \bar{x}. \phi_S(\bar{x}) \rightarrow g(\bar{x})$$

where $\phi_S$ is a conjunctive query over the source relations, and $g$ is an element of $G$.

In general, given a source database $D$, there are several databases legal wrt $G$ that satisfy $M$ wrt $D$.

However, it is easy to see that $M(D)$ is the intersection of all such databases, and therefore, is the unique "minimal" model of $I$.

GAV – Query answering via unfolding

The unfolding $M$ of a query $q$ over $G$ is the query obtained from $q$ by substituting every symbol $g$ in $q$ with the query $\phi_S$ that $M$ associates to $g$. We denote the unfolding of $q$ wrt $M$ with $\text{unf}_M(q)$.

Observations:
- Since $M(D)$ is the unique minimal model of $I$, if $q$ is a CQ or an UCQ, then $\bar{c} \in \text{cert}(q,I,D)$ iff $\bar{c} \in q^{M(D)}$.
- $\text{unf}_M(q)$ is a query expressed over the source schema $S$.
- Evaluating $q$ over $M(D)$ is equivalent to evaluating $\text{unf}_M(q)$ over $D$, i.e., $\bar{c} \in q^{M(D)}$ iff $\bar{c} \in \text{unf}_M(q)$.\Pound.
- Hence, $\bar{c} \in \text{cert}(q,I,D)$ iff $\bar{c} \in q^{M(D)}$ iff $\bar{c} \in \text{unf}_M(q)$.\Pound.
- $\sim$ Unfolding suffices for query answering in GAV without constraints.
Observations:

- If $q$ is a CQ or a UCQ, then $\text{unf}_M(q)$ is a first-order query (in fact, a CQ or UCQ).
- $|\mathcal{M}(\mathcal{D})|$ is polynomial wrt $|\mathcal{D}|$.

Hence, we obtain the following results.

**Theorem**

In a GAV data integration system without constraints, answering unions of conjunctive queries is $\logspace$ in data complexity and polynomial in combined complexity.

**Outline**

1. Query answering in GAV without constraints
2. Query answering in (G)LAV without constraints
   - (G)LAV and incompleteness
   - Approaches to query answering in (G)LAV
     - (G)LAV: Direct methods (aka view-based query answering)
     - (G)LAV: Query answering by (view-based) query rewriting
   - Constraints in $\mathcal{G}$
     | Type of mapping | Incompleteness | Inconsistency |
     |-----------------|----------------|--------------|
     | no              | GAV            | yes/ no      | no            |
     | no              | (G)LAV         | yes          | no            |
     | yes             | GAV            | yes          | yes           |
     | yes             | (G)LAV         | yes          | yes           |

Do these results extend to the case of more expressive queries?

- With more expressive queries in the mapping?
  - Same results hold if we use any computable query in the mapping.

- With more expressive user queries?
  - Same results hold if we use Datalog queries as user queries.
  - Same results hold if we use union of conjunctive queries with inequalities as user queries [vdM93].

  **Note:** The results do not extend to user queries that contain forms of negation (since it is not true anymore that $\vec{c} \in \text{cert}(q, I, \mathcal{D})$ iff $\vec{c} \in q^{\mathcal{M}(\mathcal{D})}$).
Consider $I = \langle G, S, M \rangle$, with

Global schema $G$: 
student($Code$, $Name$, $City$) 
enrolled($Scode$, $Ucode$)

Source schema $S$: 
relation $s_1($Scode$, Sname, City, Age$)

Mapping $M$:

$q_s(c, n, ci) \leftarrow s_1(c, n, ci, a)$
$q_g(c, n, ci) \leftarrow \text{student}(c, n, ci), \text{enrolled}(c, u)$

A source db $D$ and a corresponding possible global db.

**Incompleteness**

$(G)lav$ mapping assertions $\phi_S \rightarrow \phi_G$ have the logical form:

$\forall \vec{x}. \phi_S(\vec{x}) \rightarrow \exists \vec{y}. \phi_G(\vec{x}, \vec{y})$

where $\phi_S$ and $\phi_G$ are conjunctions of atoms.

Given a source database $D$, in general there are several solutions for a set of $(G)lav$ assertions (i.e., different databases that are legal wrt $G$ that satisfy $M$ wrt $D$).

Incompleteness comes from the mapping.

This holds even for the case of very simple queries $\phi_G$:

$s_1(x) \rightarrow q(x) \leftarrow \exists y. g(x, y)$

**Query answering is based on logical inference**

$q \rightarrow I \rightarrow D \rightarrow \text{Logical inference} \rightarrow \text{cert}(q, I, D)$
**(G)LAV – Approaches to query answering**

- Exploit connection with query containment.
- Direct methods (aka view-based query answering): Try to answer directly the query by means of an algorithm that takes as input the user query \( q \), the specification of \( I \), and the source database \( D \).
- By (view-based) query rewriting:
  - Taking into account \( I \), reformulate the user query \( q \) as a new query (called a rewriting of \( q \)) over the source relations.
  - Evaluate the rewriting over the source database \( D \).

>Note: In (G)LAV data integration the views are the sources.

**Query answering via query containment**

Complexity of checking certain answers under sound sources:
- The **combined complexity** is identical to the complexity of query containment under constraints.
- The **data complexity** is the complexity of query containment under constraints when the right-hand side query is considered fixed.

Hence, it is at most the complexity of query containment under constraints.

It follows that most results and techniques for query containment (under constraints) are relevant also for query answering (under constraints).

>Note: Also, query containment can be reduced to query answering. However, (in the presence of constraints) we need to allow for constants of the database to denote the same object (unique name assumption does not hold).

**(G)LAV – Canonical model**

**Def.: Canonical retrieved global database for \( I \) relative to \( D \)**

Such a database, denoted \( \text{Can}_I(D) \) (also called canonical model of \( I \) relative to \( D \)), is constructed as follows:

- Let all predicates initially be empty in \( \text{Can}_I(D) \).
- For each mapping assertion \( \phi_S \sim \phi_g \) in \( M \)
  - for each tuple \( \vec{c} \in \phi_S \) such that \( \vec{c} \notin \phi_g^{\text{Can}_I(D)} \), add \( \vec{c} \) to \( \phi_g^{\text{Can}_I(D)} \) by inventing fresh variables (Skolem terms) in order to satisfy the existentially quantified variables in \( \phi_S \).

Properties of \( \text{Can}_I(D) \):

- Unique up to variable renaming.
- Can be computed in polynomial time wrt the size of \( D \).
- Satisfies \( M \) by construction, and obviously satisfies \( G \) (since there are no constraints). Hence, \( \text{Can}_I(D) \in \text{Sem}_I(D) \).
(G)LAV – Example of canonical model

\[ q_s(c, n, ci) \leftarrow s_1(c, n, ci, a) \quad \sim \quad q_g(c, n, ci) \leftarrow \text{student}(c, n, ci) \land \text{enrolled}(c, u) \]

Example of source db \( D \) and corresponding canonical model \( \text{Can}_I(D) \).

