STATE MINIMIZATION

EXERCISE 1
Minimize the following DFA.

EXERCISE 2
Minimize the following DFA.

EXERCISE 3
Minimize the following DFA.
1) We start with the following initial partition: 
\[ C_1 = F = \{ q_0, q_7 \} \] and 
\[ C_2 = Q - F = \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \}. \]

**Step 0**

\[
\begin{align*}
\{ q_0, q_7 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \\
\{ q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \\
\{ q_6 \} & \quad \{ q_3, q_4 \} \\
\{ q_0, q_1, q_2, q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \}
\end{align*}
\]

**Step 1**

\[
\begin{align*}
\{ q_0, q_7 \} & \quad \{ q_3, q_4 \} \\
\{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \\
\{ q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \\
\{ q_0, q_1, q_2, q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \}
\end{align*}
\]

**Step 2**

\[
\begin{align*}
\{ q_0, q_1, q_2, q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4 \} \\
\{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \\
\{ q_0, q_1, q_2, q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \\
\{ q_0, q_1, q_2, q_3, q_4 \} & \quad \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \}
\end{align*}
\]

Final partition: \[ \{ q_0, q_1, q_2, q_3, q_4, q_5, q_6 \} \]

The minimized automaton looks as follows:

![Minimized Automaton Diagram]

Note: \( q_4 \) is not reachable, thus it has been omitted from the minimized automaton.
2) Initial partition:
\[ C_1 = \{q_0, q_1, q_2, q_3\}, \ C_2 = \{q_4\} \]

Step 0
\[ \{q_0, q_2, q_3\} \rightarrow_{a,b,c} \{q_4\} \]

Step 1
\[ \{q_0, q_2, q_3\} \rightarrow_{a,c} \{q_1\}, \rightarrow_{b} \{q_3, q_4\} \]
\[ \{q_0, q_2, q_3\} \rightarrow_{c} \{q_3\} \]

Final partition: \[ \{q_0, q_2\} \{q_3\} \{q_4\} \]

The minimized automaton looks as follows:

3) Initial partition:
\[ C_1 = \{q_0, q_2, q_6, q_8\}, \ C_2 = \{q_1, q_3, q_4, q_5, q_7\} \]

Step 0
\[ \{q_0, q_2, q_6, q_8\} \rightarrow_{a,b,c} \{q_1, q_3, q_4, q_5, q_7\} \]

Step 1
\[ \{q_0, q_2, q_6, q_8\} \rightarrow_{a,c} \{q_1\}, \rightarrow_{a,b,c} \{q_3, q_4\} \]
\[ \{q_0, q_2, q_6, q_8\} \rightarrow_{b} \{q_3, q_4\} \]

Final partition: \[ \{q_0, q_2, q_6, q_8\} \{q_1\} \{q_3, q_4\} \{q_7\} \]
The minimized automaton looks as follows:

Alternative solution:

One could find minimize the automata $A^*_1$ and $A^*_2$ of the last exercise in E8 (12/12/2008) and then apply the so-called product construction.