EXERCISE 1
Pick out one of the DFA's from exercise E2 (16/10/2008) and two strings of length at least five over the corresponding alphabet. Show whether the strings are accepted or not by using the extended transition function.

EXERCISE 2
Give a NFA accepting the following language over the alphabet \{a,b\}: the set of strings that end with ba, bb, or baa. Then show that the string baaba is not accepted by the NFA.

EXERCISE 3
Give NFA's accepting the following languages:

a) the set of strings over \{0,1,...,9\} such that the final digit has appeared before;

b) the set of strings over \{0,1,...,9\} such that the final digit has not appeared before;

c) the set of strings over \{0,1\} such that there are two 0's separated by a number of positions that is a multiple of four.
1) We choose the DFA from exercise 2

\[ S(q_0, e) = q_0 \]
\[ S(q_0, y) = S(S(q_0, e), y) = S(q_0, y) = q_2 \]
\[ S(q_4, y) = S(S(q_4, y), y) = S(q_4, y) = q_4 \]
\[ S(q_4, x) = S(S(q_4, x), x) = S(q_4, x) = q_5 \]
\[ S(q_9, x) = S(S(q_9, x), y) = S(q_9, x) = q_4 \]
\[ S(q_9, x) = S(S(q_9, x), y) = S(q_9, x) = q_5 \]

Note that we have underlined the recursive calls of the extended transition function \( \hat{S} \) in our calculations.
2) The NFA looks as follows:

We show that $baab$ is not accepted, i.e. that $\hat{S}(q_0, baab) = \{q_0, q_1\}$ and thus $q_3 \notin \hat{S}(q_0, baab)$.

$\hat{S}(q_0, \varepsilon) = \{q_0\}$
$\hat{S}(q_0, b) = \hat{S}(q_0, b) = \{q_0, q_2\}$
$\hat{S}(q_0, ba) = \hat{S}(q_0, a) \cup \hat{S}(q_2, a) = \{q_0\} \cup \{q_1, q_3\} = \{q_0, q_2, q_3\}$
$\hat{S}(q_0, baa) = \hat{S}(q_0, a) \cup \hat{S}(q_2, a) \cup \hat{S}(q_3, a) = \{q_0\} \cup \{q_3\} \cup \emptyset = \{q_0, q_3\}$
$\hat{S}(q_0, baab) = \hat{S}(q_0, b) \cup \hat{S}(q_3, b) = \{q_0, q_1, q_3\} \cup \emptyset = \{q_0, q_1, q_3\}$

The following graph might help to get a more intuitive understanding of what is going on.
3) The NFA's look as follows:

(a)

(b)

(c)