(G)LAV – Canonical model

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\begin{array}{|c|c|c|}
\hline
\text{Code} & \text{Name} & \text{City} \\
\hline
12 & anne & florence \\
15 & bill & oslo \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Scode} & \text{Ucode} \\
\hline
12 & x \\
15 & y \\
\hline
\end{array}
\]
```

(G)LAV – Universal solution

Let \( I = (G, S, M) \) be a data integration system, and \( D \) a source db.

**Def.: Universal solution** for \( I \) relative to \( D \)

Is a global db \( B \) that satisfies \( I \) relative to \( D \) and such that, for every global db \( B' \) that satisfies \( I \) relative to \( D \), there exists a homomorphism \( h : B \rightarrow B' \) (see \([FKMP05]\)).

Theorem

Let \( I = (G, S, M) \) be a (G)LAV data integration system without constraints in the global schema, and \( D \) a source database. Then \( \text{Can}_I(D) \) is a universal solution for \( I \) relative to \( D \) (follows from \([FKMP05]\)).

(G)LAV – Query answering

**Theorem**

Let \( I = (G, S, M) \) be a (G)LAV data integration system without constraints in the global schema, \( D \) a source database, and \( q \) a conjunctive query. Then \( \vec{c} \in \text{cert}(q, I, D) \) iff \( \vec{c} \in q(\text{Can}_I(D)) \).

**Proof.**

\[ \Rightarrow \] Trivial, since \( \text{Can}_I(D) \in \text{Sem}_I(D) \).

\[ \Leftarrow \] Consider a global db \( B \in \text{Sem}_I(D) \).

- Since \( \vec{c} \in q(\text{Can}_I(D)) \), there exists a homomorphism \( h_1 : q(\vec{c}) \rightarrow \text{Can}_I(D) \).

- Since \( \text{Can}_I(D) \) is a universal solution, there exists a homomorphism \( h_2 : \text{Can}_I(D) \rightarrow B \).

Hence, \( h_1 \circ h_2 \) is a homomorphism from \( q(\vec{c}) \) to \( B \), and \( \vec{c} \in q^B \).
(G)LAV – Complexity of query answering

From the above results, we obtain that for a CQ \( q \), we can compute \( \text{cert}(q, I, D) \) as follows:

- Compute \( \text{Can}_X(D) \) from \( D \) — polynomial in \( |D| \).
- Evaluate \( q \) over \( \text{Can}_X(D) \) — \( \text{LOGSPACE} \) in \( |D| \).

The above applies also to UCQs. Hence, we obtain the following result.

**Theorem**

In a (G)LAV data integration system without constraints, answering unions of conjunctive queries is \textbf{polynomial in data and combined complexity}.

The data complexity upper bound can actually be improved.

(G)LAV – More expressive queries?

- More expressive \textit{source queries} in the mapping?
  - Same results hold if we use \textit{any computable query} as source query in the mapping assertions.
- More expressive \textit{queries over the global schema in the mapping}?
  - Already \textit{unions} of conjunctive queries lead to intractability.
- More expressive \textit{user queries}?
  - Same results hold if we use \textit{Datalog queries} as user queries.
  - Even the simplest form of negation (inequalities) leads to intractability.

(G)LAV – “Inverse rules” technique

From [DG97]: consider mappings as “inverse” rules:

\[
\begin{align*}
\forall t, r_1(t) &\rightarrow \exists y, d, \text{movie}(t, y, d) \land \text{color}(d) \\
\forall t, r_2(t, v) &\rightarrow \exists v, \text{movie}(t, y, d) \land \text{review}(t, v)
\end{align*}
\]

Answering a query means evaluating a goal wrt to this nonrecursive logic program (which can be transformed into a union of CQs).

**Theorem**

In a (G)LAV data integration system without constraints, answering unions of conjunctive queries is \textbf{LOGSPACE in data complexity}.

(G)LAV – Intractability for views that contain union

From [vdM93], by reduction from 3-colorability.

We define the following LAV data integration system \( I = (G, S, M) \):

\[
\begin{align*}
G : & \quad \text{edge}(x, y), \quad \text{color}(x, c) \\
S : & \quad \text{sg}(x, y), \quad \text{sx}(x) \\
M : & \quad \text{sg}(x, y) \sim \text{qg}(x, y) \dashv \text{edge}(x, y) \\
& \quad \text{sx}(x) \sim \text{qd}(x) \dashv \text{color}(x, \text{RED}) \lor \text{color}(x, \text{BLUE}) \lor \text{color}(x, \text{GREEN})
\end{align*}
\]

Given a graph \( G = (N, E) \), we define the following source database \( D \):

\[
\begin{align*}
\text{sg}^D &\equiv \{ (a, b) \mid (a, b) \in E \} \\
\text{sx}^D &\equiv \{ a \mid a \in N \}
\end{align*}
\]

Consider the boolean query: \( q(1) \leftarrow \exists x, y, c. \text{edge}(x, y) \land \text{color}(x, c) \land \text{color}(y, c) \)

describing mismatched edge pairs:

- If \( G \) is 3-colorable, then \( \exists B \text{ s.t. } q^B = \text{false} \), hence \( \text{cert}(q, I, D) = \text{false} \).
- If \( G \) is not 3-colorable, then \( \text{cert}(q, I, D) = \text{true} \).

**Theorem**

In a LAV data integration system without constraints and with UCQs as views, answering CQs is \textbf{coNP-hard in data complexity}.
(G)LAV – In coNP for views and queries that are UCQs

- \( c \notin \text{cert}(q, I, D) \) if and only if there is a database \( B \) for \( I \) that satisfies \( M \) wrt \( D \), and such that \( c \notin q^B \).
- The mapping \( M \) has the form:
  \[
  \forall \vec{x}. \phi_s(\vec{x}) \rightarrow \exists \vec{y}_1. \alpha_1(\vec{x}, \vec{y}_1) \lor \cdots \lor \exists \vec{y}_n. \alpha_n(\vec{x}, \vec{y}_n)
  \]
  Hence, each tuple in \( D \) forces the existence of \( k \) tuples in any database that satisfies \( M \) wrt \( D \), where \( k \) is the maximal length of conjunctions of \( \alpha_i(\vec{x}, \vec{y}_i) \) in \( M \).
- If \( D \) has \( n \) tuples, then there is a db \( B' \subseteq B \) for \( I \) that satisfies \( M \) wrt \( D \) with at most \( n \cdot k \) tuples. Since \( q \) is monotone, \( c \notin q^B' \).
- Checking whether \( B' \) satisfies \( M \) wrt \( D \), and checking whether \( c \notin q^B' \) can be done in \( P\text{Time} \) wrt the size of \( B' \).

**Theorem**

In a LAV data integration system without constraints and with UCQs as views, answering UCQs is **coNP-complete in data complexity**.

(G)LAV – Conjectural user queries with inequalities

- coNP algorithm: guess equalities on variables in the canonical retrieved global database.
