EXERCISE 1

Write regular expressions for the following languages:

a) The set of all strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's;

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times;

c) The set of strings that either begin or end (or both) with 01;

d) The set of strings over \{x, y, z\} such that the number of y's is divisible by three;

e) The set of strings over \{0, 1\} such that at least one of the last ten positions is a 1;

f) The set of strings over \{0, 1, ..., 9\} such that the final digit has appeared before;

g) The set of strings over \{0, 1, ..., 9\} such that the final digit has not appeared before.

EXERCISE 2

Give English descriptions of the languages over the alphabet \{a, b, c\} defined by the following regular expressions:

a) \((a+b)(a+b)(a+b)\)

b) \((\epsilon+a)b(\epsilon+c)\)

c) \((cb)^* + b(cb)^* + (cb)^*c + b(cb)^*c\)

EXERCISE 3

a) Show that for every regular language \(L\) we have \((L^*)^* = L^*\).

b) Show that for all regular languages \(L\) and \(M\) we have \((L^*M^*)^* = (L \cup M)^*\). \[\text{Note: } (L \cup M)^* = L((L \cup M)^*)\]
1) a) \( a^* b^* c^* \)
1) b) \((01)(01)^* + (010)(010)^* = (01)^+ + (010)^+ \)
1) c) \((01)(0+1)^* + (0+1)^*(01)\)
   
   Note: we assume that the strings are over \{0,1\}.

1) d) \((x+1)^* y (x+1)^* y (x+1)^* y (x+1)^* \)^*

1) e) Let \( E_i = \frac{(0+1)^*... (0+1)^* 1 (0+1)^*... (0+1)^*}{i \text{ times} \ (5-i) \text{ times}} \), \( i \in \{0,1,...,9\} \).

   Then \( E = (0+1)^* (E_0 + E_1 + ... + E_9) \).

1) f) Let \( E_d = 0+1+...+9 \). Then \( E = E_d^* O E_d^* O + E_d^* 1 E_d^* 1 + ... + E_d^* 9 E_d^* 9 \).

1) g) Let \( E_0 = 1+2+...+9 \), \( E_i = 0+...+(i-1)+i+1+...+9 \) \((1 \leq i \leq 8)\),
   \( E_9 = 0+1+...+8 \), and \( E_d = 0+1+...+9 \).

   Then \( E = E_d + E_0^* O + E_1^* 1 + ... + E_9^* 9 \).
   
   (Also: \( E = E_0^* O + E_1^* 1 + ... + E_9^* 9 \).)

2) a) The set of all strings of length three that do not contain the symbol \( c \): \{aaa, aab, aba, abb, baa, bab, bba, bbb\}.

2) b) The set of all strings with exactly one \( b \), eventually preceded by an \( a \) and/or followed by a \( c \): \{b, ab, bc, abc\}.

2) c) The set of all strings consisting of alternating \( b \)'s and \( c \)'s. Alternative regular expressions for the language are:
   
   - \((c+c)(bc)^* (c+b)\)
   - \((bc)^* + (c b)^* + c(bc)^* + b(c b)^*\)
3) a) We have to show that $L^* = (L^*)^*$ and $(L^*)^* \subseteq L^*$.

$L^* \subseteq (L^*)^*$

Trivial since $(L^*)^* = \{ \varepsilon \} U_1L^*U_2L^*U_3 \ldots$.

$(L^*)^* \subseteq L^*$

given $w \in (L^*)^*$ we have to show that $w \in L^*$.

If $w \in (L^*)^*$ then there exists $n \in \mathbb{N}$ such that $w = w_1 \ldots w_n$ where $w_i \in L^*$ $(1 \leq i \leq n)$. Since, for all $i \in \{1, 2, \ldots, n\}$, there exists $m_i \in \mathbb{N}$ such that $w_i = w_{i_1} \ldots w_{i_m}$ where $w_{i_j} \in L$ $(1 \leq j \leq m_i)$ we have that $w = (w_{i_1} \ldots w_{i_m}) \ldots (w_{i_1} \ldots w_{i_m})$.

Thus $w \in L^*$.

3) b) We have to show that $(L^* M^*)^* \in (L^* U^*)^*$ and $(L^* U^*)^* \in (L^* M^*)^*$.

$(L^* M^*)^* \subseteq (L^* U^*)^*$

given $w \in (L^* M^*)^*$ we have to show that $w \in (L^* U^*)^*$.

If $w \in (L^* M^*)^*$ then $w = w_1 \ldots w_n$ where $w_i \in L^* M^*$.

Since, for all $i \in \{1, 2, \ldots, n\}$, $w_i = u_{i_1} \ldots u_{i_k} v_{i_1} \ldots v_{i_l}$ where $u_{i_j} \in L$ and $v_{i_j} \in M$ we have that:

$w = (u_{i_1} \ldots u_{i_k} v_{i_1} \ldots v_{i_l}) \ldots (u_{i_1} \ldots u_{i_k} v_{i_1} \ldots v_{i_l})$.

Thus $w \in (L^* U^*)^*$.

$(L^* U^*)^* \subseteq (L^* M^*)^*$

given $w \in (L^* U^*)^*$ we have to show that $w \in (L^* M^*)^*$.

If $w \in (L^* U^*)^*$ then $w = w_1 \ldots w_n$ where each $w_i$ is in either $L$ or $M$. If $w_i$ is in $L$ then $w_i$ is also in $L^*$ and, since $w$ is in $M^*$, $w_i = w_{i_1} \ldots w_{i_k}$ is in $L^* M^*$.

Similarly, if $w_i$ is in $M$ then $w_i$ is in $L^* M^*$.

Thus $w \in (L^* M^*)^*$.