Exercise 1 (2.5.1 from textbook)

Consider the following ε-NFA.

<table>
<thead>
<tr>
<th></th>
<th>e</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>Ø</td>
<td>{p}</td>
<td>{q}</td>
<td>{r}</td>
</tr>
<tr>
<td>q</td>
<td>{p}</td>
<td>{q}</td>
<td>{r}</td>
<td>Ø</td>
</tr>
<tr>
<td>r</td>
<td>{q}</td>
<td>{r}</td>
<td>Ø</td>
<td>{p}</td>
</tr>
</tbody>
</table>

a) Compute the ε-closure of each state.

b) Give all the strings of length three or less accepted by the automaton.

c) Convert the automaton to a DFA.

Solution:

The ε-NFA looks as follows:

![Epsilon-NFA Diagram]

a) $\text{ECLOSE}(p) = \{p\} \quad \text{ECLOSE}(q) = \{p, q\} \quad \text{ECLOSE}(r) = \{p, q, r\}$

- $q \in \text{ECLOSE}(q)$; if $p \in \text{ECLOSE}(q)$ and $p \xrightarrow{\varepsilon} q$ then $r \in \text{ECLOSE}(q)$

b) All strings except for: $\varepsilon, a, b, aa, ab, ba, aaa, aab, aba, baa$
c) From e-NFA to NFA:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>{p₁}</td>
<td>{p₁,q₁}</td>
</tr>
<tr>
<td>q</td>
<td>{p₁,q₁}</td>
<td>{p₁,q₁,r₁}</td>
</tr>
<tr>
<td>r</td>
<td>{p₁,q₁,r₁}</td>
<td>{p₁,q₁,r₁}</td>
</tr>
</tbody>
</table>

\[ \hat{\delta}_e(q_1, a) = \text{ECLOSE} \left( \bigcup_{p \in \text{ECLOSE}(q_1)} \delta(p, a) \right) \]

The NFA looks as follows:

```
\begin{tikzpicture}
  \node (p) [circle, draw] {p};
  \node (q) [circle, draw, below of=p] {q};
  \node (r) [circle, draw, right of=q] {r};

  \draw [->, bend left] (p) to node [above] {a,b,c} (q);
  \draw [->, bend left] (q) to node [above] {a,b,c} (r);
  \draw [->, bend right] (p) to node [below] {b,c} (q);
  \draw [->, bend right] (q) to node [below] {b,c} (r);
  \draw [->, bend right] (r) to node [below] {c} (p);
\end{tikzpicture}
```

From NFA to DFA:

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
<td>E</td>
</tr>
<tr>
<td>C</td>
<td>E</td>
<td>H</td>
</tr>
<tr>
<td>D</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
<td>H</td>
</tr>
<tr>
<td>F</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>G</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>H</td>
<td>H</td>
<td>H</td>
</tr>
</tbody>
</table>

The DFA looks as follows:

```
\begin{tikzpicture}
  \node (b) [circle, draw] {B};
  \node (e) [circle, draw, below of=b] {E};
  \node (h) [circle, draw, right of=e] {H};

  \draw [->, bend left] (b) to node [above] {a} (e);
  \draw [->, bend left] (e) to node [above] {b,c} (h);
  \draw [->, bend right] (h) to node [below] {a,b,c} (h);
\end{tikzpicture}
```
Exercise 2 (2.5.2 from textbook)

Repeat the previous exercise for the following E-NFA.

<table>
<thead>
<tr>
<th></th>
<th>e</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>→ p</td>
<td>q, r</td>
<td>∅</td>
<td>q</td>
<td>r</td>
</tr>
<tr>
<td>q</td>
<td>∅</td>
<td>p</td>
<td>r</td>
<td>p, q</td>
</tr>
<tr>
<td>r</td>
<td>∅</td>
<td>∅</td>
<td>∅</td>
<td>∅</td>
</tr>
</tbody>
</table>

Solution:
The E-NFA looks as follows

\[
\begin{tikzpicture}[->,>=stealth,shorten >=1pt,auto,node distance=2.8cm,thick]
  \node[state] (p) {$p$};
  \node[state] (q) [right of=p] {$q$};
  \node[state] (r) [right of=q] {$r$};

  \path
  (p) edge [loop right] node {$\varepsilon$} (p)
  (p) edge [bend left] node {$a$} (q)
  (q) edge [bend left] node {$b$} (r)
  (q) edge [loop above] node {$c$} (q)
  (r) edge [loop below] node {$\varepsilon$} (r)
  (q) edge [loop below] node {$c$} (q)

\end{tikzpicture}
\]

a) \( \text{ECLOSE}(p) = \{p, q, r\} \quad \text{ECLOSE}(q) = \{q\} \quad \text{ECLOSE}(r) = \{r\} \)

b) All strings except for: bba, bbb, bbc

c) From E-NFA to NFA:

<table>
<thead>
<tr>
<th>( \hat{S}_E )</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>{p, q, r}</td>
<td>{q, r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>q</td>
<td>{p, q, r}</td>
<td>{r}</td>
<td>{p, q, r}</td>
</tr>
<tr>
<td>r</td>
<td>∅</td>
<td>∅</td>
<td>∅</td>
</tr>
</tbody>
</table>

Ex.: \( \hat{S}_E (p, a) = \text{ECLOSE} (U_{s \in \text{ECLOSE}(p)} S(s, a)) \)

\[= \text{ECLOSE} (S(p, a) U S(q, a) U S(r, a)) = \text{ECLOSE}(\emptyset U \{ p \} U \emptyset) = \{p, q, r\} \)
The NFA looks as follows:

From NFA to DFA:

<table>
<thead>
<tr>
<th>initial construction</th>
<th>a</th>
<th>b</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A = \emptyset$</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>$B = {p}$</td>
<td>H</td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td>$C = {q}$</td>
<td>H</td>
<td>D</td>
<td>H</td>
</tr>
<tr>
<td>$D = {r}$</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>$E = {p,q}$</td>
<td>H</td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td>$F = {p,r}$</td>
<td>H</td>
<td>G</td>
<td>H</td>
</tr>
<tr>
<td>$G = {q,r}$</td>
<td>H</td>
<td>D</td>
<td>H</td>
</tr>
<tr>
<td>$H = {p,q,r}$</td>
<td>H</td>
<td>G</td>
<td>H</td>
</tr>
</tbody>
</table>

The DFA looks as follows:
Design ε-NFA's for the following languages.

a) The set of strings consisting of zero or more a's followed by zero or more b's, followed by zero or more c's.

b) The set of all strings that consist of either 01 repeated one or more times or 010 repeated one or more times.

Solution:

a) 

```
\[ \begin{array}{c}
P \\
\epsilon \\
\downarrow \\
q_1 \\
\epsilon \\
\downarrow \\
r \\
\end{array} \]
```

b) 

```
\[ \begin{array}{c}
q_0 \\
\epsilon \\
\downarrow \\
q_1 \\
\epsilon \\
\downarrow \\
q_2 \\
1 \\
\downarrow \\
q_3 \\
\epsilon \\
\end{array},
\begin{array}{c}
q_0 \\
\epsilon \\
\downarrow \\
q_5 \\
\epsilon \\
\downarrow \\
q_6 \\
1 \\
\downarrow \\
q_7 \\
\epsilon \\
\end{array},
\begin{array}{c}
q_0 \\
\epsilon \\
\downarrow \\
q_4 \\
\epsilon \\
\downarrow \\
q_8 \\
\end{array} \]
```