- coNP-hard already for a conjunctive user query with one inequality (and conjunctive view definitions) [AD98].

**Theorem**

In a (G)LAV data integration system without constraints and with CQs as views, answering CQs with inequalities is **coNP-complete in data complexity**.

Note: inequalities in the view definitions do not affect expressive power and complexity (in fact, they can be removed).

(G)LAV – Direct methods (aka view-based query answering)

Consider \( I = (G, S, M) \), and source db \( D \) (see [FKMP05]):

\[
\begin{align*}
G & : \quad g(x,y) \\
S & : \quad s(x,y) \\
M & : \quad s(x,y) \sim q(x,y) \leftarrow g(x,z) \land g(z,y) \\
D & : \quad \{ s(a,a) \}
\end{align*}
\]

- Both \( B_1 = \{ g(a,a) \} \) and \( B_2 = \{ g(a,b), g(b,a) \} \) are solutions.
- If \( B \) is a universal solution, then both \( q(a,a) \) and \( q(x,a) \) are in \( B \), with \( x \neq a \) (otherwise \( g(a,a) \) would be true in every solution).

Let \( q() \leftarrow g(x,y) \land x \neq y \)

- \( q^B \) = false, hence \( \text{cert}(q, I, D) = false \).
- But \( q^B \) true for every universal solution \( B \) for \( I \) relative to \( D \).

Hence, the notion of universal solution is not the right tool.

Query answering

In the presence of incomplete information, as is the case in (G)LAV data integration, query answering is a form of logical inference.
(G)LAV – Maximal rewritings

**Query answering by rewriting:**

- Given \( I = (G, S, M) \) and a query \( q \) over \( G \), rewrite \( q \) into a query, called \( \text{rew}_{q,I} \), over the alphabet \( A_S \) of the sources.
- Evaluate the rewriting \( \text{rew}_{q,I} \) over the source database \( D \).

**Def.: Maximal \( L \)-rewriting of \( q \) wrt \( I \)**

Given \( I = (G, S, M) \), a query \( q \) over \( G \), and a query language \( L \), a maximal \( L \)-rewriting of \( q \) wrt \( I \) is a query that:

- is expressed in \( L \);
- is sound, i.e., for every \( db D \) computes only tuples in \( \text{cert}(q, I, D) \);
- is the maximal such query among those expressible in \( L \).

We are interested in computing maximal \( L \)-rewritings.
(G)LAV – Exact rewritings

The (mappings in) a data integration system and the choice of $L$ may be such that even a maximal $L$-rewriting does not provide all answers that the query evaluated over a global db would provide.

**Def.: Exact rewriting**

An exact rewriting of a query $q$ wrt a data integration system $I = (g, S, M)$ is a rewriting that is logically equivalent to $q$, modulo the mappings $M$.

*Note:* exact rewritings may not exist for a given query.

**Example (from the previous slide)**

- $rew_{q_1, I}$ is not an exact rewriting of $q_1$ wrt $I$.
- $rew_{q_2, I}$ is an exact rewriting of $q_2$ wrt $I$.

Properties of the perfect rewriting

- Can the perfect rewriting be expressed in a certain query language?
- For a given class of queries, what is the relationship between a maximal rewriting and the perfect rewriting?
  - From a semantical point of view
  - From a computational point of view
- Which is the computational complexity of finding the perfect rewriting, and how big is it?
- Which is the computational complexity of evaluating the perfect rewriting?

Perfect rewriting

What is the relationship between answering by rewriting and certain answers?

[CDGLV05]:

- When does the (maximal) rewriting compute all certain answers?
- What do we gain or lose by focusing on a given class of queries?

Let’s try to consider the “best possible” rewriting.

Define $cert_{q, I}(\cdot)$ to be the function that, with $q$ and $I$ fixed, given source database $D$, computes the certain answers $cert(q, I, D)$.

- $cert_{q, I}$ can be seen as a query on the alphabet $A_S$.
- $cert_{q, I}$ is a (sound) rewriting of $q$ wrt $I$.
- No sound rewriting exists that is better than $cert_{q, I}$.

Hence, $cert_{q, I}$ is called the **perfect rewriting** of $q$ wrt $I$.

(G)LAV – The case of conjunctive queries

**Theorem ([LMSS95, AD98])**

Let $I = (g, S, M)$ be a (G)LAV data integration system where the queries in $M$ are CQs. Let $q$ be a CQ and let $q'$ be the union of all maximal rewritings of $q$ for the class of CQs. Then:

- $q'$ is the maximal rewriting for the class of unions of conjunctive queries (UCQs).
- $q'$ is the perfect rewriting of $q$ wrt $I$.
- $q'$ is a PTIME query.
- $q'$ is an exact rewriting (equivalent to $q$ for each database $B$ of $I$), if an exact rewriting exists.

*Does this “ideal situation” carry over to cases where $q$ and $M$ allow for union?*
(G)LAV – The case of mappings with union

When queries over the global schema in the mapping contain union:
- We have seen that view-based query answering is coNP-complete in data complexity [vdM93].
- Hence, $\text{cert}(q, I, D)$, with $q$, $I$ fixed, is a coNP-complete function.
- Hence, the perfect rewriting $\text{cert}_{(q, I)}$ is a coNP-complete query.

We do not have the ideal situation we had for conjunctive queries.

Problem:
Isolate those cases of view based query rewriting for data integration systems $I$ where mappings contain unions for which the perfect rewriting $\text{cert}_{(q, I)}$ is a PTime function (assuming $P \neq NP$) [CDGLV00c].

(G)LAV – Further references

- Inverse rules [DG97]
- Bucket algorithm for query rewriting [LRO96]
- MiniCon algorithm for query rewriting [PL00]
- Conjunctive queries using conjunctive views [LMSS95]
- Recursive queries (Datalog programs) using conjunctive views [DG97, AGK99]
- CQs with arithmetic comparison [ALM02]
- Complexity analysis [AD98, GM99]
- Variants of Regular Path Queries [CDGLV00a, CDGLV00b, CDGLV01, DT01]
- Relationship between view-based rewriting and answering [CDGLV00c, CDGLV03, CDGLV05]

(G)LAV – Data complexity of query answering

From [AD98], for sound sources:

<table>
<thead>
<tr>
<th>Global schema</th>
<th>CQ</th>
<th>CQ*</th>
<th>PQ</th>
<th>Datalog</th>
<th>FOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>CQ</td>
<td>PTime</td>
<td>coNP</td>
<td>PTime</td>
<td>PTime</td>
<td>undec.</td>
</tr>
<tr>
<td>CQ*</td>
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<td>coNP</td>
<td>PTime</td>
