Exercise 1

Write regular expressions for the following languages:

a) The set of strings over \{0,1\} that either begin on end (or both) with 01;

b) The set of strings over \{x,y,z\} such that the number of y's is divisible by three;

c) The set of strings over \{0,1,\ldots,9\} such that the final digit has appeared before;

d) The set of strings over \{0,1,\ldots,9\} such that the final digit has not appeared before.

Solution:

a) \((01)(0+1)^* + (0+1)^*(01)\)

b) \(((x+z)^*y(x+z)^*y(x+z)^*y(x+z)^*)^*\)

c) Let \(E = 0+1+\ldots+9\)

\[ E^0 E^0 O + E^*E^*1E^*1+\ldots+E^*9E^*9 \]

d) Let \(E_0 = 1+2+\ldots+9\)

\[ E_i = 0+\ldots+(i-1)+(i+1)+\ldots+9 \quad (1 \leq i \leq 8) \]

\(E_9 = 0+1+\ldots+8\)

\(E = 0+1+\ldots+9\)

\[ E + E_0^+O + E_1^+1 + \ldots + E_9^+9 \]

\[ \square \]
Exercise 2

Convert the following regular expression to a ε-NFA:

\((\varepsilon + 1)(01)^* (\varepsilon + 0)\)

Solution:

ε-NFA's for \(\varepsilon + 1\), \(\varepsilon + 0\), and \((01)^*\) are as follows:

a) \(\varepsilon + 1\)

\[
\begin{array}{c}
\varepsilon \\
\varepsilon \\
\varepsilon
\end{array}
\]

\[
\begin{array}{c}
\varepsilon \\
0 \\
1
\end{array}
\]

b) \((01)^*\)

Composition of the above ε-NFA's yields:
Exercise 3 (3.2.3 from textbook)

Convert the following DFA to a regular expression, using the state-elimination technique.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>*</td>
<td>p</td>
<td>p</td>
</tr>
<tr>
<td>q</td>
<td>p</td>
<td>s</td>
</tr>
<tr>
<td>r</td>
<td>r</td>
<td>q</td>
</tr>
<tr>
<td>s</td>
<td>q</td>
<td>r</td>
</tr>
</tbody>
</table>

Solution:

The DFA looks as follows

First, we eliminate the state r

and obtain
Second, we eliminate the state $q$

\[
\begin{align*}
S & \xrightarrow{0+10*1} q \\
q & \xrightarrow{} s \\
q & \xrightarrow{} p
\end{align*}
\]

and obtain

\[
(0+10*1) 1 \\
= 01 + 10*11
\]

Third, we eliminate the state $s$

\[
\begin{align*}
S & \xrightarrow{0+10*11} p \\
S & \xrightarrow{00+10*10} p
\end{align*}
\]

and obtain

\[
1 + 0(01 + 10*11)*(00 + 10*10)
\]

The regular expression is therefore

\[
(1 + 0(01 + 10*11)*(00 + 10*10))^
\]
Exercise 4

If we eliminate first the state $r$, but then $s$ before $q$, the solution to the previous exercise is the regular expression $(1 + (00 + 01*1)(10 + 110*1)*0)^*$. Verify that the following strings accepted by the DFA of exercise 3 are in the languages of both regular expressions.

a) 0010 1100 101
b) 01000 1 110

Solution:

Let $E_1 = (1 + 0(01+10*11)*(00+10*10))^*$

$E_2 = (1 + (00 + 01*1)(10 + 110*1)*0)^*$

a) • 0010 1100 101 $\in L(E_1)$:

\[
\begin{array}{c}
(0(01)(01)(10010)) 1 \\
\left[ \text{from (00+10*10)} \right] \\
\left[ \text{from (01+10*11)*} \right]
\end{array}
\]

b) • 0(00011)(01)(110) $\in L(E_2)$:

\[
\begin{array}{c}
(00)(10)(11001)*0 1 \\
\left[ \text{from (00+110*1)*} \right] \\
\left[ \text{from (01+01*1)} \right]
\end{array}
\]

b) • 0(100011)(01)(110) $\in L(E_2)$:

\[
\begin{array}{c}
(010001)(10)(111)*0 1 \\
\left[ \text{from (00+01*1)} \right] \\
\left[ \text{from (10+110*1)*} \right] \\
\left[ \text{from (01+10*1)} \right]
\end{array}
\]

HW: Verify that $E_2$ is obtained by eliminating first $r$, then $s$, and then $q$. 