<td>PTime</td>
<td>undec.</td>
</tr>
<tr>
<td>PQ</td>
<td>coNP</td>
<td>coNP</td>
<td>coNP</td>
<td>coNP</td>
<td>undec.</td>
</tr>
<tr>
<td>Datalog</td>
<td>coNP</td>
<td>coNP</td>
<td>coNP</td>
<td>coNP</td>
<td>undec.</td>
</tr>
<tr>
<td>FOL</td>
<td>undec.</td>
<td>undec.</td>
<td>undec.</td>
<td>undec.</td>
<td>undec.</td>
</tr>
</tbody>
</table>

Chapter III

Query answering in the presence of constraints
Chap. 3: Query answering with constraints

Outline

- The role of global integrity constraints
- Query answering in GAV with constraints
- Query answering in (G)LAV with constraints

Integrity constraints for relational schemas

- Integrity constraints (ICs) are posed over the global schema.
- Specify intensional knowledge about the domain of interest.
- Add semantics to the information.
- But data in the sources can conflict with global ICs.
- The presence of global ICs raises semantic and computational problems.

Note: global integrity constraints play the same role as an ontology in Ontology-Based Data Access.

Most important types of ICs that have been considered for the relational model:

- key dependencies (KD)
- functional dependencies (FD)
- foreign keys (FK)
- inclusion dependencies (ID)
- exclusion dependencies (ED)
Syntax and semantics of integrity constraints

We present now the syntax and semantics of the various types of integrity constraints, concentrating on KDs, IDs, and EDs.

To define their semantics, we specify when a database \( D \) satisfies a constraint \( C \), denoted \( D \models C \).

We make use the following notation: let \( r \) be a relation symbol of arity \( n \) and let \( t_1, \ldots, t_k \) be components of \( r \):

- \( r^{D}[i_1, \ldots, i_k] \) denotes the projection of relation \( r^{D} \) on the components \( i_1, \ldots, i_k \).
- Given a tuple \( t \in r^{D} \), we have that \( t[i_1, \ldots, i_k] \) denotes the tuple constituted by the components \( i_1, \ldots, i_k \) of \( t \).

Def.: Syntax of inclusion dependencies:

\[
\text{key}(r) = \{i_1, \ldots, i_k\}
\]

with \( i_1, \ldots, i_k \) components of \( r \).

Semantics: \( D \models \text{key}(r) = \{i_1, \ldots, i_k\} \) if for all \( t_1, t_2 \in r^{D} \), we have that \( t_1[i_1, \ldots, i_k] = t_2[i_1, \ldots, i_k] \) implies \( t_1 = t_2 \).

Example

For \( r \) of arity 3, the KD \( \text{key}(r) = \{1\} \) corresponds to the FOL sentence

\[
\forall x, y, y', z. r(x, y, z) \land r(x, y', z') \rightarrow y = y' \land z = z'.
\]

Note: KDs are a special form of equality-generating dependencies.

Key dependencies (KD)

A key dependency (KD) states that a set of attributes functionally determines all the attributes of a relation.

Def.: Syntax of key dependencies:

\[
\text{key}(r) = \{i_1, \ldots, i_k\}
\]

with \( i_1, \ldots, i_k \) components of \( r \).

Semantics: \( D \models \text{key}(r) = \{i_1, \ldots, i_k\} \) if for all \( t_1, t_2 \in r^{D} \), we have that \( t_1[i_1, \ldots, i_k] = t_2[i_1, \ldots, i_k] \) implies \( t_1 = t_2 \).

Example

For \( r \) of arity 3, the KD \( \text{key}(r) = \{1\} \) corresponds to the FOL sentence

\[
\forall x, y, y', z. r(x, y, z) \land r(x, y', z') \rightarrow y = y' \land z = z'.
\]

Note: KDs are a special form of equality-generating dependencies.

Inclusion dependencies (IDs)

An inclusion dependency (ID) states that the presence of a tuple \( t_1 \) in a relation implies the presence of a tuple \( t_2 \) in another relation, where \( t_2 \) contains a projection of the values contained in \( t_1 \).

Def.: Syntax of inclusion dependencies:

\[
r[i_1, \ldots, i_k] \subseteq s[j_1, \ldots, j_k]
\]

with \( i_1, \ldots, i_k \) components of \( r \), and \( j_1, \ldots, j_k \) components of \( s \).

Semantics: \( D \models r[i_1, \ldots, i_k] \subseteq s[j_1, \ldots, j_k] \) if \( r^{D}[i_1, \ldots, i_k] \subseteq s^{D}[j_1, \ldots, j_k] \).

Example

For \( r \) of arity 3 and \( s \) of arity 2, the ID \( r[1] \subseteq s[2] \) corresponds to the FOL sentence:

\[
\forall x, y, w. r(x, y, w) \rightarrow \exists z. s(z, x)
\]

Note: IDs are a special form of tuple-generating dependencies.

Exclusion dependencies (EDs)

An exclusion dependency (ED) states that the presence of a tuple \( t_1 \) in a relation implies the absence of a tuple \( t_2 \) in another relation, where \( t_2 \) contains a projection of the values contained in \( t_1 \).

Def.: Syntax of exclusion dependencies:

\[
r[i_1, \ldots, i_k] \cap s[j_1, \ldots, j_k] = \emptyset
\]

with \( i_1, \ldots, i_k \) components of \( r \), and \( j_1, \ldots, j_k \) components of \( s \).

Sem.: \( D \models r[i_1, \ldots, i_k] \cap s[j_1, \ldots, j_k] = \emptyset \) if \( r^{D}[i_1, \ldots, i_k] \cap s^{D}[j_1, \ldots, j_k] = \emptyset \).

Example

For \( r \) of arity 3 and \( s \) of arity 2, the ED \( r[1] \cap s[2] = \emptyset \) corresponds to the FOL sentence:

\[
\forall x, y, z. r(x, y, z) \rightarrow \neg s(z, x)
\]

Note: EDs are a special form of denial constraints.
Outline

1. The role of global integrity constraints
2. Query answering in GAV with constraints
   - Incompleteness and inconsistency in GAV systems
   - Query answering in GAV under inclusion dependencies
   - Rewriting CQs under inclusion dependencies in GAV
   - Query answering in GAV under IDs and KDs
   - Query answering in GAV under IDs, KDs, and EDs
3. Query answering in (G)LAV with constraints

Semantics of GAV systems with integrity constraints

Given a source db $D$, a global db $B$ (over $\Delta$) satisfies $I$ relative to $D$ if:
- It is legal wrt the global schema, i.e., it satisfies the ICs.
- It satisfies the mapping, i.e., $B$ is a superset of the retrieved global database $M(D)$ (sound mappings).

Recall:
- $M(D)$ is obtained by evaluating, for each relation in $A_D$, the corresponding mapping query over the source database $D$.
- We are interested in certain answers to a query, i.e., those that hold for all global databases that satisfy $I$ relative to $D$.

GAV system with integrity constraints

We consider a data integration system $I = (\mathcal{G}, S, M)$ where:
- $\mathcal{G}$ is a global schema with constraints.
- $M$ is a set of GAV mappings, whose assertions have the form $\phi_S \sim g$ and are interpreted as $\forall \bar{x}. \phi_S(\bar{x}) \rightarrow g(\bar{x})$.

where $\phi_S$ is a conjunctive query over $S$, and $g$ is an element of $\mathcal{G}$.

Basic observation: Since $\mathcal{G}$ does have constraints, the retrieved global database $M(D)$ may not be legal for $\mathcal{G}$.

GAV with constraints – Example

Consider $I = (\mathcal{G}, S, M)$, with:
- $\mathcal{G}$: student(Code, Name, City) key(student) = \{Code\}
  university(Code, Name) key(university) = \{Code\}
  enrolled(Scode, Ucode) enrolled[Scode] \subseteq student[Code]
  enrolled[Ucode] \subseteq university[Code]
- Source schema $S$: $s_1(Scode, Sname, City, Age),$ $s_2(Ucode, Uname),$ $s_3(Scode, Ucode)$
- Mapping $M$: $\{ (c, n, ci) | s_1(c, n, ci, a) \} \sim student(c, n, ci)$
  $\{ (c, n) | s_2(c, n) \} \sim university(c, n)$
  $\{ (s, u) | s_3(s, u) \} \sim enrolled(s, u)$
**GAV with constraints – Example of retrieved global db**

<table>
<thead>
<tr>
<th>university</th>
<th>student</th>
<th>enrolled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Code</td>
<td>Name</td>
<td>City</td>
</tr>
<tr>
<td>AF</td>
<td>bocconi</td>
<td></td>
</tr>
<tr>
<td>BN</td>
<td>ucla</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>anne</td>
<td>florence</td>
</tr>
<tr>
<td>15</td>
<td>bill</td>
<td>oslo</td>
</tr>
<tr>
<td>12</td>
<td>AF</td>
<td>16</td>
</tr>
<tr>
<td>15</td>
<td>bill</td>
<td>oslo</td>
</tr>
<tr>
<td>12</td>
<td>AF</td>
<td>16</td>
</tr>
</tbody>
</table>

Example of source database $D$ and corresponding retrieved global database $M(D)$.

**GAV with constraints – Example of incompleteness**

$s^D_1(16, BN)$ and the mapping imply $enrolled_1^{BN}(16, BN)$ for all $B \in Sem_I(D)$.

 Due to the inclusion dependency $enrolled[\text{Code}] \subseteq student[\text{Code}]$ in $G$, 16 is the code of some student in all $B \in Sem_I(D)$.

Since $D$ does not provide information about name and city of the student with code 16, a global database that is legal for $I$ wrt $D$ may contain arbitrary values for these.

**GAV with constraints – Unfolding is not sufficient**

Mapping $M$: 

$\{ (c, n, ci) \mid s_3(c, n, ci, a) \} \sim student(c, n, ci) \\
\{ (c, n) \mid s_2(c, n) \} \sim university(c, n) \\
\{ (s, u) \mid s_4(s, u) \} \sim enrolled(s, u)$

$\begin{align*}
s^D &= \{ 12 \text{ anne florence 21} \\
&15 \text{ bill oslo 24} \}
\end{align*}$

Consider the query: $q = \{ (c) \mid student(c, n, ci) \}$

Unfolding of $q$ wrt $M$: $unf_M(q) = \{ (c) \mid s_3(c, n, ci, a) \}$

The query $unf_M(q)$ retrieves from $D$ only the answer $\{12, 15\}$, while the correct answer would be $\{12, 15, 16\}$.

The simple unfolding strategy is not sufficient for GAV with constraints.

**GAV with constraints – Example of inconsistency**

The tuples in $s^D_1$ and the mapping imply $student_1^{BN}(12, \text{anne.florence})$ and $student_1^{BN}(12, \text{bill.oslo})$, for all $B$ that satisfy the mapping.

Due to the key dependency $key(student) = \{Code\}$ in $G$, there is no global database that satisfies the mapping and is legal wrt the global schema, i.e., $Sem_I(D) = \emptyset$.
Global integrity constraints

Query answering in GAV with constraints

Query answering in (G)LAV with constraints

Chap. 3: Query answering with constraints

Incompleteness and inconsistency in GAV systems

Incompleteness and inconsistency in (G)LAV systems

Chap. 3: Query answering with constraints

GAV data integration systems with constraints

<table>
<thead>
<tr>
<th>Constraints in G</th>
<th>Type of mapping</th>
<th>Incompleteness</th>
<th>Inconsistency</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>GAV</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>no</td>
<td>(G)LAV</td>
<td>yes / no</td>
<td>no</td>
</tr>
<tr>
<td>IDs</td>
<td>GAV</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>KDs</td>
<td>GAV</td>
<td>yes / no</td>
<td>yes</td>
</tr>
<tr>
<td>IDs + KDs</td>
<td>GAV</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>yes</td>
<td>(G)LAV</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Constraint satisfaction – Incompleteness and inconsistency

Inclusion dependencies – Example

Global schema $G$:

- player($Pname, YOB, Pteam$)
- team($Tname, Tcity, Tleader$)

Constraints: $\text{team}[Tleader, Tname] \subseteq \text{player}[Pname, Pteam$]

Sources $S$: $s_1$ and $s_3$ store players
- $s_2$ stores teams

Mapping $M$: $\{ (x, y, z) \mid s_1(x, y, z) \lor s_3(x, y, z) \} \leadsto \text{player}(x, y, z)$

$\{ (x, y, z) \mid s_2(x, y, z) \} \leadsto \text{team}(x, y, z)$

Inclusion dependencies – Example retrieved global db

Source database $D$:

- $s_1$: Totti 1971 Roma
- $s_2$: Juve Torino Del Piero
- $s_3$: Buffon 1978 Juve

Retrieved global database $M(D)$:

- player: Totti 1971 Roma
- Buffon 1978 Juve
- team: Juve Torino Del Piero
Inclusion dependencies – Example retrieved global db

The ID on the global schema tells us that Del Piero is a player of Juve. All global databases satisfying $I$ have at least the tuples shown above, where $\alpha$ is some value of the domain $\Delta$.

**Warnings**
- There may be an infinite number of databases satisfying $I$.
- In case of cyclic IDs, a database satisfying $I$ may be of infinite size.

Intuitive strategy: Add new facts until IDs are satisfied.

Problem: Infinite construction in the presence of cyclic IDs.

**Example**
- Let $r$ be binary with $r(2) \subseteq r(1)$.
- Suppose $M(D) = \{ r(a, b) \}$.
  - add $r(b, c_1)$
  - add $r(c_1, c_2)$
  - add $r(c_2, c_3)$
  - $\ldots$ (ad infinitum)

**Example**
- Suppose $M(D) = \{ r(a, b) \}$.
  - add $s(a, c_1)$
  - add $r(c_1, c_2)$
  - add $s(c_1, c_2)$
  - add $r(c_2, c_3)$
  - $\ldots$ (ad infinitum)

Chasing inclusion dependencies – Infinite construction

The chase of a database

**Def.: Chase of a database**

The chase of a database is the exhaustive application of a set of rules that transform the database, in order to make it consistent with a set of integrity constraints.

Typically, there will be one or more chase rules for each different type of constraint.
The ID-chase rule

The chase for IDs has only one rule, the **ID-chase rule**.

Let $D$ be a database:

- the schema contains the ID $r[i_1, \ldots, i_k] \subseteq s[j_1, \ldots, j_k]$ and there is a fact in $D$ of the form $r(a_1, \ldots, a_n)$ and there are no facts in $D$ of the form $s(b_1, \ldots, b_m)$ such that $a_{i_\ell} = b_{j_\ell}$ for each $\ell \in \{1, \ldots, k\}$, then add to $D$ the fact $s(c_{i_1}, \ldots, c_{i_k})$, where for each $h \in \{1, \ldots, m\}$, if $h = j_\ell$ for some $\ell$ then $c_h = a_{i_\ell}$ otherwise $c_h$ is a new constant symbol (not in $D$ yet).

**Notice:** New existential symbols are introduced (skolem terms).

Limiting the chase

Why don’t we use a finite number of existential constants in the chase?

**Example**

Consider $r[1] \subseteq s[1]$ and $s[2] \subseteq r[1]$, and suppose $\mathcal{M}(D) = \{ r(a, b) \}$. Compute chase($\mathcal{M}(D)$) with only one new constant $c_1$:

- $0 \ r(a, b)$
- $1 \ add \ s(a, c_1)$
- $2 \ add \ r(c_1, c_1)$
- $3 \ add \ s(c_1, c_1)$

This database is **not** a canonical model for $\mathcal{I}$ wrt $D$. E.g., for query $q = \{ x \mid s(x, y), s(y, y) \}$, we have $a \notin \chi_{\text{chase}}(\mathcal{M}(D))$ while $a \notin \text{cert}(q, \mathcal{I}, D)$.

Arbitrarily limiting the chase is **unsound**, for any finite number of new constants.

Properties of the chase

- **Bad news:** the chase is in general infinite.
- **Good news:** the chase identifies a **canonical model**. A canonical model is a database that “represents” all the models of the system.
- We can use the chase to prove soundness and completeness of a query processing method . . .
- . . . but only for positive queries!

Chasing the query

When chasing the data, the termination condition would need to take into account the query.

We consider an alternative approach, based on the idea of a **query chase**.

- Instead of chasing the data, we chase the query.
- Is the dual notion of the database chase.
- IDs are applied from right to left to the query atoms.
- Advantage: much easier termination conditions, which imply:
  - decidability properties
  - efficiency

This technique provides an algorithm for rewriting UCQs under IDs.
Query rewriting under inclusion dependencies

- Given a query \( q \) over the global schema \( G \), we look for a rewriting \( \text{rew} \) of \( q \) expressed over \( S \).
- A rewriting \( \text{rew} \) is perfect if \( \text{rew}^\ast = \text{cert}(q, I, D) \), for every source database \( D \).
- With a perfect rewriting, we can do query answering by rewriting.
  ~ We avoid actually constructing the retrieved global database \( M(D) \).

Iterative Rewriting for IDs – Algorithm ID-\( \text{rewrite} \)

**Input**: relational schema \( G \), set \( \Psi_{ID} \) of IDs, UCQ \( Q \)

**Output**: perfect rewriting of \( Q \)

\[ Q' = Q; \]

**repeat**

\[ Q_{aux} := Q'; \]

**for each** \( g \in Q_{aux} \) **do**

\[ (a) \text{ for each } g_1, g_2 \in \text{body}(q) \text{ do} \]

\[ \text{if } g_1 \text{ and } g_2 \text{ unify then } Q' := Q' \cup \{ \tau(\text{reduce}(q, g_1, g_2)) \}; \]

\[ (b) \text{ for each } g \in \text{body}(q) \text{ do} \]

\[ \text{for each } ID \in \Psi_{ID} \text{ do} \]

\[ \text{if } ID \text{ is applicable to } g \text{ then } Q' := Q' \cup \{ q[g/\text{rewrite}(g, ID)] \} \]

**until** \( Q_{aux} = Q' \)

**return** \( Q' \)
Query answering in GAV under IDs

Properties of ID-rewrite
- ID-rewrite terminates.
- ID-rewrite produces a perfect rewriting of the input query.

More precisely, let $unf_M(q)$ be the unfolding of the query $q$ wrt the GAV mapping $M$.

**Theorem**
$unf_M(ID-rewrite(q))$ is a perfect rewriting of the query $q$.

**Theorem**
Query answering in GAV systems under IDs is in PTIME in data complexity (actually in LogSpace).

Non-key-conflicting IDs

**Def.: Non-key-conflicting ID (NKCID)**
Is an ID of the form $r_1[x_1] \subseteq r_2[x_2]$ where $x_2$ is not a strict superset of $key(r_2)$.

**Example**
Let $r$ be of arity 3 and $s$ of arity 4 with $key(s) = \{1, 2\}$.
- The following are NKCIDs:
  - $r[2] \subseteq s[2]$, since $\{2\}$ is a strict subset of $key(s)$.
  - $r[2, 3] \subseteq s[1, 2]$, since $\{1, 2\}$ coincides with $key(s)$.
  - $r[1, 2] \subseteq s[2, 3]$, since $1 \in key(s)$ but $1 \not\in \{2, 3\}$.
- The following is not a NKCID: $r[1, 2, 3] \subseteq s[1, 2, 4]$.

**Note:** Foreign keys (FKs) are a special case of NKCIDs.

Query answering under IDs and KDs

We have already seen that in GAV systems under sound mappings:
- Key dependencies may give rise to inconsistencies.
- When $M(D)$ violates the KDs, no legal database exists and query answering becomes trivial.

How do KDs interact with IDs?

**Theorem**
Query answering under IDs and KDs is undecidable.

**Proof:** By reduction from implication of IDs and KDs.

We need to look for syntactic restrictions on the form of the dependencies that ensures decidability.

Separation for IDs and KDs

**Theorem (IDs-KDs separation)**
Under KDs and NKCIDs, if $M(D)$ satisfies the KDs, then the KDs can be ignored wrt certain answers of a user query $q$.

**Intuition:** For NKCIDs, when applying the ID-chase rule to a tuple $\vec{t}_1 \in r_1^D$, we can choose the tuple $\vec{t}_2$ to introduce in $r_2^D$ so that it does not violate key($r_2$):
- When key($r_2$) $\subseteq x_2$, fresh constants in $\vec{t}_2$ are chosen for key attributes, and so there is no other tuple in $r_2^D$ coinciding with $\vec{t}_2$ on all key attributes.
- When key($r_2$) $\not\subseteq x_2$, if there is already a tuple $\vec{t}$ in $r_2^D$ such that $\vec{t}_1[x_1] = \vec{t}[x_2]$, we choose $\vec{t}$ for $\vec{t}_2$.

Query answering becomes undecidable as soon as we extend the language of the IDs.
Query processing under separable KDs and IDs

Overall query answering algorithm:
- Verify consistency of $M(D)$ with respect to KIDs.
- Compute ID-rewrite of the input query.
- Unfold wrt $M$ the query computed at previous step.
- Evaluate the unfolded query over the sources.

Note:
- The KD consistency check can be done by suitable CQs with inequality.
- The computation of $M(D)$ can be avoided (by unfolding the queries for the KD consistency check).

Checking KD consistency – Example

Relation: $player(Pname, Pteam)$

Key dependency: $key(player) = \{Pname\}$

Query to check (in)consistency of the KD:

$q = \{() | player(x,y), player(x,z), y \neq z\}$

is true iff the instance of player violates the KD.

Mapping $M$: \[
\{(x,y) | s_1(x,y) \lor s_2(x,y) \} \sim player(x,y)\]

Unfolding of $q$ wrt $M$:

$\{() | s_1(x,y), s_1(x,z), y \neq z \lor \$

$s_1(x,y), s_2(x,z), y \neq z \lor \$

$s_2(x,y), s_1(x,z), y \neq z \lor \$

$s_2(x,y), s_2(x,z), y \neq z\}$

Query answering in GAV under separable IDs + KDs

Theorem (CaLR03)

Answering conjunctive queries in GAV systems under KDs and NKCIDs is in $PTIME$ in data complexity (actually in $LOGSPACE$).

Can we extend these results to more expressive user queries?
- The rewriting technique extends immediately to unions of CQs $ID-\text{rewrite}(q_1) \lor \ldots \lor q_n).$
- This is not the case for recursive queries.

Theorem (CaRo03)

Answering recursive queries under KDs and FKs is undecidable.

Answering recursive queries under IDs is undecidable.

Query answering under IDs and EDs

Under EDs:
- Possibility of inconsistencies.
- When $M(D)$ violates the EDs, no legal database exists and query answering becomes trivial.

Under IDs and EDs:
- How do EDs and IDs interact?
- Is query answering separable?
- Is query answering decidable?
Exclusion dependencies – Example

Global schema $G$: player($Pname$, YOB, $Pteam$)
         team($Tname$, $Tcity$, $Tleader$)
         coach($Cname$, $Cteam$)

Constraints: $team[Tleader, Tname] \subseteq player[Pname, Pteam]$
         coach[$Cname]\cap player[Pname] = \emptyset$

Sources $S$: $s_1$ and $s_3$ store players
$s_3$ stores teams
$s_4$ stores coaches

Mapping $M$: $\{ (x,y,z) | s_1(x,y,z) \lor s_3(x,y,z) \} \rightarrow player(x,y,z)$
$\{ (x,y,z) | s_2(x,y,z) \} \rightarrow team(x,y,z)$
$\{ (x,y) | s_4(x,y) \} \rightarrow coach(x,y)$

Retrieved global database $\mathcal{M}(D)$:

player: Totti 1971 Roma
         Buffon 1978 Juve
         Del Piero α Juve

coach: Del Piero Viterbese

“Repair” of $team[Tleader, Tname] \subseteq player[Pname, Pteam]$.

Violation of $coach[Cname]\cap player[Pname] = \emptyset$.

Can we detect such situations without actually constructing $\mathcal{M}(D)$?

Retrieved global database under EDs – Example

Source database $D$:

$s_1$: Totti 1971 Roma
$s_2$: Juve Torino Del Piero
$s_3$: Buffon 1978 Juve
$s_4$: Del Piero Viterbese

Retrieved global database $\mathcal{M}(D)$:

player: Totti 1971 Roma
         Buffon 1978 Juve
         Del Piero α Juve

coach: Del Piero Viterbese

Deductive closure of EDs under IDs – Example

Can we saturate (close) the EDs by adding all the EDs that are logical consequences of the EDs and IDs?

Example

From

$team[Tleader, Tname] \subseteq player[Pname, Pteam]$
$coach[Cname]\cap player[Pname] = \emptyset$

it follows that

$coach[Cname]\cap team[Tleader] = \emptyset$.

This constraint is violated by the retrieved global database $\mathcal{M}(D)$.
Global integrity constraints

Query answering in (G)LAV with constraints

Chap. 3: Query answering with constraints

Deductive closure of EDs under IDs

**Def.:** Derivation rule of EDs under EDs and IDs

From the ID \( r[i_1, \ldots, i_k, i_{k+1}, \ldots, i_h] \subseteq s[j_1, \ldots, j_k] \) and the ED \( s[j_1, \ldots, j_k] \cap t[\ell_1, \ldots, \ell_k] = \emptyset \)

derive the ED \( r[i_1, \ldots, i_k] \cap t[\ell_1, \ldots, \ell_k] = \emptyset \).

Corresponds to a simple application of **resolution** on the FOL sentences corresponding to EDs and IDs.

**Theorem**

If the set of EDs is closed with respect to the above rule, it contains all EDs that are logical consequences of the initial EDs and IDs.

Query answering in GAV under IDs, KDs, and EDs

**Theorem (ID-KD-ED Separation)**

Under KDs, NKCIDs, and EDs, if \( M(D) \) satisfies all the KDs and satisfies all EDs derived from the IDs and the original EDs, then the KDs and EDs can be ignored wrt certain answers of a query.

We obtain a method for query answering in GAV under KDs, NKCIDs, and EDs:

1. Close the set of EDs with respect to the IDs.
2. Verify consistency of \( M(D) \) with respect to KDs and EDs.
3. Compute ID-rewrite of the input query.
4. Unfold the query computed at the previous step.
5. Evaluate the query over the sources.

The ED consistency check can be done by suitable CQs.

Query answ. in GAV under IDs, KDs and EDs – Complexity

**Note:**

1. Closing the set of EDs wrt the IDs is independent of the data.
2. Consistency of \( M(D) \) wrt KDs and EDs can be verified through suitable queries over the source database \( D \).

**Theorem (Lemb04)**

Answering conjunctive queries in GAV systems under KDs, NKCIDs, and EDs is in \( \text{PTime} \) in data complexity (actually in \( \text{LogSpace} \)).
We consider a data integration system $\mathcal{I} = \langle \mathcal{G}, \mathcal{S}, \mathcal{M} \rangle$ where:

- $\mathcal{G}$ is a global schema with constraints.
- $\mathcal{M}$ is a set of LAV mappings, whose assertions have the form $\phi_\mathcal{S} \leadsto \phi_\mathcal{G}$ and are interpreted as
  \[ \forall \vec{x}. \phi_\mathcal{S}(\vec{x}) \rightarrow \phi_\mathcal{G}(\vec{x}), \]
  where $\phi_\mathcal{S}$ is a CQ over $\mathcal{S}$, and $\phi_\mathcal{G}$ is a CQ over $\mathcal{G}$.

Basic observation: Since $\mathcal{G}$ does have constraints, the canonical retrieved global database $\text{Can}_I(\mathcal{D})$ may not be legal for $\mathcal{G}$.
Transforming LAV into GAV

Consider a LAV mapping:

\[ s(x_1, \ldots, x_k) \sim \{ (x_1, \ldots, x_k) \mid \text{conj}(x_1, \ldots, x_k, x_{k+1}, \ldots, x_h) \} \]

where \( \text{conj}(x_1, \ldots, x_k, x_{k+1}, \ldots, x_h) \) is a conjunction of atoms over the variables \( x_1, \ldots, x_h \), whose predicate symbols are global relations.

We transform it into a GAV mapping and a set of IDs as follows:

- We introduce two new global relations: image \( s_{im/k} \), and expand \( s_{exp/h} \).
- We replace the LAV mapping with the GAV mapping

\[ \{ (x_1, \ldots, x_k) \mid s(x_1, \ldots, x_k) \} \sim s_{im}(x_1, \ldots, x_k) \]

- We introduce the following IDs:

\[ s_{im}[1, \ldots, k] \subseteq s_{exp}[1, \ldots, k] \]
\[ s_{exp}[1, \ldots, i] \subseteq s[g[1, \ldots, i]] \]

for each atom \( g(x_{i_1}, \ldots, x_{i_l}) \) in \( \text{conj}(x_1, \ldots, x_k, x_{k+1}, \ldots, x_h) \).

(G)LAV systems under IDs

Under IDs only, we can exploit also for (G)LAV the previous results for GAV, by turning the (G)LAV mappings into GAV mappings:

- We transform a (G)LAV integration system \( I = (G, S, M) \) with IDs only into a GAV system \( I' = (G', S, M') \).

- With respect to \( I \), the transformed system \( I' \) contains auxiliary IDs and auxiliary global relation symbols.

- The transformation is query-preserving:

For every CQ \( q \) and for every source database \( D \), the certain answers to \( q \) wrt \( I \) and \( D \) are equal to the certain answers to \( q \) wrt \( I' \) and \( D \).

Transforming LAV into GAV – Example

Initial LAV mappings:

\[ s(x, y) \sim \{ (x, y) \mid r_1(x, z), r_2(y, w) \} \]
\[ t(x, y) \sim \{ (x, y) \mid r_1(x, z), r_3(y, x) \} \]

We introduce two new global relations for each mapping assertion:

- \( s_{im/2}, s_{exp/4} \), and \( t_{im/2}, t_{exp/3} \)

Transformed GAV mappings:

\[ \{ (x, y) \mid s(x, y) \} \sim s_{im}(x, y) \]
\[ \{ (x, y) \mid t(x, y) \} \sim t_{im}(x, y) \]

IDs introduced by the transformation:

\[ s_{im}[1, 2] \subseteq s_{exp}[1, 2] \quad s_{exp}[1, 3] \subseteq s_{im}[1, 2] \quad s_{exp}[2, 4] \subseteq r_{im}[1, 2] \]
\[ t_{im}[1, 2] \subseteq t_{exp}[1, 2] \quad t_{exp}[1, 3] \subseteq r_{im}[1, 2] \quad t_{exp}[2, 1] \subseteq r_{im}[1, 2] \]
**Query answering in (G)LAV systems under IDs**

Method for query answering in a (G)LAV system $I$ with IDs:

1. Transform $I$ into a GAV system $I'$.
2. Apply the query answering method for GAV systems under IDs (The unfolding step must take into account the presence of auxiliary global symbols).

**Theorem**

Answering conjunctive queries in (G)LAV systems under IDs is in $\text{PTime}$ in data complexity (actually in $\text{LogSpace}$).

---

**Query answering in (G)LAV systems under IDs and EDs**

Method for query answering in a (G)LAV system $I$ with IDs and EDs:

1. Transform $I$ into a GAV system $I'$.
2. Apply the query answering method for GAV systems under IDs and EDs (The unfolding step must take into account the presence of auxiliary global symbols).

**Theorem**

Answering conjunctive queries in (G)LAV systems under IDs and EDs is in $\text{PTime}$ in data complexity (actually in $\text{LogSpace}$).

---

**Query answering in (G)LAV systems under KDs**

We consider a (G)LAV system with only KDs in the global schema:

- The transformation of (G)LAV into GAV is still correct in the presence of KDs.
- More precisely, starting from a (G)LAV system $I$ with KDs, we obtain a GAV system $I'$ with KDs and IDs.
- But in general, $I'$ is such that the IDs added by the transformation are key-conflicting IDs (i.e., these IDs are not NKCIDs), and hence the KDs are in general not separable.

Therefore, it is not possible to apply the query answering method for (G)LAV systems under separable KDs and IDs.

**Question:** Can we find some analogous query answering method based on query rewriting?
Problem: KDs and LAV mappings derive new equality-generating dependencies (not simple KDs).

Theorem (AbDu98)

Given a LAV data integration system $I$ with KDs in the global schema and a conjunctive query $q$, in general there does not exist a first-order query $rew$ such that $rew^D = cert(q, I, D)$ for every source database $D$.

In other words, in LAV with KDs, conjunctive queries are not first-order rewritable, and one would need to resort to more powerful relational query languages (e.g., Datalog).

Can query answering in integration systems be performed by first-order (UCQ) rewriting?

- GAV with IDs + EDs: yes
- GAV with IDs + KDs + EDs: only if KDs and IDs are separable
- (G)LAV with IDs + EDs: yes
- (G)LAV with KDs: no
Further issues and open problems

- Further forms of constraints, e.g.,
  - KDs with restricted forms of key-conflicting IDs
  - ontology languages, description logics, RDF (cf. OBDA)
- Semistructured data and XML
  - constraints (DTDs, XML Schema, …)
  - query languages (transitive closure)
- Finite models vs. unrestricted models \[Ros06\]
- Data exchange and materialization
References I

Complexity of answering queries using materialized views.

Answering queries using materialized views with disjunction.

Answering queries using views with arithmetic comparisons.

On the expressive power of data integration systems.

References II

On the role of integrity constraints in data integration.

On the decidability of query containment under constraints.

Inconsistency tolerance in P2P data integration: an epistemic logic approach.

References III

Data complexity of query answering in description logics.

Logical foundations of peer-to-peer data integration.

Answering regular path queries using views.

Query processing using views for regular path queries with inverse.

References IV

View-based query processing and constraint satisfaction.

View-based query answering and query containment over semistructured data.
In Proc. of the 8th Int. Workshop on Database Programming Languages (DBPL 2001), 2001.

View-based query containment.


Dealing with Inconsistency and Incompleteness in Data Integration.

Data integration: A theoretical perspective.

Answering queries using views.

Query answering algorithms for information agents.
In Proc. of the 13th Nat. Conf. on Artificial Intelligence (AAAI’96), pages 40–47, 1996.

A scalable algorithm for answering queries using views.
In Proc. of the 26th Int. Conf. on Very Large Data Bases (VLDB 2000), pages 484–495, 2000.

[Rei84] R. Reiter.
Towards a logical reconstruction of relational database theory.

[Ros06] R. Rosati.
On the decidability and finite controllability of query processing in databases with incomplete information.

Querying logical databases.

Recursively indefinite databases.

Logical approaches to incomplete information.